SIGNAL RECOVERY CHEATSHEET

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Notes

Follow the lessons: not everything is written here. This is just a recap with the most important things to know according to me. Most qualitative aspects and explanations are contained in the slides/videos. This is especially true for the photodetector section

This is just a brief guide when doing written exercises so do not expect to study theory here.

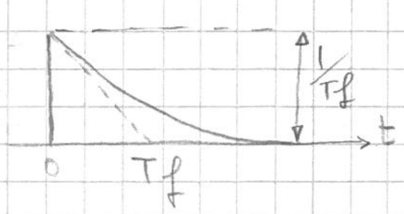
Do a lot of exercises so everything that is written here becomes natural and automatic.

There can be errors, do not take for granted every thing that is written here. Check slides/lessons/tutorials/book.

PART 1 - Filters

LPF - RC $\rightarrow$ Simplest filter

- Used in the 1st question to compute noise w/o filtering $\rightarrow$ preamplifier



$T_f = RC$ time constant $\rightarrow$ exp is zero after $\sim 5T_f$

$$h(t) = 1(t)\,\frac{1}{T_f}\cdot e^{-t/T_f} \quad \text{where} \quad 1(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Autocorrelation:

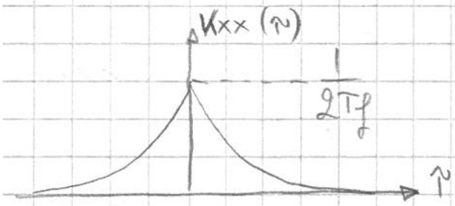
$$\underbrace{K_{xx}(0)}_{\text{time}} = \int_{-\infty}^{+\infty} x^2(\tau)\,d\tau = \frac{1}{T_f^2}\int_{0}^{+\infty} e^{-\frac{2\tau}{T_f}}\,d\tau = -\frac{1}{T_f^2}\cdot\frac{T_f}{2}\left[e^{-\frac{2\tau}{T_f}}\right]_0^{\infty} = \frac{1}{2T_f}$$

$0 \rightarrow$ because of $1(t)$

$$|W_m(f)|^2 = \frac{1}{1+(2\pi f T_f)^2}$$

Autocorr. can be computed in freq:

$$\underbrace{K_{xx}(0)}_{\text{frequency}} = \int_{-\infty}^{+\infty} |W_m(f)|^2\,df = \frac{1}{2\pi T_f}\int_{-\infty}^{+\infty}\frac{\frac{1}{2\pi T_f}}{1+2\pi f T_f}\,df = \frac{1}{2\pi T_f}\left[\operatorname{atan} 2\pi f T_f\right]_{-\infty}^{+\infty} = \frac{\pi}{2\pi T_f} = \frac{1}{2T_f}$$



$$K_{xx}(\tau) = \frac{1}{2T_f}\,e^{-|\tau|/T_f}$$

$\rightarrow$ Computation is similar to $K_{xx}(0)$

Signal filtering: heavily depends on signal shape

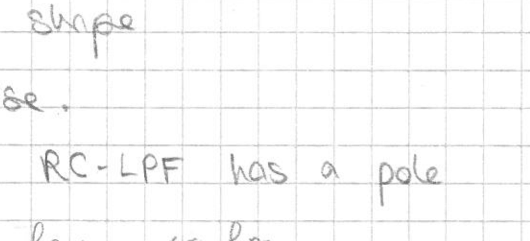
Noise filtering: suppose wide-band white noise.

e.g: preamp limits noise at 50 MHz and RC-LPF has a pole at $f_{RC} = \frac{1}{2\pi T_f} = 5\,\text{MHz}$ $\quad T_{RC} = 10\,T_{PA} \rightarrow f_{POLE_{RC}} \ll f_{POLE_{PA}}$

In this case we can neglect PA pole, so $R_{nn}(\tau) = \frac{S_{uni}}{2}\cdot\delta(\tau)$

$$\sigma_n^2 = \int_{-\infty}^{+\infty} R_{nn}(\tau)\,K_{xx}(\tau)\,d\tau = \frac{S_{uni}}{2}\,K_{xx}(0)$$

$R_{nn}(\tau) = \frac{S_{uni}}{2}\,\delta(\tau)$

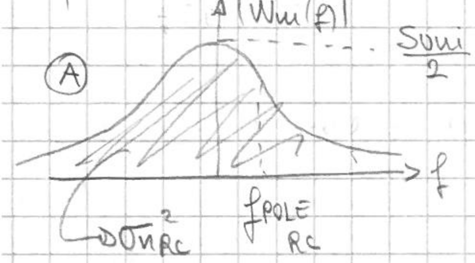


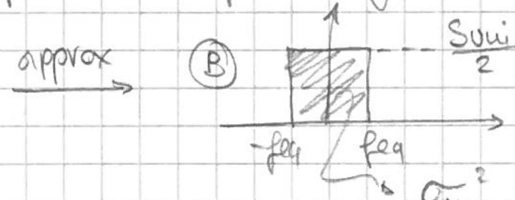
$\sigma_n^2 = \frac{S_{uni}}{2}\,K_{xx}(0) \rightarrow$ delta narrows $-\infty/+\infty$ integration to just the value in zero example: $T_{RC} = 10\, T_{PA}$ $\quad \sigma_n \Big|_{PA} = \sqrt{S_{uni} \cdot \frac{1}{4T_{PA}}}$ $\quad \sigma_n \Big|_{RC} = \sqrt{S_{uni} \cdot \frac{1}{4T_{RC}}}$

$SNR = \frac{V_p}{\sigma_n}$ ⤳ Suppose signal isn't affected by RC-LPF

$SNR\Big|_{RC} = \sqrt{10}\; SNR\Big|_{PA}$ ⤳ SNR increases by ~ 3 times

Equivalent bandwidth for RC filtering





area of rectangle

$$\sigma^2_{n_{eq}} = \int_{-\infty}^{+\infty} |W_{eq}(f)|^2 df = \frac{S_{uni}}{2} \cdot 2 f_{eq}$$

Noise integration in (A) has to be equal (equivalent) to case (B)

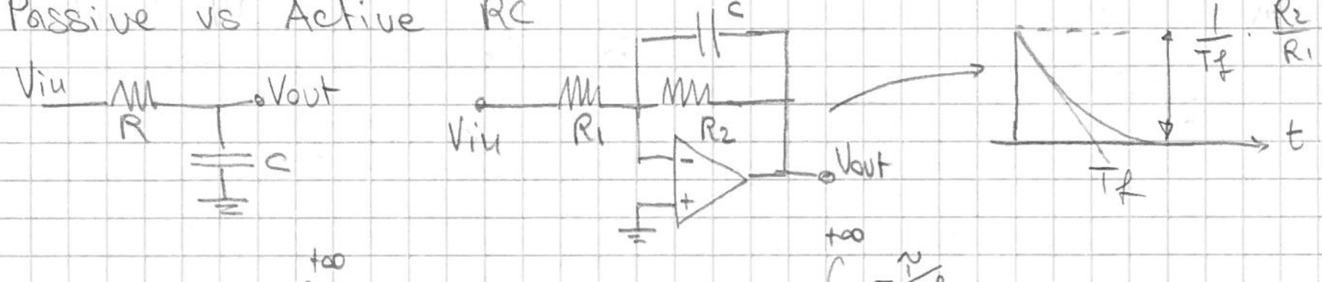
$$\sigma_{RC}^2 = \sigma_{eq} \longrightarrow \frac{\cancel{S_{uni}}}{\cancel{2}} \cdot \frac{1}{2T_f} = \frac{\cancel{S_{uni}}}{\cancel{2}} \cdot 2 \cdot f_{eq} \qquad f_{eq} = \frac{1}{4T_f}$$

Knowing that, for a signal, $f_{POLE} = \frac{1}{2\pi T_f}$ then we can do the following:

$$\frac{1}{4T_f} = f_{eq} = \frac{\pi}{2}\, f_{POLE}\Big|_{RC}$$

Therefore the equivalent noise BW is $\frac{\pi}{2}$ higher than the RC pole frequency

Passive vs Active RC



$$\text{DC Gain} = \int_{-\infty}^{+\infty} h(t)\, dt = \frac{1}{\cancel{T_f}} \cdot \frac{R_2}{R_1} \left(-\cancel{T_f}\right) \int_0^{+\infty} e^{-\frac{\tilde{\tau}}{T_f}} d\tilde{\tau} = \frac{R_2}{R_1}$$

Note: DC gain does not depend on frequency and capacitor value (This will not be true for Ratemeter integrator)

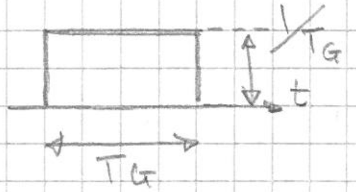
What about SNR? ⤳ input signal is not affected by RC-LPF

$$SNR = \frac{V_p \cdot \frac{R_2}{R_1}}{\sqrt{\frac{S_{uni}}{2} K_{ww}(0)}} = \frac{V_p\, R_2/R_1}{\sqrt{\frac{S_{uni}}{2} \cdot \frac{1}{2T_f} \cdot \left(\frac{R_2}{R_1}\right)^2}} = \frac{V_p\, \cancel{R_2/R_1}}{\sqrt{S_{uni} \cdot \frac{\pi}{2} f_{POLE}}\; \cancel{\frac{R_2}{R_1}}} = SNR\Big|_{PASSIVE}$$

SNR is not affected by DC gain.

When asked during the exam, answer is: Gain does not increase SNR but it helps to keep the noise of following stages low enough to be negligible (because signal will be increased thanks to the gain). Moreover, it exploits the full dynamic range of a possible ADC connected to the measurement system

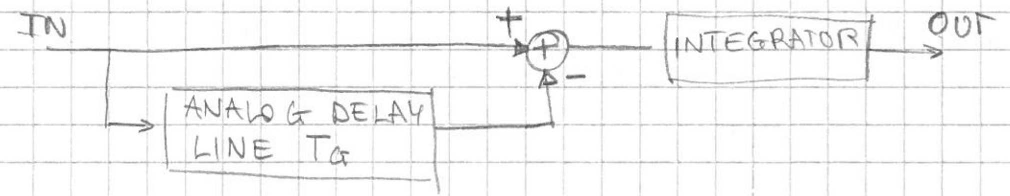
GATED INTEGRATOR AND MOBILE MEAN FILTER



MMF: makes use of analog delay lines → → be careful! T_G can't be really high

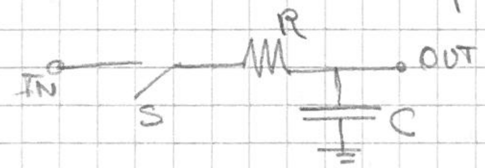
eg: $T_G = 20ns$ length of delay line $= c_0 \cdot 20ns = 6m$ (c_0: speed of light)

