CAMPI ELETTROMAGNETICI

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Campi Scalari

Un campo scalare é una funzione che associa ad ogni punto dello spazio n-dimensionale un valore scalare (es: la temperatura in una stanza $T(x,y,z)$)

Campi Vettoriali

Un campo vettoriale associa ad ogni punto dello spazio n-dimensionale un valore vettoriale (m valori scalari) (es: il campo elettrostatico $\vec{E}(x,y,z) = E_x(x,y,z)\vec{u}_x + E_y(x,y,z)\vec{u}_z + E_z(x,y,z)\vec{u}_z$)

Operatori sui campi:

Gradiente

Divergenza

Rotore

- Il Gradiente opera su un campo scalare e restituisce un campo vettoriale

$$\vec{\nabla}\phi(x,y,z) = \frac{\partial\phi(x,y,z)}{\partial x}\vec{u}_x + \frac{\partial\phi(x,y,z)}{\partial y}\vec{u}_y + \frac{\partial\phi(x,y,z)}{\partial z}\vec{u}_z$$

Significato fisico: direzione di massima variazione

- La Divergenza opera su un campo vettoriale e restituisce un campo scalare

$$\vec{\nabla}\cdot\vec{F}(x,y,z) = \frac{\partial F_x(x,y,z)}{\partial x} + \frac{\partial F_y(x,y,z)}{\partial y} + \frac{\partial F_z(x,y,z)}{\partial z}$$

Significato fisico: sorgenti "pozzo" del campo

- Il Rotore opera su un campo vettoriale (3D) e restituisce un altro campo vettoriale

$$\vec{\nabla}\times\vec{F}(x,y,z) = \det\begin{bmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{bmatrix} =$$

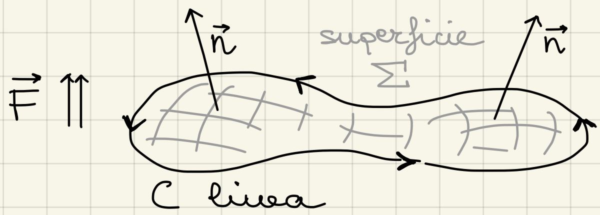
$$= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\vec{u}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\vec{u}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\vec{u}_z$$

Significato fisico: sorgenti vorticose del campo

Il campo elettrico (statico) é un campo irrotazionale, cioé ammette solo sorgenti di tipo pozzo ($\vec{\nabla} \times \vec{E} = 0$)

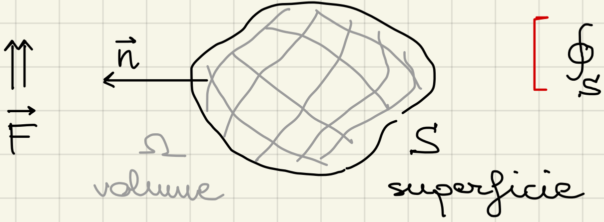
Il campo magnetico (statico) é un campo solenoidale, cioé ammette solo sorgenti di tipo vorticoso ($\vec{\nabla} \cdot \vec{H} = 0$)

Teorema di Stokes



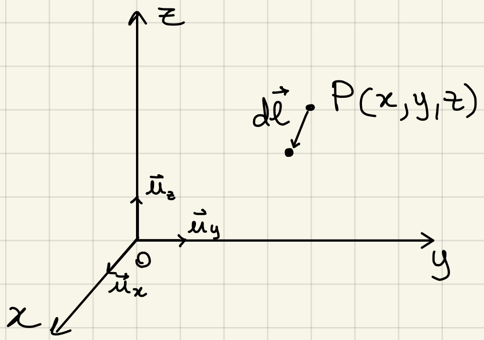
$$\left[\oint_c \vec{F} \cdot d\vec{\ell} = \int_\Sigma \vec{\nabla} \times \vec{F} \cdot d\vec{\Sigma} \right] \text{ (rotore)}$$

Teorema di Gauss



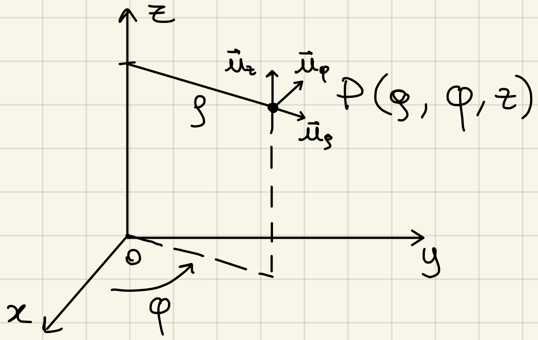
$$\left[\oint_S \vec{F} \cdot d\vec{S} = \int_\Omega \vec{\nabla} \cdot \vec{F} \, d\Omega \right] \text{ (divergenza)}$$

Coordinate cartesiane



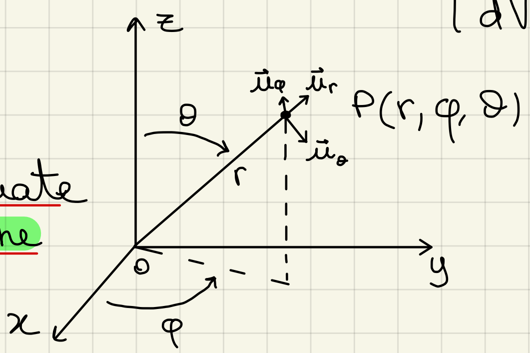
$$\begin{cases} d\vec{\ell} = dx\,\vec{u}_x + dy\,\vec{u}_y + dz\,\vec{u}_z \\ dV = dx\,dy\,dz \end{cases}$$