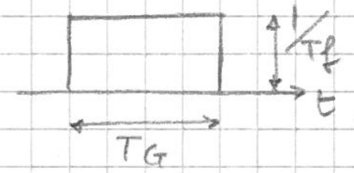
Do not go above $10/20ns = T_G$ for MMF



When T_G has to be higher than $10/20ns$ → GI filter

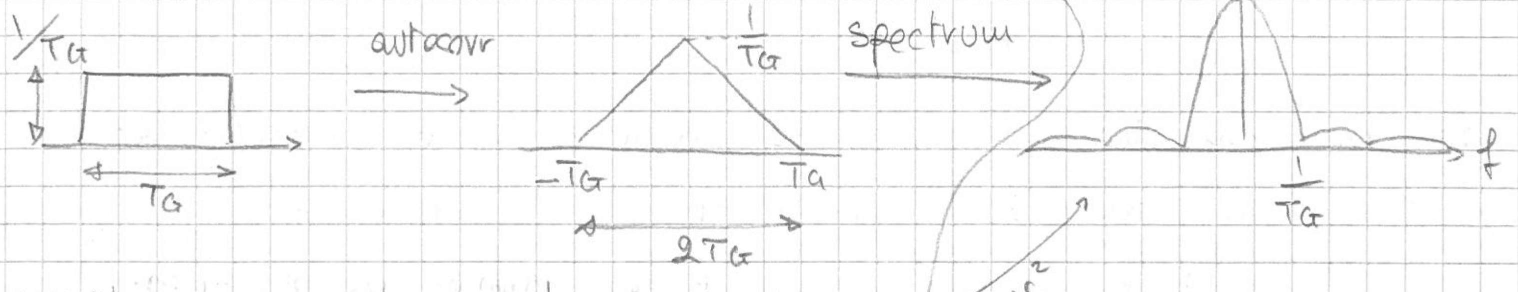
It's a switched parameter filter





Usually, to keep $G|_{DC} = 1$ $T_f = T_G$ is used

$$K_{ww}(0) = \int_{-\infty}^{+\infty} [w(t)]^2 dt = \left(\frac{1}{T_f}\right)^2 \int_0^{T_G} dt = \frac{T_G}{T_f^2} = \frac{1}{T_G}$$



$$|W(f)| = \text{sinc}(2\pi f T_G) = \frac{|\sin 2\pi f T_G|}{2\pi f T_G}$$

↳ $\mathcal{F}[\text{rect}]$

Filtering noise: $\sigma_n^2 = \frac{S_{uni}}{2} \cdot K_{ww}(0) = \frac{S_{uni}}{2} \cdot \frac{1}{T_G} \rightarrow f_{eq}|_{GI} = \frac{1}{2T_G}$

Comparison between GI and RC

Taking into account noise filtering only:

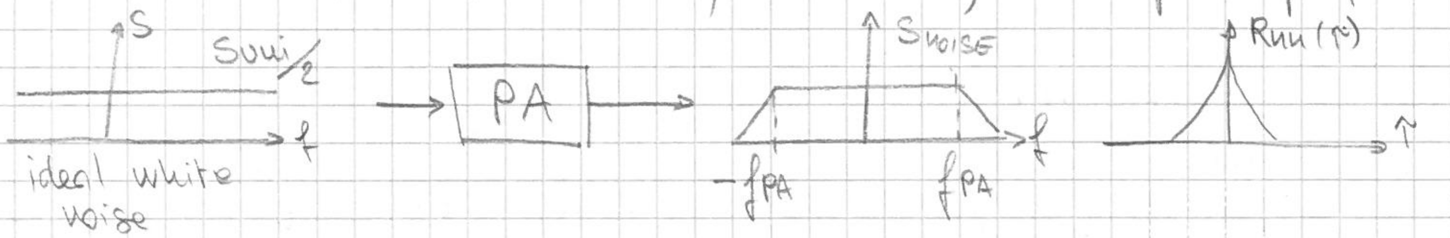
$$\left.\begin{aligned}\sigma_{RC}^2 = \frac{S_{ni}}{2} \cdot \frac{1}{2T_f} \qquad \sigma_{GI}^2 = \frac{S_{ni}}{2} \cdot \frac{1}{T_G}\end{aligned}\right] \rightarrow T_{RC}\Big|_{eq} = \frac{T_G}{2}$$

So the equivalent bandwidth of a GI seen from a RC-LPF perspective would be half of TG

Note on noise filtering

We assumed for the sake of simplicity that $R_{nn}(\tau) = \frac{S_{ni}}{2}\delta(\tau)$

In reality deltas do not exist → we have the shortest autocorrelation time that usually is set by the preamplifier



Rnn(τ) resembles a delta but it really is an exponential.

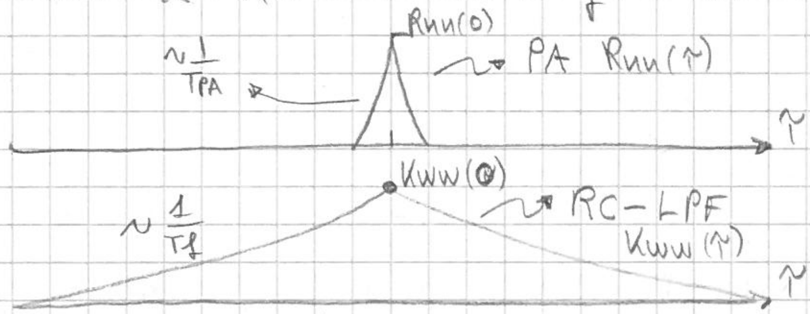
On the other hand, if we take a RC-LPF with $T_f = 10T_{PA}$

It will be:

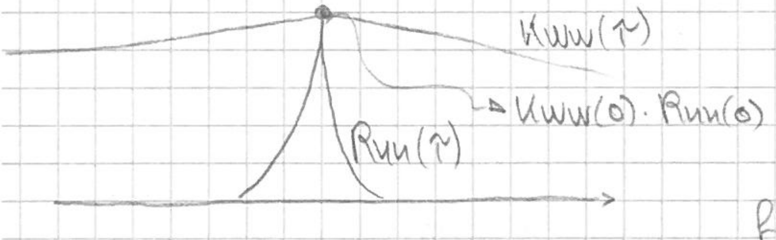
$$\sigma_n^2 = \int_{-\infty}^{+\infty} R_{nn}(\tau)\, K_{ww}(\tau)\, d\tau$$

RC-LPF

exp from PA



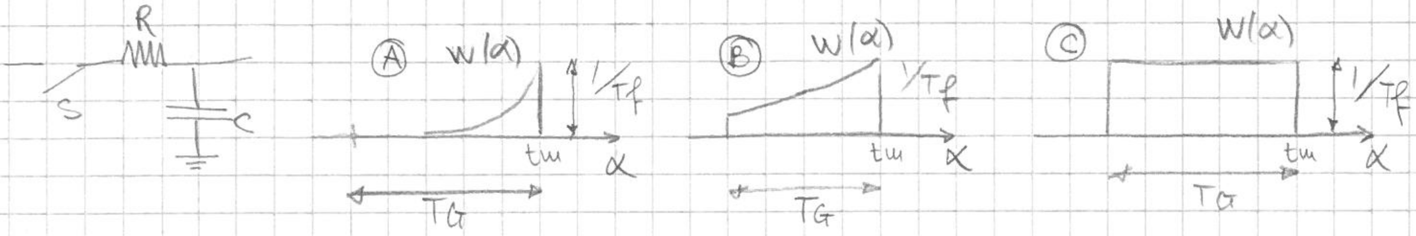
Suppose $T_f = 10\,T_{PA}$ then PA autocorrelation time $\left(\sim \frac{1}{T_{PA}}\right)$ would narrow the integral from $-\infty$ to $+\infty$ to an integral that considers the very short autocorrelation of PA with respect to the autocorrelation of RC-LPF. That said, if we zoom:



we can approximate (by accepting a small error) the Rnn(τ) to a delta → integral narrows from $\pm\infty$ to just zero evaluation:

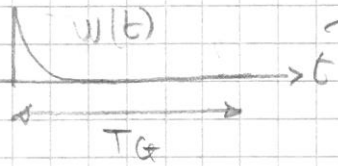
Therefore $\sigma_n^2 = \frac{S_{ni}}{2} \cdot \frac{1}{2T_f}$

OTHER SWITCHED PARAMETER FILTERS



Ⓐ Sample & Hold: used to sample signals before an ADC by

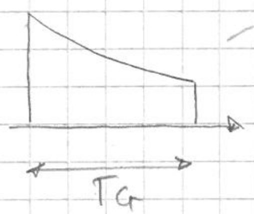
$T_G \gg T_f$ opening/closing the switch at $f_{acquisition}$.



$$\to K_{ww}(0) = \int_0^{T_G} [w(t)]^2 dt \cong \int_0^{+\infty} \left(\frac{1}{T_f} e^{-\frac{\tau}{T_f}} \right)^2 d\tau = \frac{1}{2T_f}$$

Noise filtering is the same of a RC-LPF (T_G has no effect)
Usually T_f is short so noise reduction is poor. It's never used to limit noise.

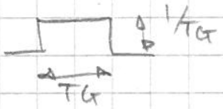
Ⓑ Switched RC: mild filtering. $T_G \sim T_f$.

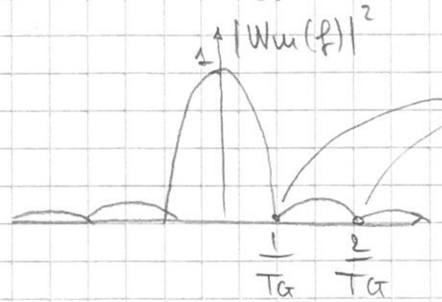


$$\to K_{ww}(0) = \int_0^{T_G} \left(\frac{1}{T_f}\right)^2 e^{-2\tau/T_f} d\tau = \frac{1}{2T_f}\left(1 - e^{-\frac{2T_G}{T_f}}\right)$$

Mild noise filtering. It is barely used because of the ease of calculations that a GI can give

Ⓒ Gated Integrator: $T_f \gg T_G$ now noise filtering can be heavy if $\frac{T_G}{T_f^2}$ is a high number.

Note on GI: consider  , its spectrum will have



→ zeros at multiples of $\frac{1}{T_G}$ → It offers specific harmonic cancellation. This is valid for MMF or GI only.

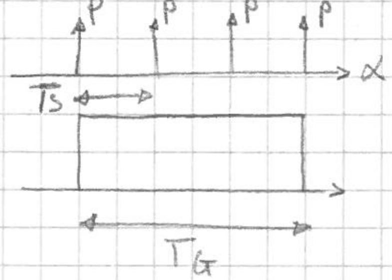
A RC-LPF can not exclude specific harmonics.

e.g: signal has a 50Hz/100Hz/200Hz/.... disturb/interference that needs to be cancelled out → $\frac{1}{T_G} = 50\,Hz \to T_G = 20\,ms$

In this way it is possible to cancel out the disturb.

Digital filters

Discrete integrator



Note: if noise was ideal white $\rightarrow$ ∞ noise

Approx: take into account that noise is wideband white noise limited by the preamp (autocorrelation time of the noise is T_n).

Consider sampling frequency $f_S = \frac{1}{T_S}$. Hypothesis: $2T_n \ll T_S$ to keep uncorrelated the samples.

Input sequences: S_x is the input signal series while n_x is the input noise one.

Equivalent analog GI $\rightarrow$ $T_G = N T_S$ DC gain $= N \cdot P$

It is called discrete time averager if $P = \frac{1}{N}$.

Sampling signal: $S_y = \sum_{k=1}^{N} P \cdot S_x = N \cdot P \cdot S_x$

Sampling noise $n_y = \sum_{k=1}^{N} P \cdot n_{xk} \rightarrow \overline{n_y^2} = P^2 \left(\overline{n_{x1}^2} + \overline{n_{x2}^2} + \overline{n_{x1} n_{x2}} + \dots \right)$

But if $2T_n \ll T_S$ (sampling freq is much lower than autocorrelation time) then all the samples are uncorrelated:

$\overline{n_{x1}} = \overline{n_{x2}} = \overline{n_x}$ because noise is stationary

$\overline{n_{x1} n_{x2}} = 0$ $\rightarrow$ uncorrelated noise samples

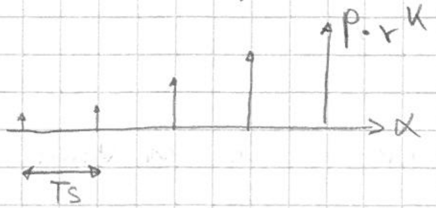
Therefore $\overline{n_y^2} = N \cdot P^2 \cdot \overline{n_x^2}$

$$SNR = \frac{S_y}{\sqrt{\overline{n_y^2}}} = \frac{N \cdot P \cdot S_x}{\sqrt{N P^2 \cdot \overline{n_x^2}}} = \frac{S_x}{n_x} \cdot \sqrt{N} = SNR\Big|_{IN} \cdot \sqrt{N}$$

$SNR\Big|_{IN}$ = ratio computed before the DI, so it will take into account the preamplifier filtering only. e.g.:

$$SNR\Big|_{IN} = \frac{V_P}{\sqrt{\frac{S_{uui}}{2} \cdot \frac{1}{2T_{PA}}}}$$ $\rightarrow$ voltage amplitude of a DC signal

Discrete exponential averager



Weight decreases with sample number

$w_K = P \cdot r^K$ where $(1-r) \ll 1$

preamp autocorrelation time

As always $2T_n \ll T_s$ $\rightsquigarrow$ uncorrelated noise samples

Signal $S_y = S_x \cdot P \cdot \sum_{K=0}^{\infty} r^K = S_x \cdot \frac{P}{1-r}$ DC gain $= \frac{P}{1-r}$

Integrated signal is the sum of the samples but averaged

Noise $\overline{n_y^2} = \overline{n_x^2}\, P^2 \sum_{K=0}^{\infty} r^{2K} = \overline{n_x^2} \frac{P^2}{1-r^2}$ $\rightsquigarrow$ Same reasoning of DI

$$SNR\Big|_{out} = SNR\Big|_{in} \frac{\frac{1}{1-r}}{\sqrt{\frac{1}{1-r^2}}} = \sqrt{\frac{1+r}{1-r}} \rightsquigarrow \begin{array}{l} 1-r \ll 1 \\ 1+r \simeq 2 \end{array}$$

$$SNR_{out} = SNR\Big|_{IN} \cdot \sqrt{\frac{2}{1-r}}$$

If we take $r = e^{-T_s/T_e}$ then $1-r \simeq \frac{T_s}{T_e}$ therefore

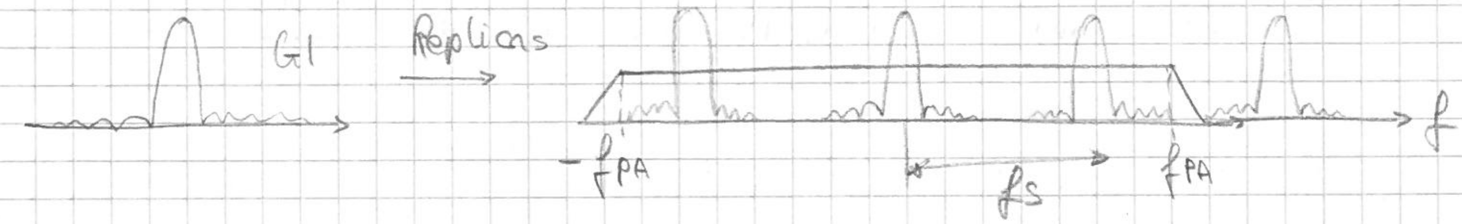
$$SNR\Big|_{out} = SNR\Big|_{IN} \cdot \sqrt{\frac{2T_s}{T_e}}$$

Removing uncorrelated noise samples hypothesis

$2T_n \ll T_s$ can also be not true $\rightarrow$ this means that, for a DI, the $\sqrt{N}$ improvement factor will change if N is noise samples are correlated ($\rightarrow T_s \sim T_n$)

Frequency demonstration

DI is a sampled GI] $\rightarrow$ Sampling in time $\Longleftrightarrow$ replica in spectrum

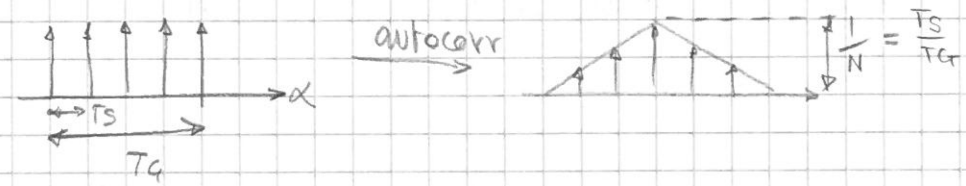


If $f_s \uparrow\uparrow$ $\rightarrow$ replicas are more spaced between each other

The more $f_s \nearrow$; the more replicas will fall out of noise band limit f_{PA}. Therefore we can conclude that for $f_s \rightarrow \infty$ only the center replica survives $\rightarrow$ it's the analog GI

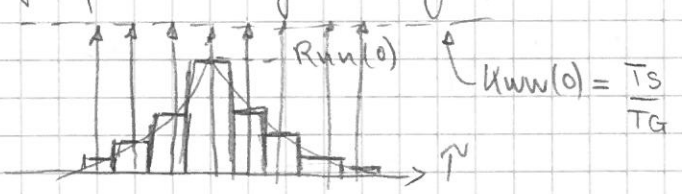
Conclusion: $SNR \sim \sqrt{N}$ for $2T_n \ll T_s$, for smaller T_s the improvement factor $\sqrt{N}$ starts to change, with an upper boundary set at its analog counterpart $\longrightarrow$ GI is the limit.

Time demonstration



Autocorr of DI is the sampled autocorr of GI

If f_s is high enough, more deltas will cut $R_{nn}(\tau)$:

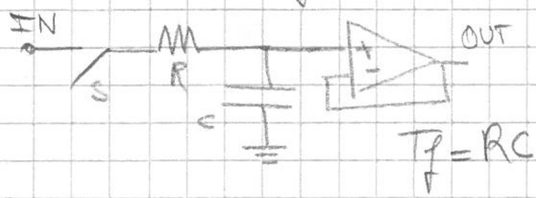


$\sigma_n^2 = \frac{1}{T_G}$ (area of the scaloid) $\rightarrow$ like in the analog domain

It is possible to demonstrate that area of the scaloid is always higher than the area of $R_{nn}(\tau)$ $\rightarrow$ noise will be higher than GI for $f_s \rightarrow \infty$ Exactly the analog GI is obtained

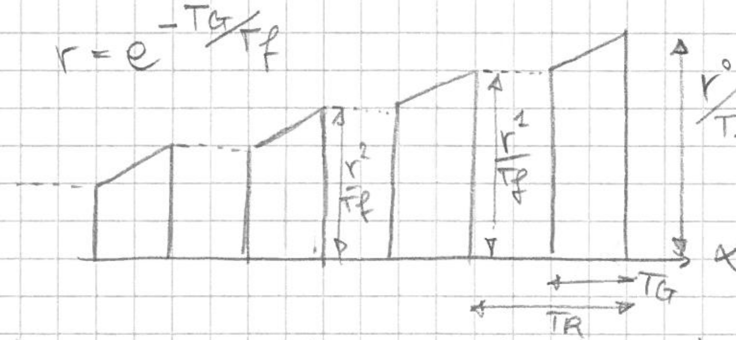
Exploit repetition rates

Boxcar integrator

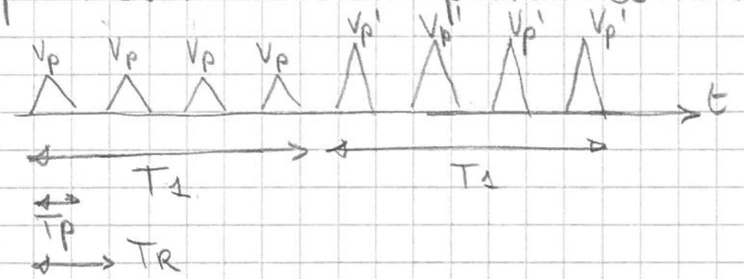


$T_f = RC$

$r = e^{-T_G/T_f}$



Boxcar is: GI + exponential averager. A sync signal just integrates the time replicas of a pulse (e.g: train of pulses coming from a modulated laser). The replicas need to have a fairly stable peak over a time period so that repetition can be exploited. E.g:



Input signal (duration T_p) is repeated each T_R and peak does not change for a time T_1.

Exponential transient has to be lower than T_1, so

$$\underbrace{5T_f\Big|_{BI}}_{\substack{\text{transient} \\ (5\tau)}} = \underbrace{\frac{T_1}{T_R}}_{\text{Number of replicas inside the wanted time period}} \cdot T_p \rightsquigarrow \text{duration of integration for each replica}$$

Note: T_R could be a statistical variable → it can change over time. Since BI maintains its charge when switch is open, T_R can change (for example jitter or different arrival time) without issues. This is because DC gain does not depend on frequency

$$\text{DC gain} = \int_{-\infty}^{+\infty} w_{BI}(t)\,dt = 1 \rightsquigarrow \text{it's the same of a RC-LPF}$$

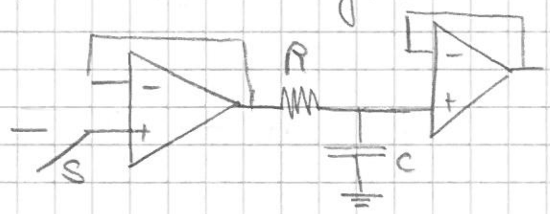
GI integration

$T_G \ll T_f \to r \simeq 1$

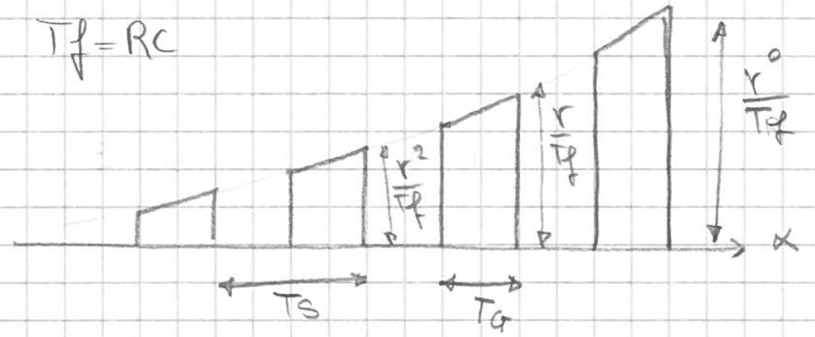
$$SNR\Big|_{OUT} = SNR\Big|_{GI} \frac{\sum_k r^k}{\sqrt{\sum r^{2k}}} = SNR\Big|_{GI} \frac{\frac{1}{1-r}}{\sqrt{\frac{1}{1-r^2}}} = SNR\Big|_{GI}\sqrt{\frac{1+r}{1-r}} \simeq$$

$$\simeq SNR\Big|_{GI}\sqrt{\frac{2T_f}{T_G}}$$

Ratemeter Integrator



$T_f = RC$



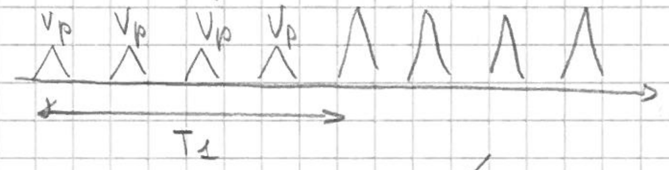
$$r = e^{-T_S/T_f}$$

↳ Note: r depends on repetition T_S !!!