Coordinate cilindriche



$$\begin{cases} d\vec{\ell} = d\rho\,\vec{u}_\rho + \rho\,d\varphi\,\vec{u}_\varphi + dz\,\vec{u}_z \\ dV = \rho\,d\rho\,d\varphi\,dz \end{cases}$$

Coordinate sferiche



$$\begin{cases} d\vec{\ell} = dr\,\vec{u}_r + r\,d\theta\,\vec{u}_\theta + r\sin\theta\,d\varphi\,\vec{u}_\varphi \\ dV = r^2 \sin\theta\,d\theta\,d\varphi\,dr \end{cases}$$

$\vec{E}$	$\left[\frac{V}{m}\right]$	campo elettrico	ε	$\left[\frac{F}{m}\right]$	costante dielettrica
$\vec{H}$	$\left[\frac{A}{m}\right]$	campo magnetico	μ	$\left[\frac{H}{m}\right]$	permeabilitá magnetica
$\vec{B}$	$[T]$	densitá di flusso magnetico			
$\vec{D}$	$\left[\frac{C}{m^2}\right]$	densitá di flusso elettrico			

CAMPO ELETTROSTATICO

Carica elettrica $(q_e = 1{,}6 \cdot 10^{-19}\,C)$

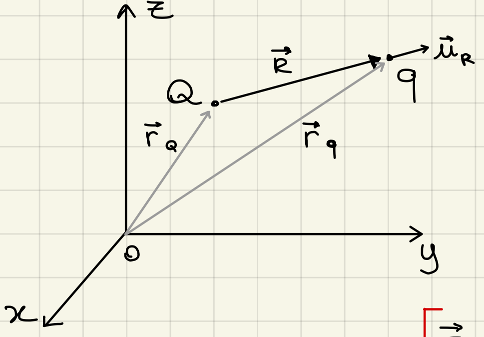
Carica puntiforme $Q\ [C]$

Densitá di carica lineare $\rho_l\ \left[\frac{C}{m}\right]$

" " " superficiale $\rho_s\ \left[\frac{C}{m^2}\right]$

" " " volumetrica $\rho_v\ \left[\frac{C}{m^3}\right]$

Legge di Coulomb (1785)



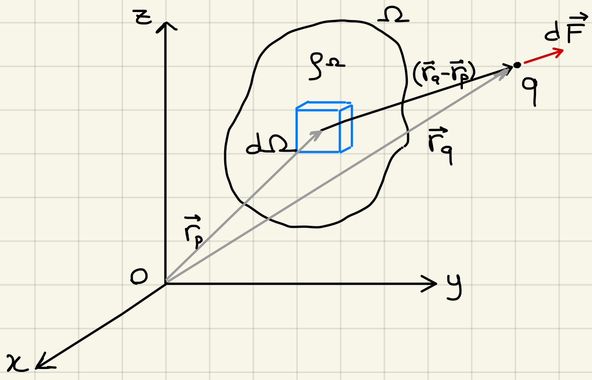
$$\vec{F} = \frac{Q \cdot q}{4\pi\varepsilon_0 R^2}\,\vec{u}_R \quad [N]$$

$$\varepsilon_0 = 8{,}854 \cdot 10^{-12}\,\frac{F}{m} \qquad \left(\mu_0 = 4\pi \cdot 10^{-7}\,\frac{H}{m}\right)$$

$$\vec{F} = \frac{Q \cdot q}{4\pi\varepsilon_0}\,\frac{(\vec{r}_q - \vec{r}_Q)}{|\vec{r}_q - \vec{r}_Q|^3}$$ forza elettrostatica

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\,\frac{(\vec{r}_q - \vec{r}_Q)}{|\vec{r}_q - \vec{r}_Q|^3} = \frac{Q}{4\pi\varepsilon_0 R^2}\,\vec{u}_R$$ campo elettrostatico

Caso di un corpo rigido (carica non puntiforme):

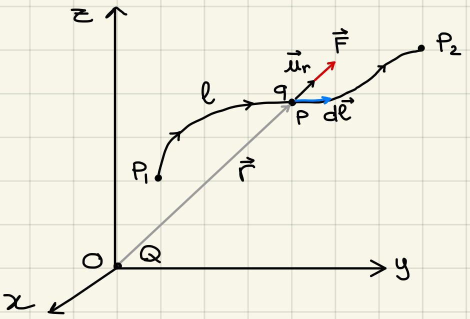


$$d\vec{F} = \frac{q}{4\pi\varepsilon_0}\,\frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|^3}\,\rho_\Omega(\vec{r}_p)\,d\Omega$$

$$\vec{F} = \int_\Omega d\vec{F} = \int_\Omega \frac{q}{4\pi\varepsilon_0}\,\frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|^3}\,\rho_\Omega(\vec{r}_p)\,d\Omega$$

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q} = \int_\Omega \frac{1}{4\pi\varepsilon_0}\,\frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|^3}\,\rho_\Omega(\vec{r}_p)\,d\Omega$$

POTENZIALE ELETTROSTATICO



$$\vec{F} = q\vec{E} = \frac{qQ}{4\pi\varepsilon_0 r^2}\vec{u}_r$$

$$W = -\int_\ell \vec{F}\cdot d\vec{\ell} = -\int_\ell q\vec{E}\cdot d\vec{\ell} =$$

$$= -\int_{P_1}^{P_2}\frac{qQ}{4\pi\varepsilon_0 r^2}\vec{u}_r\cdot d\vec{\ell} = -\int_{P_1}^{P_2}\frac{qQ}{4\pi\varepsilon_0 r^2}dr =$$

$$= \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

lavoro compiuto per spostare la carica q da P_1 a P_2 lungo ℓ attraverso il campo elettrico generato da Q

$V = \frac{W}{q}$ potenziale (lavoro per unità di carica)