There is no hold state → capacitor (unlike BI) can now discharge.

We still have GI + exponential averaging.

Considering the same situation as before;



$5T_{RI} = 5T_f = T_1$

↑ RI

$$SNR\Big|_{OUT} = SNR\Big|_{GI} \frac{\sum \dots}{\sum \dots} = SNR\Big|_{GI} \cdot \sqrt{\frac{2T_f}{T_S}} = SNR\Big|_{GI} \sqrt{2T_f \cdot f_S}$$

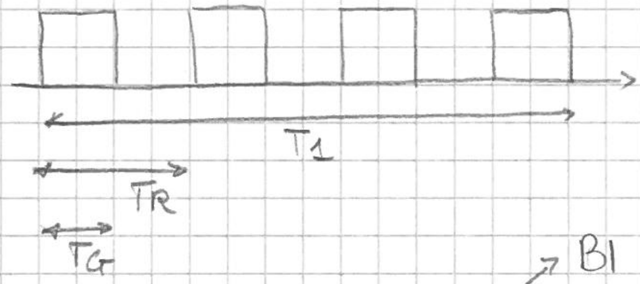
Where f_S = repetition frequency

This means that now SNR depends on frequency → if T_R is affected by jitter SNR will change as well.

This can be a drawback compared with the BI (therefore RI can not be used in these kind of situations).

Ratemeter becomes useful when a DC signal is fed and switch is closed with f_S: a frequency-to-voltage is obtained.

Note: if T_R is fixed and T_G is the same, using a BI and/OR a RI will lead to the same improvement factor. Proof:



$$T_{f_{BI}} = \frac{T_1 T_G}{5T_R}$$

$$T_{f_{RI}} = \frac{T_1}{5}$$

→ They are equivalent unless T_R has a σ in time

Improvement factor

$$BI = \sqrt{\frac{2T_f}{T_G}} = \sqrt{2\frac{T_1 \cancel{T_G}}{5T_R} \cdot \frac{1}{\cancel{T_G}}} = \sqrt{\frac{2T_1}{5T_R}}$$

$$RI = \sqrt{\frac{2T_f}{T_R}} = \sqrt{2\frac{T_1}{5} \cdot \frac{1}{T_R}} = \sqrt{\frac{2T_1}{5T_R}}$$

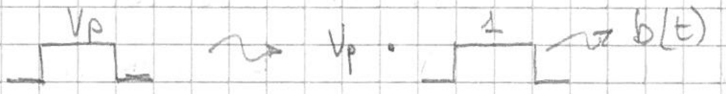
Ratemeter DC gain $= |W_{R1}(0)| = \int_{-\infty}^{+\infty} w_{R1}(t)\,dt \approx \frac{T_G}{T_R} = T_G \cdot f_P$

As we can see, DC gain (unlike BI) now depends on repetition rate.

OPTIMUM FILTERING

Note: OF theory works for white noise only → if noise spectrum is not white (e.g: SI going into an integrator), a whitening filter will be needed.

Example signal :



Separate peak from signal shape $b(t)$

$$\text{Signal} = V_p \int_{-\infty}^{+\infty} b(\tau)\, w(\tau)\, d\tau = K_{bw}(0) \cdot V_p$$

$$R_{nn} = \frac{S_{uni}}{2}\,\delta(\tau)$$

$$\text{Noise} = \int_{-\infty}^{+\infty} R_{nn}(\tau)\, K_{ww}(\tau)\, d\tau = \frac{S_{uni}}{2} K_{ww}(0)$$

$$SNR = \frac{V_p \cdot K_{bw}(0)}{\sqrt{\frac{S_{uni}}{2} K_{ww}(0)}}$$ = maximum (because of OF theory) if

$K_{bw}(0) = K_{bb}(0)$ and $K_{ww}(0) = K_{bb}(0)$ → filter has same signal shape

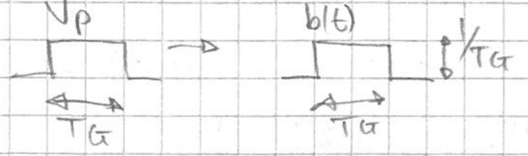
$$SNR\Big|_{OF} = \frac{V_p\, K_{bb}(0)}{\sqrt{\frac{S_{uni}}{2} K_{bb}(0)}} = \frac{V_p}{\sqrt{\frac{S_{uni}}{2}}}\sqrt{K_{bb}(0)}$$

Example:

$K_{bb}(0) = T_G$

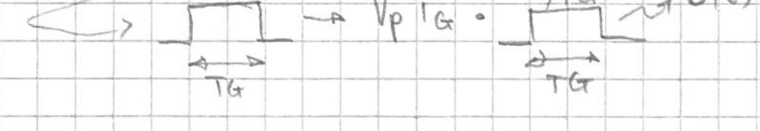
$$SNR\Big|_{OF} = \frac{V_p}{\sqrt{\frac{S_{uni}}{2}}}\sqrt{T_G}$$

It also works for whatever $b(t)$ amplitude

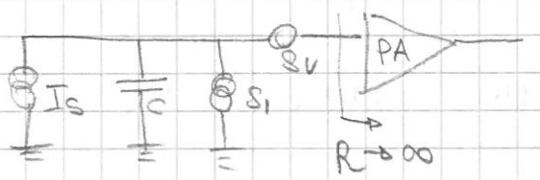


$K_{bb}(0) = \frac{1}{T_G}$

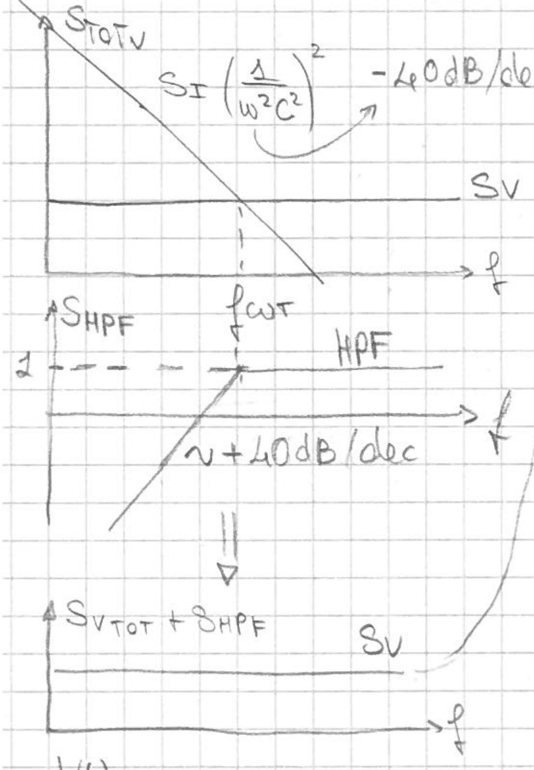
$$SNR\Big|_{OF} = \frac{V_p \cdot T_G}{\sqrt{\frac{S_{uni}}{2}}}\sqrt{\frac{1}{T_G}} = \frac{V_p \sqrt{T_G}}{\sqrt{\frac{S_{uni}}{2}}}$$



Whitening filter - example

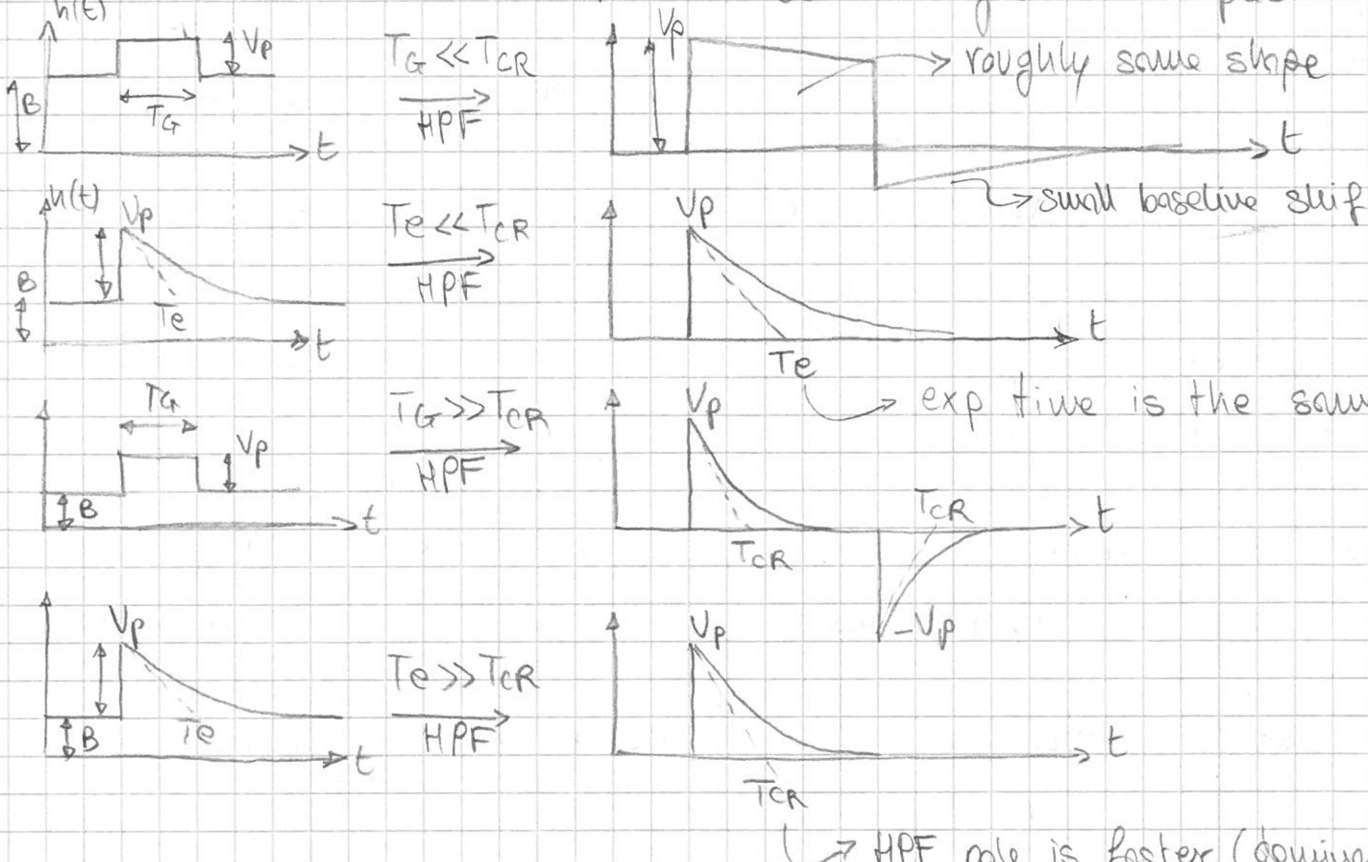


Sv directly transfers through the out.
SI sees the capacitor as load (no resistance), so spectrum will be



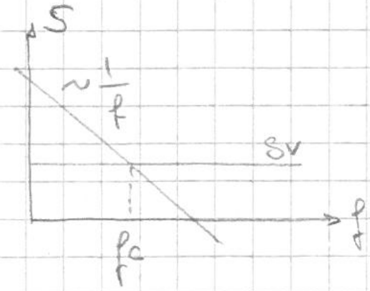
SI gets integrated through $\left(\frac{1}{sC}\right)^2$
Spectrum is no more white.
There's the need to place a HPF
With $f_{CUT} = f_{POLE}\big|_{HPF}$ so that spectrum returns white.

Note: HPF filtering means that signal can be affected by the low frequency cut. If Is has a baseline, that will be removed. If Is is slow enough, it will be changed. Examples:



HPF pole is faster (dominant)
exp. time is therefore changed

Flicker noise



It's defined through the slope $\left(\frac{1}{f^\alpha}\right)$ and the corner frequency with white noise.
If there is no upper/lower bound, integrated noise will be ∞. Consider:

- RC-LPF higher limit sets $f_{HI} = \frac{1}{2\pi T_{fRC}}$
- CR-HPF lower limit sets $f_{LO} = \frac{1}{2\pi T_{fCR}}$

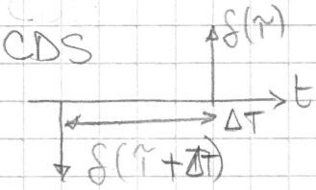
If CR-HPF freq is low enough, then $\sigma_w = \sqrt{S_{wni} \frac{\pi}{2} f_{HI}}$

While integrated $1/f$ noise will be $\sigma_{1/f} = \sqrt{S_{wni} f_c \cdot \ln\left(\frac{f_{HI}}{f_{LO}}\right)}$

Total noise will be $\sigma_{TOT} = \sqrt{\sigma_w^2 + \sigma_{1/f}^2}$

Goal: reduce $1/f$ noise by setting f_{HI}/f_{LO} in order to have negligible flicker noise when summed quadratically with σ_w

CDS



We could do a zero setting by double sampling:
① - uncorrelated fast samples will be sampled two times (delta sign does not matter in terms of noise)
② - correlated (very slow) samples will be "added and then subtracted" by the two deltas $\longrightarrow$ High pass action of $1/f$ and low frequency signals.

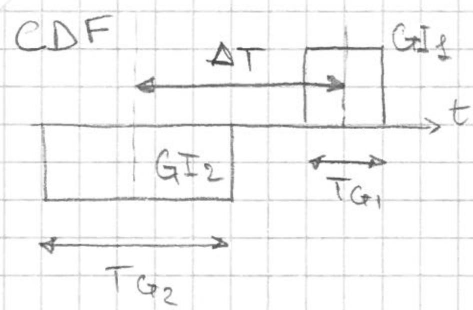
By doing computations, HPF cut is set to $f_{LO} = \frac{1}{2\pi \Delta T}$

While no high limit is imposed because of deltas $\rightarrow$ only the preamp limits noise on the higher side.
Because of double acquisition of all frequencies (δ) at ①, noise in CDS is doubled:

$\sigma_w \overset{\sim}{=} \sqrt{2 S_{wni} \cdot \frac{\pi}{2} f_{PA}}$ ~ f_{LO} should be considered as low limit but it is usually negligible
$\hookrightarrow$ noise doubling

$\sigma_{1/f} = \sqrt{2 \frac{S_{wni}}{2} f_c \ln\left(f_{PA}/f_{LO}\right)}$

CDF



To solve noise doubling caused by deltas, it is possible to use a double GI system. ΔT is defined as the difference between middle positions of each GI. (Be careful when selecting GI, see the example later).

Noise will still be integrated two times, but now there's freedom to reduce noise introduced by zero setting.

GI_1: filter that integrates useful signal $\rightarrow$ set this like there is no CDF action going on $\rightarrow$ specs set by SNR and signal

GI_2: filter used for HP action on $1/f$ and low freq signals:

Characteristic frequencies:

$$f_{GI_1} = \frac{1}{2T_{G_1}} \qquad f_{GI_2} = \frac{1}{2T_{G_2}} \qquad f_{Lo} = \frac{1}{2\pi \Delta T}$$

$$\sigma_W = \sqrt{S_{uni}\left[\left(f_{GI_1} - \cancel{f_{Lo}}\right) + \cancel{\left(f_{GI_2} - f_{Lo}\right)}\right]} \approx \sqrt{S_{uni}\, f_{GI_1}}$$

$f_{Lo} \ll f_{GI_1}, f_{GI_2}$; $f_{GI_1} \gg f_{GI_2}$

Always check for simpler calculations

$$\sigma_{1/f} = \sqrt{\frac{S_{uni}}{2} f_c \left[\ln\left(\frac{f_{GI_1}}{f_{Lo}}\right) + \ln\left(\frac{f_{GI_2}}{f_{Lo}}\right)\right]} \approx \sqrt{\frac{S_{uni}}{2} f_c \ln\left(\frac{f_{GI_1}}{f_{Lo}}\right)}$$

Note: ideally $T_{G2} \rightarrow \infty$ to have negligible noise. On the other hand if $T_{G2} \rightarrow \infty$, $\left.\Delta T \propto \frac{T_{G1}}{2} + \frac{T_{G2}}{2} \rightarrow \infty \right] \Rightarrow f_{Lo} \ll 1$

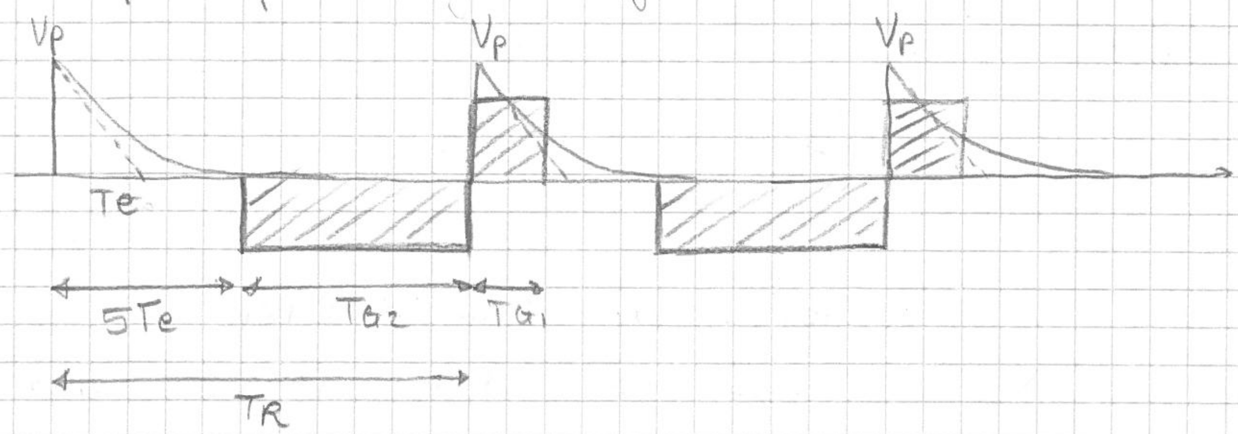
If f_{Lo} is too low $\rightarrow$ high pass action on $1/f$ would be too scarse. This would lead to good σ_W but $\sigma_{1/f}$ would increase too much. Remember that:

$f_{HI} = 100MHz \quad f_{Lo} = 1Hz \quad \ln\left(\frac{100MHz}{1Hz}\right) = 18,4$

- If f_{HI} halves: $100MHz \rightarrow 50MHz$ ($50MHz$ loss) $\ln\left(\frac{50M}{1}\right) = 17,7$
- If f_{Lo} doubles: $1Hz \rightarrow 2Hz \rightarrow 1Hz$ more leads to same result

Conclusion: controlling lower freq limit is way more important than lowering higher limit $\rightarrow$ ratio is the thing that matters

Example of a CDF setting



TG1 : set by single pulse integration to max SNR out.

for example, for an exp signal $T_{G1_1} = 1{,}25\,T_e$

TR : repetition period of a periodic signal

Te : exponential time constant

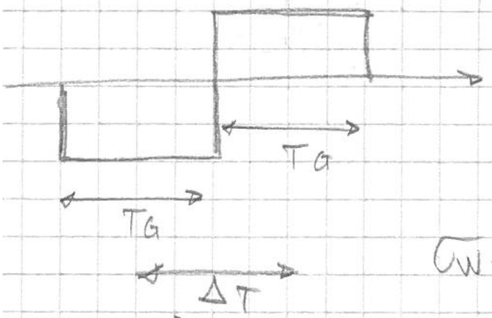
5Te : exponential transient has passed

To have good zero setting signal has to be constant, so

$T_{G2} = T_R - 5\,T_e$

TG1 is already set by other specs to fulfill SNR requirements

Particular case: symmetric CDF



$$\Delta T = \frac{T_G}{2} + \frac{T_G}{2} = T_G \qquad f_{HI} = \frac{1}{2T_G} \qquad f_{LO} = \frac{1}{2\pi\,\Delta T}$$

Symmetric GI ⟶ Noise is doubled

$$\sigma_W = \sqrt{S_{wi}\left[(f_{HI} - f_{LO}) + (f_{HI} - f_{LO})\right]} = \sqrt{2\,S_{wi}\,(f_{HI} - f_{LO})}$$

We approx $\frac{1}{f}$ noise integration to just the triangle.