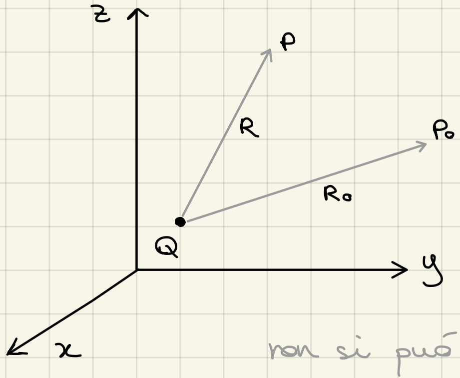
$$-\int_{P_1}^{P_2}\vec{E}\cdot d\vec{\ell} = V(P_2) - V(P_1)$$

$$V(P) = -\int_{P_0}^{P}\vec{E}\cdot d\vec{\ell} + V_0$$ → $V(P_0)$ costante additiva

$$\oint_c \vec{E}\cdot d\vec{\ell} = 0 \longrightarrow \lim_{S\to 0}\oint_c \vec{E}\cdot d\vec{\ell} = 0 \longrightarrow \boxed{\vec{\nabla}\times\vec{E} = 0}$$

↓ il campo elettrostatico è irrotazionale (non esistono sorgenti vorticose)

$$dV = -\vec{E}\cdot d\vec{\ell} \longrightarrow \boxed{\vec{E} = -\vec{\nabla} V}$$



$$V(P) = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 R_0}$$ costante additiva

$R_0 \to +\infty$, $V(P) = \frac{Q}{4\pi\varepsilon_0 R}$ $\quad \vec{E} = -\vec{\nabla} V$

non si può sempre fare!

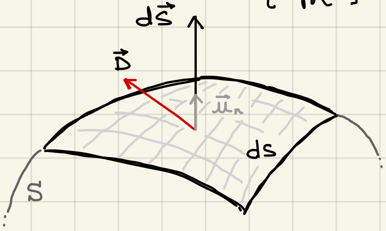
poiché il potenziale dipende solo dalla distanza radiale è conveniente usare un sistema di coordinate polari

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\vec{u}_r + \frac{1}{r}\cancel{\frac{\partial f}{\partial \theta}}\vec{u}_\theta + \frac{1}{r\sin\theta}\cancel{\frac{\partial f}{\partial \varphi}}\vec{u}_\varphi$$

$$\Longrightarrow \vec{E} = -\frac{\partial V}{\partial r}\vec{u}_r = \frac{Q}{4\pi\varepsilon_0 R^2}$$ ✓

$$\vec{D} = \varepsilon_0\vec{E} \quad \left[\frac{C}{m^2}\right]$$

$$\vec{D} = \frac{Q}{4\pi}\frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|^3} = \frac{Q}{4\pi R^2}\vec{u}_R$$



$$d\phi = \vec{D}\cdot d\vec{s} \qquad \phi = \oint_S \vec{D}\cdot d\vec{s}$$

$$\phi = \oint_S \frac{Q}{4\pi R^2}\vec{u}_r\cdot d\vec{s} = \oint_S \frac{Q}{4\pi}\frac{\vec{u}_r\cdot d\vec{s}}{R^2}$$

$$\oint_S d\theta = 4\pi$$ angolo giro solido

$$\boxed{\phi = \oint_S \frac{Q}{4\pi}d\theta = \frac{Q}{4\pi}\oint_S d\theta = Q}$$ (il flusso di $\vec{D}$ è uguale alla carica che la genera)

$$d\theta = \frac{\vec{u}_r\cdot d\vec{s}}{R^2}$$

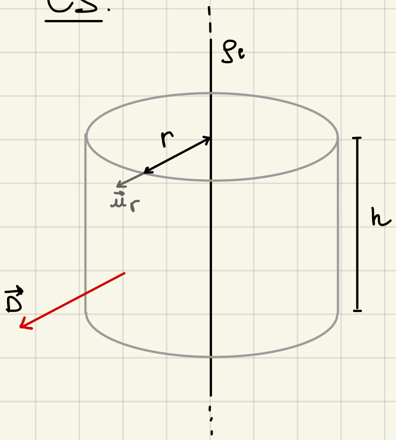
↳ $\sin\theta\, d\varphi\, d\theta$

$$\phi = \oint_S \vec{D} \cdot d\vec{s} = \oint_S \vec{D}_1 + \vec{D}_2 \cdot d\vec{s} = \oint_S \vec{D}_1 \cdot d\vec{s} + \oint_S \vec{D}_2 \cdot d\vec{s} = Q_1 + Q_2$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q = \int_\Omega \rho_\Omega d\Omega$$

$$\lim_{\Omega\to 0} \oint_S \vec{D} \cdot d\vec{s} = \lim_{\Omega\to 0} \int_\Omega \rho_\Omega \, d\Omega = \lim_{\Omega\to 0} \rho_\Omega \cdot \Omega \longrightarrow \left[\vec{\nabla} \cdot \vec{D} = \rho_\Omega \right]$$

Es:



$\vec{D}$ é esclusivamente radiale per ragioni di simmetria (considerando il filo infinitamente esteso)

$$\phi = \oint_S \vec{D}_r \cdot d\vec{s} = D_r(r) \cdot 2\pi r \cdot h = Q$$

$$Q = \int_\Omega \rho_\Omega \, d\Omega = \int_h \rho_\ell \, d\ell = \rho_\ell \cdot h$$

$$\vec{D}(r) = \frac{\rho_\ell}{2\pi r} \vec{u}_r$$

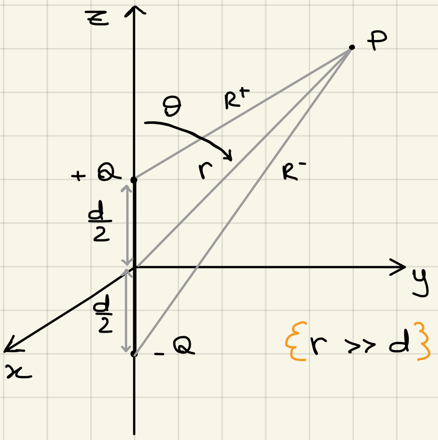
$$\vec{E}(R) = \frac{\vec{D}(R)}{\varepsilon_0} = \frac{\rho_\ell}{2\pi \varepsilon_0 \underline{r}} \vec{u}_r$$

la dipendenza da r non é più quadratica inversa

$$\left[V(R) = -\int_{R_0}^{R} \vec{E} \cdot d\vec{\ell} = -\int_{R_0}^{R} \frac{\rho_\ell \, dr}{2\pi \varepsilon_0 r} = \frac{\rho_\ell}{2\pi \varepsilon_0} \underbrace{\ln\left(\frac{R_0}{R}\right)} \right]$$

non posso più stabilire un potenziale per $R_0 \to \infty$ in quanto divergerebbe

Es (dipolo elettrostatico):



$\{r >> d\}$

$$V(P) = \frac{Q}{4\pi\varepsilon_0 R^+} - \frac{Q}{4\pi\varepsilon_0 R^-}$$

$$R^+ = \sqrt{r^2 + \left(\frac{d}{2}\right)^2 - 2r\frac{d}{2}\cos\theta} \overset{*}{\simeq} r - \frac{d}{2}\cos\theta$$

teo. di Carnot

$$R^- = \sqrt{r^2 + \left(\frac{d}{2}\right)^2 + 2r\frac{d}{2}\cos\theta} \overset{*}{\simeq} r + \frac{d}{2}\cos\theta$$

$$\lim_{\substack{d\to 0 \\ (Q\to\infty)}} V(P) \simeq \lim_{\substack{d\to 0 \\ (Q\to\infty)}} \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta} \right) = \frac{Q d \cos\theta}{4\pi\varepsilon_0 r^2}$$

per mantenere ← il prodotto Q·d (dipolo) costante

il potenziale decresce più rapidamente che nel caso della singola carica ($\frac{1}{r^2}$ vs $\frac{1}{r}$) a causa dell'azione distruttiva dei campi delle due cariche

$\vec{p} = Q \cdot \vec{d}$ dipolo elettrico

$$\left[V(P) = \frac{Q d \cos\theta}{4\pi\varepsilon_0 r^2} \right]$$ potenziale del dipolo elettrico

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{u}_\theta + \frac{1}{r\,\mathrm{sen}\theta}\frac{\partial f}{\partial \varphi}\vec{u}_\varphi$$

il potenziale del dipolo NON dipende da φ!

$$\vec{E}(P) = -\vec{\nabla}V = -\left(\frac{\partial V(P)}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V(P)}{\partial \theta}\vec{u}_\theta\right)$$

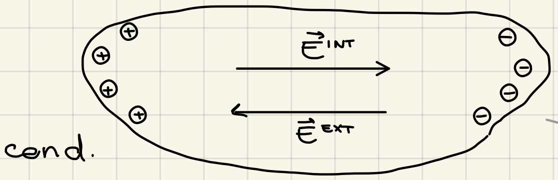
$$\lim_{d\to 0}\left[\vec{E}(P) = \frac{Qd}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\vec{u}_r + \mathrm{sen}\theta\,\vec{u}_\theta\right)\right]$$

In generale, per calcolare il campo generato da una particolare distribuzione di carica, lo ricavo attraverso il potenziale secondo l'uguaglianza $\vec{E} = -\vec{\nabla}V$

Campo Elettrostatico nella Materia

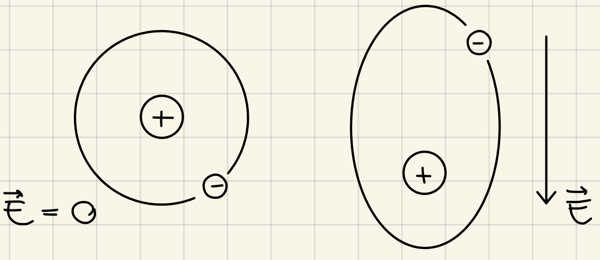
→ Conduttori

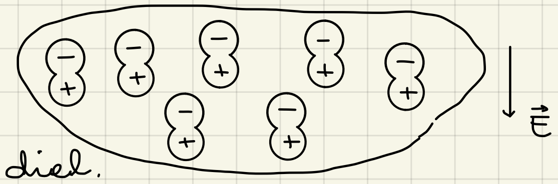
- campo $\vec{E}$ sempre nullo all'interno
- campo $\vec{E}$ tangente alla superficie nullo (sup. equipotenziale) → non compio lavoro a scorrere le cariche lungo la superficie



→ all'equilibrio (dopo un certo tempo di rilassamento)

→ Dielettrici





Il campo $\vec{E}$ induce nella materia una distribuzione volumetrica di dipoli.