Approx $1/f$ noise will be:

$$\sigma_{\frac{1}{f}} \cong \sqrt{2{,}11\,S_{wi}\,f_c\,\ln\left(\frac{\frac{1}{2\pi\Delta T}}{\frac{1}{2T_G}}\right)} = \sqrt{---\,\ln\left(\frac{\frac{1}{\cancel{2}\pi \cancel{T_G}}}{\frac{1}{\cancel{2T_G}}}\right)} = \sqrt{2{,}11\,S_{wi}\,f_c\,\ln(\pi)}$$

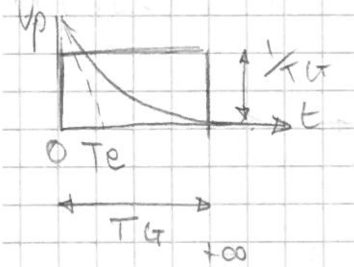
This symm. CDF is useful when there is not much room left for GI2

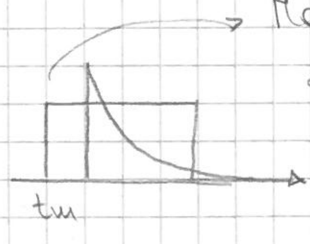
BLR - Base Line Restorer

It is very specific, it will not be discussed here (check theory as always ☺)

FILTERING SIGNALS

- Exp × GI filter





→ Moving tm before does not integrate signal → It does not make sense since it just integrates noise

$$sig = \int_{-\infty}^{+\infty} s(\tau)\, w(\tau)\, d\tau = \int_0^{T_G} \frac{V_p}{T_G} \cdot e^{-\tau/T_e} d\tau = -\frac{V_p T_e}{T_G}\left[e^{-\tau/T_e}\right] = \frac{T_e}{T_G}\left(1 - e^{-T_G/T_e}\right) \cdot V_p$$

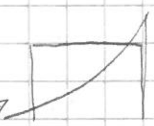
$$noise = \int_{-\infty}^{+\infty} R_{nn}(\tau) K_{ww}(\tau)\, d\tau = \frac{S_{uni}}{2} \cdot \frac{1}{T_G}$$

→ SNR max for $x = 1{,}25$

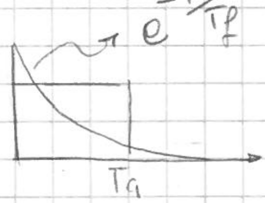
$$SNR = \frac{V_p \frac{T_e}{T_G} \cdot \left(1 - e^{-T_G/T_e}\right)}{\sqrt{\frac{S_{uni}}{2} \frac{1}{T_G}}} \frac{\sqrt{T_e}}{\sqrt{T_e}} = \frac{V_p}{\sqrt{\frac{S_{uni}}{2}}} \cdot \sqrt{T_e} \cdot \frac{1 - e^{-x}}{\sqrt{x}} \qquad T_G = 1{,}25\, T_e$$

$x = \frac{T_G}{T_e}$

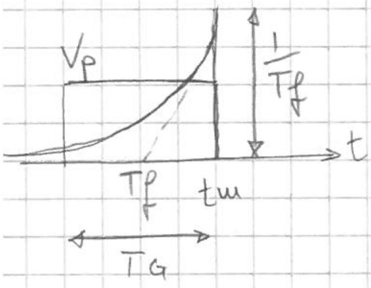
$$SNR\Big|_{T_G = 1{,}25 T_e} \cong \frac{V_p}{\sqrt{\frac{S_{uni}}{2}}} \cdot \sqrt{T_e} \cdot 0{,}638$$



integral is the same as →



- Rect pulse × RC-LPF



$$sig = \int_0^{T_G} \frac{V_p \cdot e^{-\tau/T_f}}{T_f} d\tau = V_p \left(1 - e^{-T_G/T_f}\right)$$

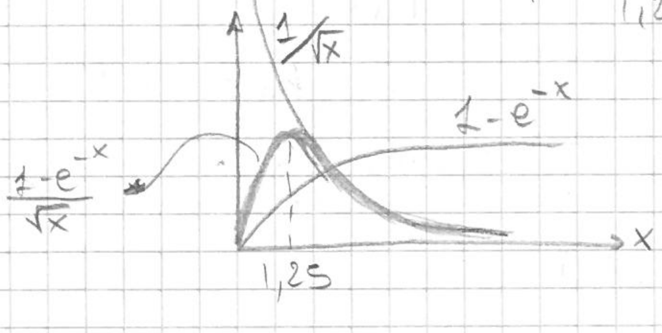
$$noise = \sigma_n^2 = \frac{S_{uni}}{2} \cdot \frac{1}{2T_f}$$

$$SNR = \frac{V_p}{\sqrt{\frac{S_{uni}}{4}}} \frac{1 - e^{-T_G/T_f}}{\sqrt{\frac{1}{T_f}}} \frac{\sqrt{T_G}}{\sqrt{T_G}} = \frac{V_p}{\sqrt{\frac{S_{uni}}{4}}} \sqrt{T_G} \frac{1 - e^{-x}}{\sqrt{x}}$$

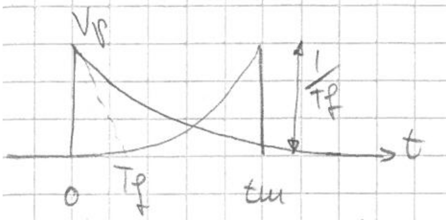
$x = 1{,}25$ so

$$T_f = \frac{T_G}{1{,}25} = 0{,}8\, T_G$$

$$SNR\Big|_{max} \cong \frac{V_p}{\sqrt{\frac{S_{uni}}{4}}} \sqrt{T_G} \cdot 0{,}638$$



- Exp pulse $\times$ RC-LPF



For semplicity, both exp time constant are the same:

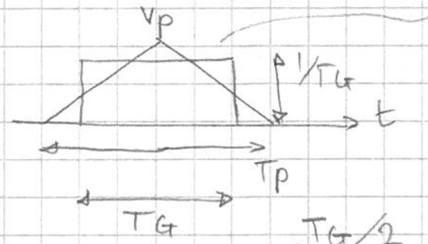
$$\text{Signal} = \int_0^{t_m} V_p e^{-\tau/T_f} \frac{1}{T_f} e^{+\frac{\tau - t_m}{T_f}} d\tau = \frac{V_p}{T_f} \cdot t_m e^{-t_m/T_f}$$

$$\text{Noise} = \sigma_n^2 = \frac{S_{vni}}{2} \cdot \frac{1}{2T_f} = \sqrt{S_{vni} \frac{\pi}{2} f_{pole}}$$

$$SNR = \frac{(V_p/T_f)}{\sqrt{S_{vni}\frac{\pi}{2} f_{pole}}} \cdot \underbrace{t_m e^{-t_m/T_f}}$$

SNR is max for $t_m = T_f$

- Tri pulse $\times$ GI



It makes sense to choose t_m so that GI is centered on the peak (to max SNR). Now we need to find T_G that maxes SNR.

Take half of the triangle from 0 to T_G

$$\text{Signal} = 2\int_0^{T_G/2} \frac{1}{T_G} V_p \left(1 - \frac{2\tau}{T_p}\right) d\tau = \frac{2V_p}{T_G}\left[\tau - \frac{2\tau^2}{T_p}\right]_0^{T_G/2} = V_p\left(1 - \frac{T_G}{2T_p}\right)$$

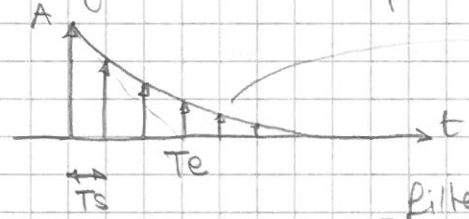
$$\text{noise} = \sigma_n^2 = \sqrt{\frac{S_{vni}}{2} \cdot \frac{1}{T_G}}$$

$$SNR = \frac{V_p\left(1 - \frac{T_G}{4T_p}\right)}{\sqrt{\frac{S_{vni}}{2} \cdot \frac{1}{T_G}}} = \frac{V_p}{\sqrt{\frac{S_{vni}}{2}}} \sqrt{T_G}\left(1 - \frac{T_G}{2T_p}\right)$$

Max for $T_G = \frac{2}{3} T_p$

$$SNR\Big|_{MAX} = \frac{V_p}{\sqrt{\frac{S_{vni}}{2}}} \cdot \sqrt{\frac{2}{3} T_p} \cdot \frac{2}{3}$$

- Digital OF of exponential signal



→ K sample has r^K weight where $r = e^{-Ts/Te}$

filter has same shape of the signal → autocorr of digital filter

$$\text{signal} = A \sum_{k}^{\infty} r^{2k} = \frac{A}{1-r^2} \qquad \text{noise} = \sigma_n^2 = \sigma_{pA}^2 \sum_{k}^{\infty} r^{2k} = \frac{\sigma_{pA}^2}{1-r^2}$$

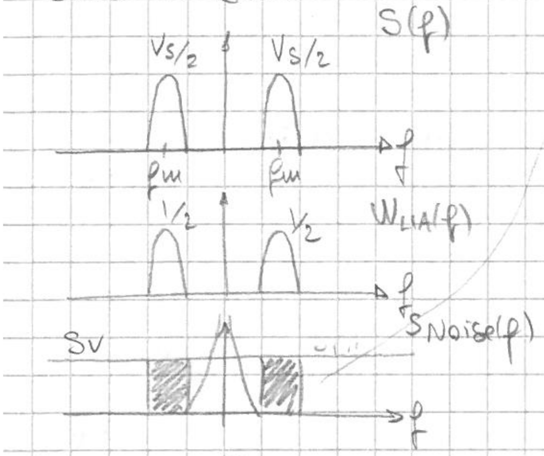
$$SNR\Big|_{out} = \frac{A}{\sigma_{pA}} \cdot \frac{\frac{1}{1-r^2}}{\sqrt{\frac{1}{1-r^2}}} = \frac{A}{\sigma_{pA}} \cdot \frac{1}{\sqrt{1-r^2}}$$

if $2Ts \ll Te$ then $1-r^2 = 1-e^{-\frac{2Ts}{Te}} \approx \frac{2Ts}{Te}$

$$SNR\Big|_{out} \approx \frac{A}{\sigma_{pA}} \sqrt{\frac{Te}{2Ts}} = SNR\Big|_{input} \sqrt{\frac{Te}{2Ts}}$$