Si definisce la densità di momento di dipolo:

$$\vec{P} = \lim_{\Delta\Omega\to 0} \sum_i \frac{\vec{p}_i}{\Delta\Omega}$$

Si dimostra che risulta:

$$\vec{\nabla}\cdot(\varepsilon_0\vec{E} + \vec{P}) = \rho_e$$

→ densità volumetrica di carica reale

↓ campo densità momento di dipolo

campo densità di flusso elettrico ← $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ $\quad\quad$ $\vec{\nabla}\cdot\vec{D} = \rho_e$

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$ (mezzi lineari)

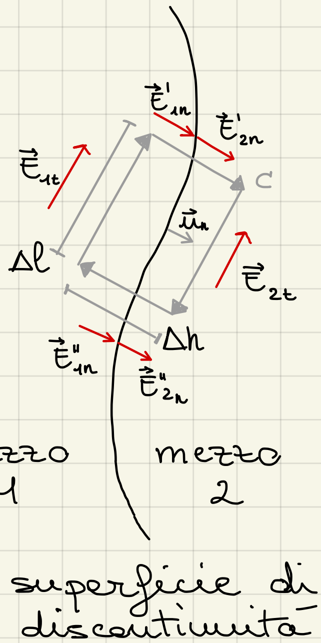
suscettività elettrica ←

$$\vec E = \vec E_e + \vec E_i \qquad \varepsilon = \varepsilon_0(1+\chi_e) = \varepsilon_0\varepsilon_r \qquad \varepsilon_r = 1 + \overset{\overset{\text{0 nel vuoto}}{\uparrow}}{\chi_e}$$

$$\vec D = \varepsilon\vec E = \varepsilon_0\varepsilon_r\vec E = \varepsilon_0(1+\chi_e)\vec E = \varepsilon_0\vec E + \vec P$$

χ_e è un numero puro; uno scalare (>0) nei mezzi isotropi (che mantengono le stesse proprietà in tutte le direzioni), una matrice (3×3) in quelli non isotropi

Condizioni al contorno per $\vec E$



$$\oint_c \vec E \cdot d\vec l = 0 = \text{(considerando } \Delta l \text{ e } \Delta h \text{ piccoli)}$$

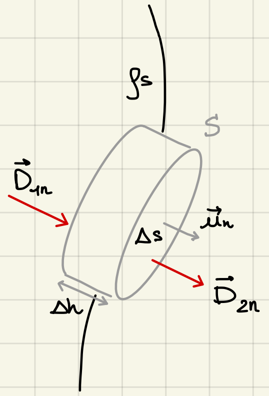
$$= E_{1t}\Delta l + E'_{1n}\frac{\Delta h}{2} + E'_{2n}\frac{\Delta h}{2} - E_{2t}\Delta l - E''_{2n}\frac{\Delta h}{2} - E''_{1n}\frac{\Delta h}{2}$$

$$\Delta h \to 0, \quad (E_{1t} - E_{2t})\Delta l = 0 \implies \boxed{E_{1t} = E_{2t}}$$

il campo elettrostatico tangente alla superficie di frontiera a due sostanze diverse si conserva sempre

$$\vec u_n \times (\vec E_{1t} + \vec E_{2t}) = 0$$

$$\boxed{\frac{D_{1t}}{D_{2t}} = \frac{\varepsilon_1}{\varepsilon_2}}$$



carica collocata sulla superficie di discontinuità

$$\oint_s \vec D \cdot d\vec s = D_{2n}\cdot\Delta s - D_{1n}\Delta s = \rho_s\Delta s \implies \boxed{D_{2n} - D_{1n} = \rho_s}$$

la densità di flusso di campo elettrostatico normale alla superficie di frontiera si conserva sempre

$$\vec u_n \cdot (\vec D_{2n} - \vec D_{1n}) = \rho_s$$

$$\boxed{\varepsilon_2\vec E_{2n} - \varepsilon_1\vec E_{1n} = \rho_s}$$

Dielettrico – Dielettrico: (normalmente $\rho_s = 0$)

$$E_{2t} = E_{1t} \qquad D_{2t} = \frac{\varepsilon_2}{\varepsilon_1}D_{1t}$$

$$E_{2n} = \frac{\varepsilon_1}{\varepsilon_2}E_{1n} \qquad D_{2n} = D_{1n}$$

Conduttore$_1$ – Dielettrico$_2$:

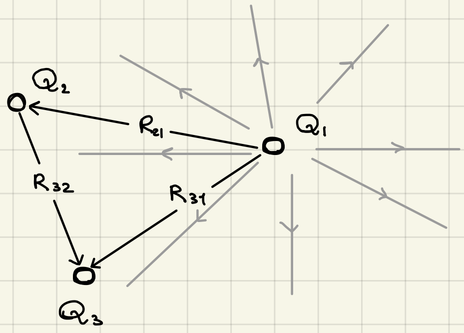
$$E_{2t} = 0 \qquad D_{2t} = 0 \qquad E_{1t} = 0$$

$$E_{2n} = \frac{\rho_s}{\varepsilon_2} \qquad D_{2n} = \rho_s \qquad E_{1n} = 0$$

NB: se la componente tangenziale o normale cambia e l'altra no, sia il modulo che il verso del vettore cambiano

Energia del Campo Elettrostatico

$$W_{21} = Q_2 \frac{Q_1}{4\pi\varepsilon R_{21}} = Q_2 V_{21} = Q_1 V_{12}$$



$$W_{31} + W_{32} = Q_3 \frac{Q_1}{4\pi\varepsilon R_{31}} + Q_3 \frac{Q_2}{4\pi\varepsilon R_{32}}$$
$$= Q_3 V_{31} + Q_3 V_{32}$$

$$W_e = W_{21} + W_{31} + W_{32} = Q_2 V_{21} + Q_3 V_{31} + Q_3 V_{32}$$

oppure equivalentemente

$$W_e = W_{12} + W_{13} + W_{23} = Q_1 V_{12} + Q_1 V_{13} + Q_2 V_{23}$$

$$\longrightarrow \quad W_e = \frac{1}{2}\left[Q_1(V_{12}+V_{13}) + Q_2(V_{21}+V_{23}) + Q_3(V_{31}+V_{32})\right]$$

$$\Longrightarrow \quad \boxed{W_e = \frac{1}{2}\sum_{i=1}^{3} Q_i V_i} \qquad \boxed{W_e = \frac{1}{2}\int_\Omega \rho_\Omega V \, d\Omega}$$