LIA

- Sine × Sine



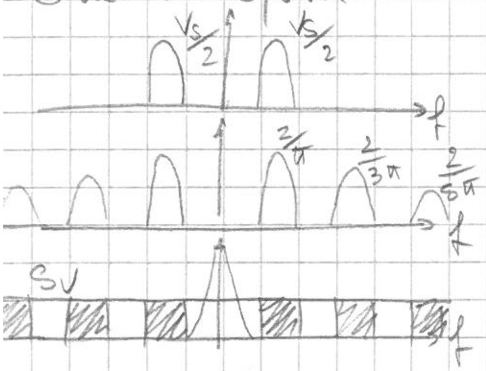
$$\text{signal} = \frac{V_s}{2} \cdot 2 = V_s \qquad \Delta f_n = 2 \cdot \frac{\pi}{2} f_{LPF}$$

$$\sigma_n = \sqrt{2 \cdot \frac{S_{TOT}}{2} \cdot \Delta f_n} = \sqrt{2}\, \sigma_{LPF}$$

$$\text{Where } \sigma_{LPF} = \sqrt{\frac{S_V}{2} \cdot 2 \cdot \frac{\pi}{2} f_{LPF}} = \sqrt{S_V \frac{\pi}{2} f_{LPF}}$$

$$\underline{SNR = \frac{V_s}{\sqrt{2}\, \sigma_{LPF}}}$$

- Sine × Square



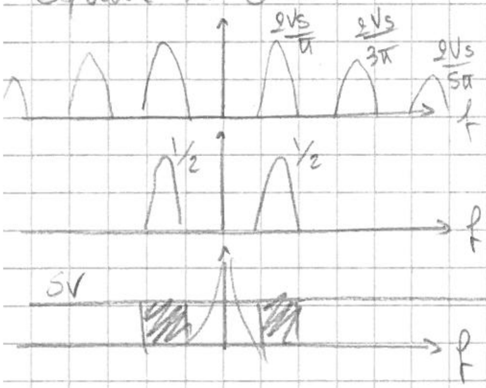
$$\text{signal} = 2 \cdot \frac{V_s}{2} \cdot \frac{2}{\pi} = \frac{2}{\pi} V_s$$

$$\sigma_n = \sqrt{2 \cdot \frac{S_V}{2} \Delta f_n \left(\sum_k^{\infty} \frac{2}{\pi} \frac{1}{2k+1} \right)^2}$$

$$= \sqrt{2 \frac{S_V}{2} \cdot 2 \cdot \frac{\pi}{2} f_{LPF} \cdot \frac{4}{\pi^2} \cdot \frac{\pi^2}{8}} = \sqrt{S_V \cdot \frac{\pi}{2} f_{LPF}}$$

$$\underline{SNR = \frac{2 V_s}{\pi\, \sigma_{LPF}}}$$

- Square × Sine

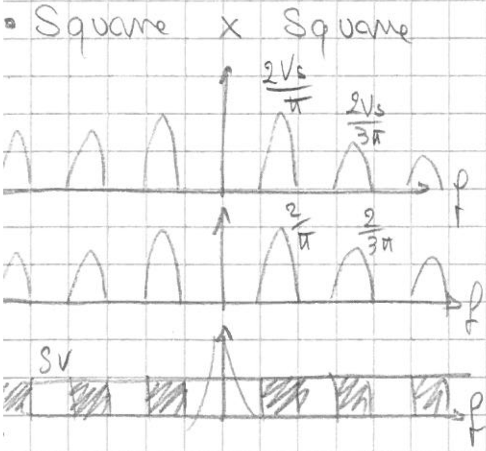


$$\text{signal} = 2 \cdot \frac{2}{\pi} V_s \cdot \frac{1}{2} = \frac{2}{\pi} V_s$$

$$\sigma_n = \sqrt{2}\, \sigma_{LPF}$$

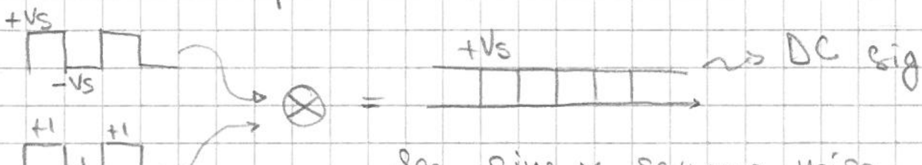
$$\underline{SNR = \frac{2}{\pi\sqrt{2}} \frac{V_s}{\sigma_{LPF}}}$$

- Square × Square



$$\text{signal} = 2 V_s \left(\sum_k^{\infty} \frac{2}{\pi} \cdot \frac{1}{2k+1} \right)^2 = \cancel{2} V_s \cdot \frac{4}{\cancel{\pi^2}} \cdot \frac{\cancel{\pi^2}}{\cancel{8}} = V_s$$

Intuitive explanation:

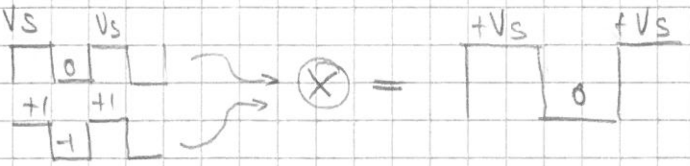


See sine × square noise

$$\sigma_n = \sigma_{LPF} \qquad \underline{SNR = V_s / \sigma_{LPF}}$$

Note:

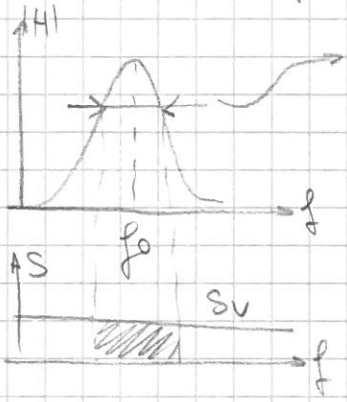
if signal



DC signal $= \frac{V_s}{2}$

BPF - RLC filter

In case we don't want a LIA implementation we could use a RLC bandpass filter



$\Delta f = \frac{f_0}{Q}$ ⤳ to integrate the lowest noise we would like a high Q

$\sigma_n = \sqrt{\frac{S_V}{2} \cdot \frac{\pi}{2} \Delta f} = \sqrt{\frac{S_V}{2} \frac{\pi}{2} \frac{f_0}{Q}}$

For $f_0 \gg 100MHz$ → $Q > 10$ (best technology $Q \sim 100$)

For $1MHz < f_0 < 100MHz$ → $Q \gtrsim 10$

For $f_0 < 1MHz$ → $Q \gtrsim 5$ max

Note: for a given Q, $\Delta f_{noise} = \frac{\pi}{2} \frac{f_0}{Q}$ is reduced as f_0 decreases

SECOND PART - Photo detectors

$P_P = n_p h\nu$ ⤳ optical power n_p = photon generation rate $\frac{\text{\# photon}}{\text{\# second}}$

$I_D = n_e \cdot q$ ⤳ photogenerated current n_e = electron gen. rate

$\eta_D = \frac{n_e}{n_p} = \frac{N_e}{N_p}$ = quantum detection efficiency N_e = # electrons generated

N_p = # photons generated

$= \frac{\text{\# photogenerated } e^- \text{ or } e^-/\text{hole pair}}{\text{\# photons arrived to PD}}$

$S_D = \frac{I_D}{P_P}$ or $\frac{V_D}{P_L}$ = $\frac{\text{output current/voltage}}{\text{input power on PD area}}$ = Radiant sensitivity $\left[\frac{A}{W}\right], \left[\frac{V}{W}\right]$

$S_D = \frac{I_D}{P_P} = \frac{n_e q}{n_p h\nu} = \frac{n_e}{n_p} \cdot \frac{\lambda}{hc/q} = \eta_D \cdot \frac{\lambda[\mu m]}{1,24}$ wavelength expressed in micrometers

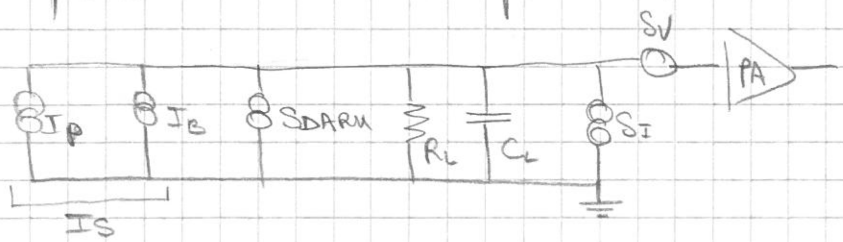
NEP = noise equivalent power (detector noise referred to input).

It takes into account just detector intrinsic noise

Note: to compute NEP SNR = 1 and noise band = 1Hz

$SNR\Big|_{NEP} = \frac{I_S}{\sqrt{2qI_B \cancel{\Delta f}_{\to 1}}} = \frac{S_D P_P}{\sqrt{2qI_B}} = 1$ ⤳ $NEP = P_P\Big|_{min} = \frac{\sqrt{2qI_B}}{S_D}$ dark current

Acquisition chain for Photo detectors



I_P = pulse current $\quad I_B$ = baseline current (ambient light, etc...)

$R_L C_L$ = sensor load where $C_L = C_{PA} + C_{sensor}$

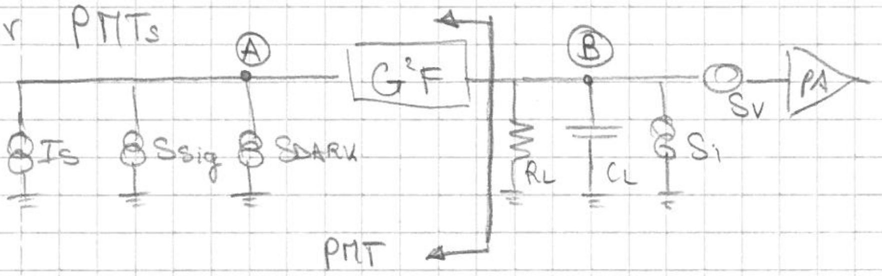
$S_{DARK} = \sqrt{2qI_D}$ = shot noise induced by dark current

S_I, S_V = preamplifier noise sources $\left(\sqrt{S_I}\big|_{voltage} = \sqrt{\frac{S_V}{R^2}}\right)$

$\sqrt{4kT/R_L}$ = resistor voltage noise

$\sqrt{2qI_S}$ = shot noise induced by signal current itself

For PMTs



PA input referred noises (current):

$\sqrt{2qI_S G^2F}$ = signal noise

$\sqrt{2qI_D G^2F} = \sqrt{2q^2 n_B G^2 F}$ where n_B = dark count rate

$\sqrt{\frac{4kT}{R}}$ = resistor noise

$\left.\begin{array}{l}\sqrt{S_i} = \\ \sqrt{S_V/R_L^2} = \end{array}\right] \rightarrow$ PA input current noises

It is possible to refer all the noises in A instead of B by correctly multiplying by G^2F

PMTs

$G \sim 10^{3} \div 10^{6}$ easily

$F \approx 2$ or less for most PMTs ($F = 1$ best tech)

η = depends a lot from PMT type

S20: range from 300 to ~800 nm
$\eta = 20\%$ at 350nm (highest), $0{,}5\%$ at 800nm

S11: range from 300 to ~600 nm
$\eta = 15\%$ at 450 nm (highest), 1% at 800nm

Silicon PDs

- Absorption lengths:

at 400nm $\rightarrow$ $\underline{La = 100nm}$
500nm $\rightarrow$ $La = 1\,\mu m$
600 nm $\rightarrow$ $La = 2\,\mu m$
700 nm $\rightarrow$ $La = 5\,\mu m$
800 nm $\rightarrow$ $\underline{La = 10\,\mu m}$

- Neutral region $\underline{W_n = 200\,nm \div 2\,\mu m}$
(100nm best tech available)

- Depleted region has to be $W_d \geq La$
$W_d \sim 5La$ is preferrable

- Reflectivity $R = 0{,}2 \div 0{,}4$
(0,1 best tech available)

$$\eta_D = \underbrace{(1-R)}_{A}\ \underbrace{e^{-\frac{W_n}{La}}}_{B}\ \underbrace{\left(1-e^{-\frac{W_d}{La}}\right)}_{C}$$

$$C\big|_{PD} = \varepsilon_0 \overset{12}{\varepsilon_{si}} \cdot \frac{A}{W_d}$$

A → area of detector
Wd → depleted layer

(A) Reflect the smallest amount of photons as possible

(B) low number of photons must be absorbed here (W_n has to be very thin)

(C) Absorb the highest amount of photons as possible (W_d has to be thick, typically $\approx 5La$)

Single Electron Response (SER): few tens of picoseconds

APDs

Same acquisition scheme for PMTs, but:

$G =$ up to 500 max $\rightarrow$ strongly varies with temperature
$G = 1000$ can not be feasible

$F = 2$ lowest possible

$F \sim 2,5$ for $G = 100$

$F \sim 5$ for $G = 500$

Low noise preamplifier

$\sqrt{S_{V,u}} = 2 \div 5 \, nV/\sqrt{Hz}$
$\sqrt{S_{I,u}} = 0,5 \, pA/\sqrt{Hz}$
$\rightarrow$ Reasonable values

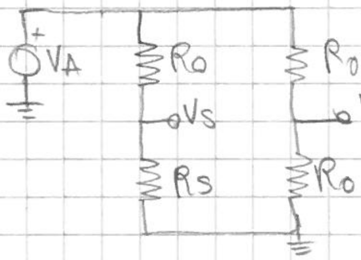
Best LNA values available $\sqrt{S_{V,u}} \sim 1 nV/\sqrt{Hz}$ $\sqrt{S_{I,u}} = 0,01 \, pA/\sqrt{Hz}$

Temperature sensors

Thermistors: non linear response with temperature $\rightarrow$ can be compensated using look-up tables in microprocessors

PT100: very linear response through a larger temperature range ($\sim 13,8K$ to $903K$)

Wheatstone bridge consider $R_S = R_0 (1 + \alpha \Delta T)$ $\rightarrow \Delta T = T - T_0$



$R_S = R_0 + \Delta R_S$

$\Delta R_S = \alpha \Delta T R_0$

$$\begin{cases} V_0 = V_A \dfrac{R_0}{2R_0} = \dfrac{V_A}{2} \\ V_S = V_A \cdot \dfrac{R_S}{R_0 + R_S} \end{cases}$$

$$\Delta V_S = V_S - V_A = V_A \left(\frac{R_0 + \Delta R_S}{2R_0 + \Delta R_S} - \frac{1}{2} \right) = \underbrace{\frac{V_A}{2} \frac{\Delta R_S}{2R_0 + \Delta R_S}}_{\text{large signal expression}}$$

For small variations $\Delta R_S \ll 2R_0$, so $\Delta V_S = \dfrac{V_A}{4} \cdot \dfrac{\Delta R_S}{R_0} = \dfrac{V_A}{4} \dfrac{\alpha \Delta T \cancel{R_0}}{\cancel{R_0}}$

$\Delta V_S = \dfrac{V_A}{4} \cdot \alpha \Delta T$ $\rightarrow$ small signal expression

Resulting noise from the wheatstone bridge will be

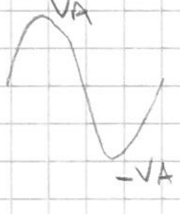
$\sqrt{S_V}$, $\sqrt{S_I R_S^2}$, $\sqrt{4KTR_S}$, $1/f$ components

Usually, a power requirement is set not to cause selfheating of the system, for example $\left(\dfrac{V_A/2}{R_0}\right)^2 \leq P_{MAX}$

We find V_A value when solved

For sinusoidal V_A, voltage bias can be $\sqrt{2}$ higher to its DC equivalent

Note: when using LIAs, sine/square V_A can change SNR by a lot $\rightarrow$ remember to always check power requirement.



$\rightarrow \dfrac{V_A}{\sqrt{2}}$ in DC equivalence

$\rightarrow V_A/2$ in DC equivalence

This can be misleading in the type of wave to use in the LIA, but the end result (considering P_{MAX} requirement) will be the same regardless of the approach used. Example:

- sine $V_A \rightarrow \dfrac{\left(\frac{V_A}{2\sqrt{2}}\right)^2}{R_0} \leq P_{MAX} \rightarrow V_A\Big|_{MAX} = \sqrt{8 R_0 P_{MAX}}$ $\quad SNR = \dfrac{\frac{\sqrt{8 R_0 P_{MAX}}}{4} \alpha \Delta T}{\sqrt{2}\ \sigma_{LPF}}$

- square $V_A \rightarrow \dfrac{\left(\frac{V_A}{2}\right)^2}{R_0} \leq P_{MAX} \rightarrow V_A\Big|_{MAX} = 2\sqrt{R_0 P_{MAX}}$ $\quad SNR = \dfrac{\frac{2\sqrt{R_0 P_{MAX}}}{4} \cdot \alpha \Delta T}{\sigma_{LPF}}$

We can see $SNR\Big|_{\substack{\text{sine} \\ \text{LIA}}} = SNR\Big|_{\substack{\text{square} \\ \text{LIA}}}$

STRAIN GAUGES

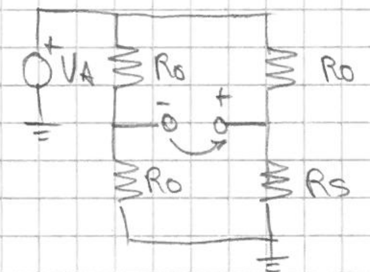
Strain $\varepsilon = \frac{N}{E}$ → Force applied (N), Young Modulus (E)

$$R = \rho \frac{L}{A} \rightarrow \frac{\Delta R}{R_0} = \frac{\Delta L}{L_0} - \frac{\Delta A}{A_0} + \frac{\Delta \rho}{\rho_0} = \varepsilon + 2\nu\varepsilon + \beta E \varepsilon = G\varepsilon$$

where G = gauge factor

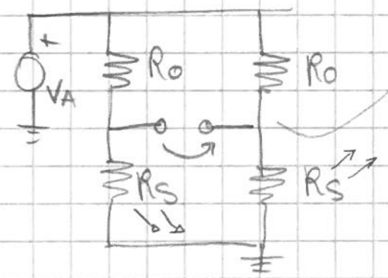
When an extension of the resistor is applied → length increases, area decreases and resistivity changes because of the piezoresistive effect $\rho = \rho_0 (1 + \beta N)$

Wheatstone bridge



→ $\Delta V_S = \frac{V_A}{4} \frac{\Delta R_S}{R_0} = \frac{V_A}{4} \cdot G\varepsilon$ exactly like the thermoresistors

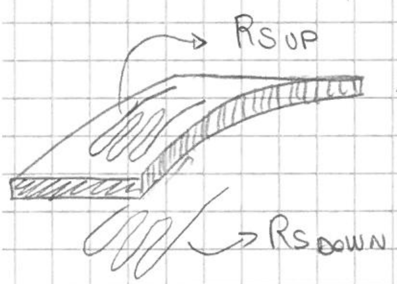
If two SG are used:

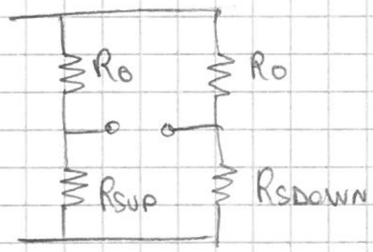


$\Delta V_S = \frac{V_A}{2} G\varepsilon$ amplitude is doubled

Note: SGs need to experience opposite stress so that $\left(\frac{V_A}{4} G\varepsilon - \left(-\frac{V_A}{4} G\varepsilon\right)\right) = \frac{V_A}{2} G\varepsilon$

How can they be positioned?





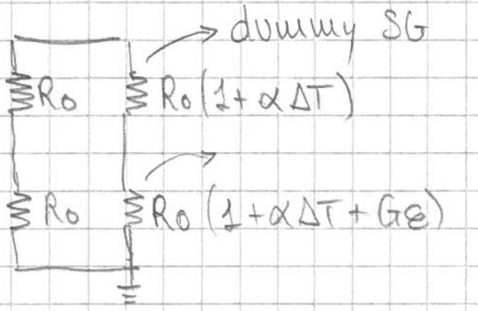
Using this configuration it is possible to measure compression and extension

Temperature dependancy

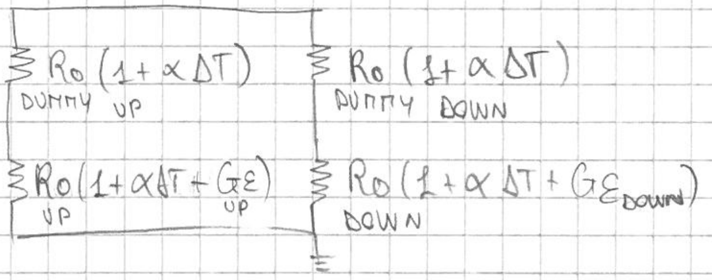
SGs are often placed in hot places or on pieces which temperature changes rapidly

$$\Delta V_s\Big|_{temperature} = \frac{V_A}{4}\alpha\,\Delta T$$ ⤳ same expression for the PT100

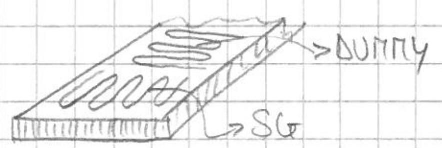
To solve this, dummy SGs are used:

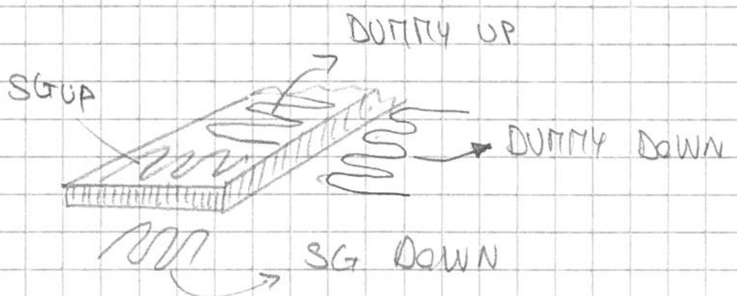


↳ Single SG



↳ Comp/extension double SG configuration





Dummy orientation is perpendicular to SG so that it does not experience the wanted stress

If $\Delta T\big|_{dummy} = \Delta T\big|_{SG} \rightarrow \Delta V_S = 0$ but $\Delta T_D \neq \Delta T_{SG}$

(They usually differ by $\pm 0{,}1\,^\circ C$ or something similar) So

$$\Delta V_s\Big|_{temperature} = \frac{V_A}{4}\alpha\,(\Delta T_{SG} - \Delta T_D) \quad \text{or} \quad \frac{V_A}{2}\alpha\,(\Delta T_{SG} - \Delta T_D)$$

example:

$V_A = 50\,mV$, $\alpha = 4\,m \cdot \frac{1}{^\circ C}$, $\Delta T = 100\,^\circ C$, $(\Delta T_{SG} - \Delta T_D) = 0{,}1\,^\circ C$

$$\Delta V_s\Big|_{w/o\ dummy} = \frac{V_A}{4}\alpha\,\Delta T = 5 \cdot 10^{-3}\ \text{strain}$$

$$\Delta V_s\Big|_{with\ dummy} = \frac{V_A}{4}\alpha\,(\Delta T_{SG} - \Delta T_D) = 5 \cdot 10^{-6}\ \text{strain}$$

→ Changes by 3 orders of magnitude