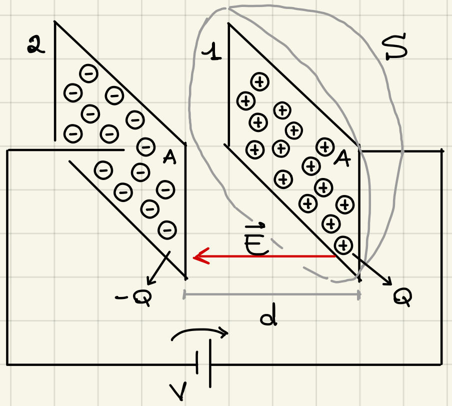
Si può dimostrare anche che $W_e = \frac{1}{2}\int_{spazio} \vec{D}\cdot\vec{E}\, d\Omega$

$$= \frac{1}{2}\int_{spazio} \varepsilon |\vec{E}|^2 \, d\Omega$$

$$\boxed{w_e = \frac{1}{2}\vec{D}\cdot\vec{E} \quad \text{oppure} \quad w_e = \frac{1}{2}\varepsilon|\vec{E}|^2}$$

energia puntuale o densità di energia per unità di volume del campo elettrostatico

CAPACITÁ ELETTRICA

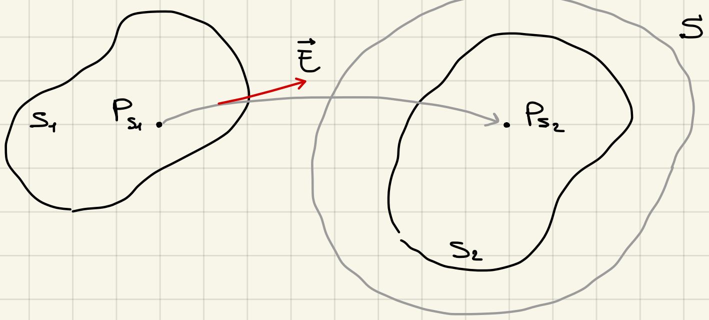


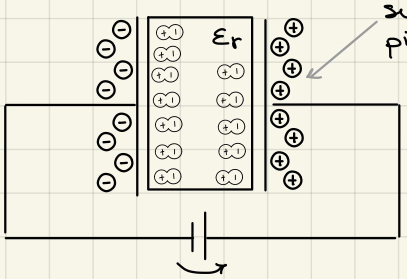
$$\oint_S \vec{D}\cdot d\vec{s} = \varepsilon_0 E A = Q \qquad \left(E = \frac{V}{d}\right)$$

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon_0 E A}{V} = \varepsilon_0 \frac{A}{d}}$$

$$C = \frac{Q}{V} = \frac{\oint_S \vec{D}\cdot d\vec{s}}{-\int_{P_{S_1}}^{P_{S_2}} \vec{E}\cdot d\vec{\ell}}$$

d piccolo per poter trascurare gli effetti di bordo





si accumula più carica

$$C = \frac{\varepsilon_0 \boxed{\varepsilon_r} A}{d}$$

L'inserimento di un dielettrico fra le armature ne aumenta la capacità.

Energia immagazzinata nel condensatore

$$W_e = \frac{1}{2}\int_{S_1} \rho_{s_1} V \, ds_1 + \frac{1}{2}\int_{S_2} \rho_{s_2} V \, ds_2$$

$$= \frac{V_1}{2}\int_{S_1} \rho_{s1} \, ds_1 + \frac{V_2}{2}\int_{S_2} \rho_{s_2} \, ds_2$$

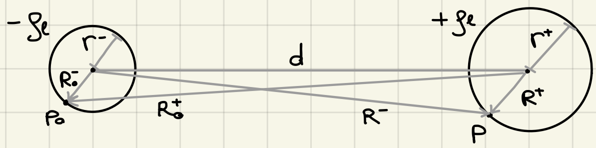
$$= \frac{1}{2} Q \underbrace{(V_1 - V_2)}_{\Delta V} \Longrightarrow \left[W_e = \frac{1}{2} C V^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} Q V \right]$$

ΔV: differenza di potenziale fra le armature del condensatore

$$W_e = \frac{1}{2}\int_{\text{spazio}} \varepsilon_0 |\vec{E}|^2 d\Omega = \frac{1}{2}\varepsilon_0 \int_{\substack{\text{spazio}\\ \text{tra le armature}}} V^2/d^2 \, d\Omega = \frac{1}{2}\underbrace{\varepsilon_0 \frac{V^2}{\cancel{d^2}} \cdot A}_{C} \cdot \cancel{d} = \frac{1}{2} C V^2$$

↓ ricavata dall'energia del campo elettrostatico

Es (capacità (per u.l.) di una linea bifilare):



$$\{ d \gg r^-; r^+ \}$$

$$V(P) = \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{R_0^+}{R^+}\right) - \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{R_0^-}{R^-}\right)$$

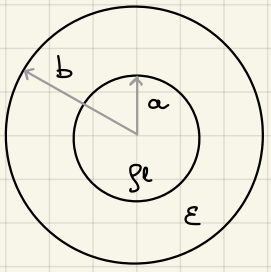
$$= \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{R_0^+ R^-}{R_0^- R^+}\right)$$

$$V \overset{*}{\simeq} \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{d^2}{r^+ r^-}\right)$$

potenziale generato da un cavo con densità lineare di carica

$$\left[C = \frac{\rho_l}{V} = \rho_l \frac{2\pi\varepsilon_0}{\rho_l \ln\left(\frac{d^2}{r^+ r^-}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{d^2}{r^+ r^-}\right)} \right]$$

Es (capacità per u.l. di un cavo coassiale):



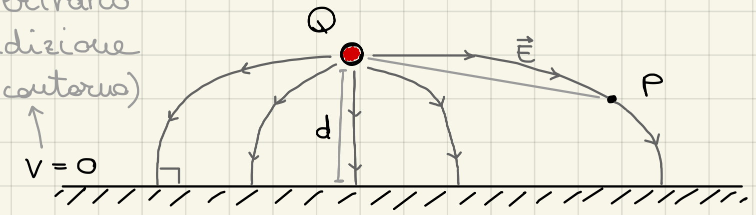
$$E(r) = \frac{\rho_l}{2\pi\varepsilon r}$$

$$V = -\int_b^a E(r) dr = -\int_b^a \frac{\rho_l}{2\pi\varepsilon r} dr = \frac{\rho_l}{2\pi\varepsilon} \ln\left(\frac{b}{a}\right)$$

$$\left[C = \frac{\rho_l}{V} = \frac{2\pi\varepsilon}{\ln(b/a)} \right]$$

Metodo delle immagini

arbitrario (condizione al contorno)



$$V(P) = ?$$

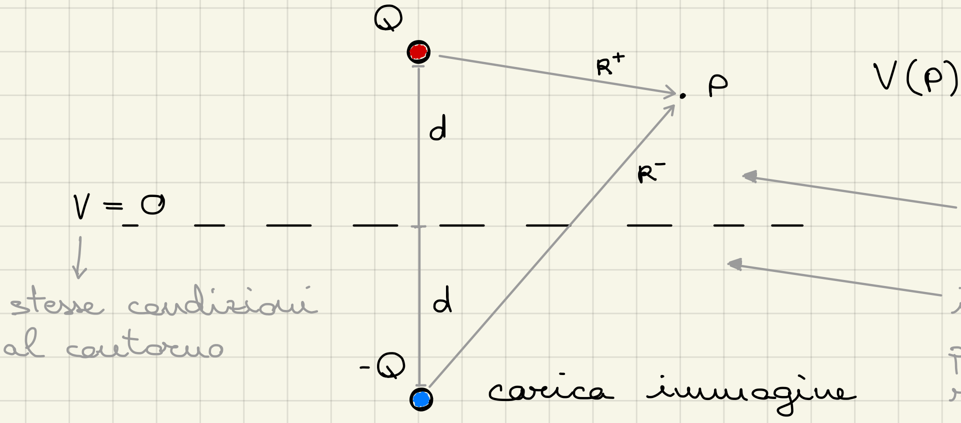
$$\vec{E}(P) = ?$$

piano conduttore

solo componente normale

→ Teorema di Unicità

Data una regione di spazio, delimitata da una superficie chiusa, sulla quale siano assegnate le condizioni al contorno (potenziale o sua derivata normale), la soluzione del problema elettrostatico (che soddisfa le condizioni al contorno) è **unica**.



$$V(P) = \frac{Q}{4\pi\varepsilon R^+} - \frac{Q}{4\pi\varepsilon R^-}$$

il campo elettrico è come prima

il campo elettrico non è più come prima ($\neq 0$) ma non ci interessa perché P non può stare da questa parte

Corrente elettrica e Legge di Ohm

$\vec{v}_d = \mu_q \vec{E}$

↓ mobilità elettronica

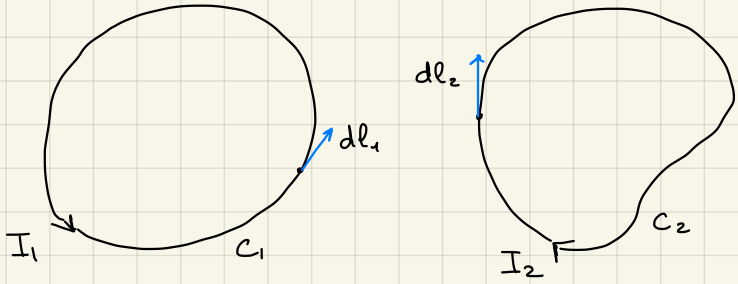
$\vec{J} = qN\vec{v}_d = qN\mu_q\vec{E} = \sigma\vec{E}$

σ → conducibilità $\left[\frac{S}{m}\right]$

N → densità volumetrica di portatori

La velocità di deriva v_d è generalmente dell'ordine di solo qualche millimetro al secondo, perché misura lo spostamento medio delle cariche nel conduttore, che è molto inferiore rispetto all'effettiva velocità delle cariche (10^6 m/s) e alla propagazione della corrente e dell'energia nel circuito (10^8 m/s).

CAMPO MAGNETOSTATICO



Legge di Biot-Savart

$$d\vec{F}_{12} = \frac{\mu_0}{4\pi}\frac{I_1 d\vec{l}\times[I_2 d\vec{l}_2\times\vec{u}_{12}]}{R^2}$$

$$\vec{F}_{12} = \frac{\mu_0 I_1 I_2}{4\pi}\oint_{c_1}\oint_{c_2}\frac{d\vec{l}_1\times(d\vec{l}_2\times\vec{u}_{12})}{R^2}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}\int_{c_1}\int_{c_2}\frac{\vec{u}_{12}(d\vec{l}_1\cdot d\vec{l}_2)}{R_2} = -\vec{F}_{21}$$

$$d\vec{F}_{12} = I_1 d\vec{\ell}_1 \times \vec{B}_2 \qquad d\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{I_2 d\vec{\ell}_2 \times \vec{u}_{12}}{R^2}$$

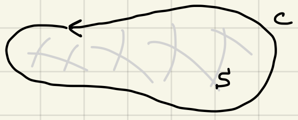
$$\vec{B}_2 = \oint_{C_1} \frac{\mu_0}{4\pi}\frac{I_2 d\vec{\ell} \times \vec{u}_{12}}{R^2} = \int_{\Omega} \frac{\mu_0}{4\pi}\frac{\vec{J} \times u_R}{R^2} d\Omega$$

Legge di Gauss

$$\oint_S \vec{B}\cdot d\vec{s} = 0 \qquad \lim_{\Omega \to 0} \oint_S \vec{B}\cdot d\vec{s} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

Legge di Ámpere

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

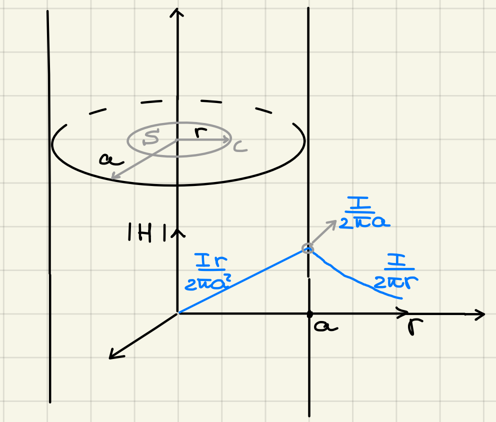


$$\oint_c \vec{H}\cdot d\vec{\ell} = \int_S \vec{J}\cdot d\vec{s}$$

$$\left(\oint_c \vec{H}\cdot d\vec{\ell} = \int_S \vec{\nabla}\times\vec{H}\cdot d\vec{s} = \int_S \vec{J}\cdot d\vec{s} \quad \text{teo. di Stokes} \right)$$

$$\lim_{S\to 0} \oint_c \vec{H}\cdot d\vec{\ell} = \lim_{S\to 0}\int_S \vec{J}\cdot d\vec{s} = \vec{J}\cdot\vec{S} = I$$

Es (filo percorso da corrente):



$$J = I/(\pi a^2)$$

$$\oint_c \vec{H}\cdot d\vec{\ell} = \int_S \vec{J}\cdot d\vec{s}$$

$r < a$ $\quad \vec{H}$ interno

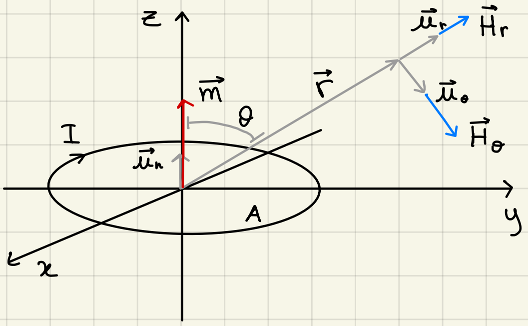
$$\oint_c \vec{H}\cdot d\vec{\ell} = H_\varphi 2\pi r = \int_S \vec{J}\cdot d\vec{s} = \frac{I}{\pi a^2}\pi r^2$$

$$\left[\vec{H}(r) = H_\varphi \vec{u}_\varphi = \frac{Ir}{2\pi a^2}\vec{u}_\varphi \right]$$

$r > a$ $\quad \vec{H}$ esterno

$$\oint \vec{H}\cdot d\vec{\ell} = H_\varphi 2\pi r = \int \vec{J}\cdot d\vec{s} = I \implies \left[\vec{H}(r) = \frac{I}{2\pi r}\vec{u}_\varphi \right]$$

Es (dipolo magnetico):



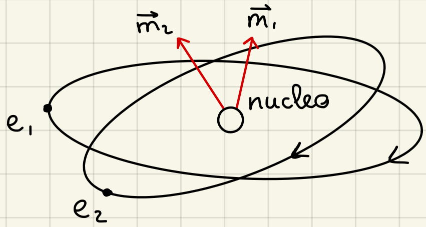
$$\vec{m} = A \cdot I \cdot \vec{u}_n$$

$$\left[\vec{B} = \frac{\mu_0 |\vec{m}|}{4\pi r^3}\left(2\cos\theta\,\vec{u}_r + \text{sen}\,\theta\,\vec{u}_\theta\right) \right]$$

$$\left(\vec{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\vec{u}_r + \text{sen}\,\theta\,\vec{u}_\theta\right) \right)$$ dip. elettrico

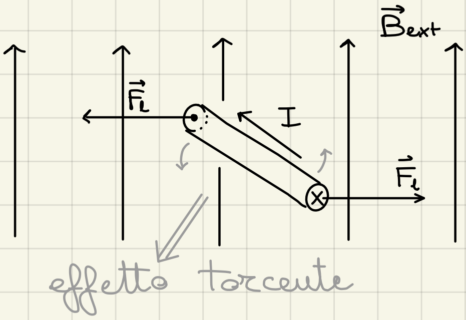
Campo magnetico nei materiali

un campo magnetico esterno allinea i momenti di dipolo degli atomi



$$\vec M = \lim_{\Delta\Omega\to 0} \frac{\sum_i \vec m_i}{\Delta\Omega} \qquad \vec M = \chi_m \vec H$$

χ_m é detta suscettività magnetica



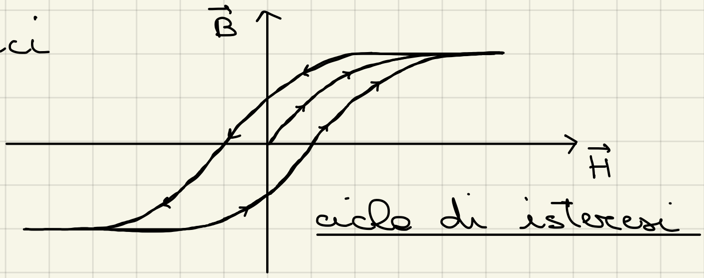
$$\mu = \mu_0 \mu_r = \mu_0 (1+\chi_m)$$

$\mu_r = 1+\chi_m$ é la permeabilità relativa

$$\vec B = \mu \vec H = \mu_0 \mu_r \vec H = \mu_0 (1+\chi_m)\vec H = \mu_0(\vec H + \vec M)$$

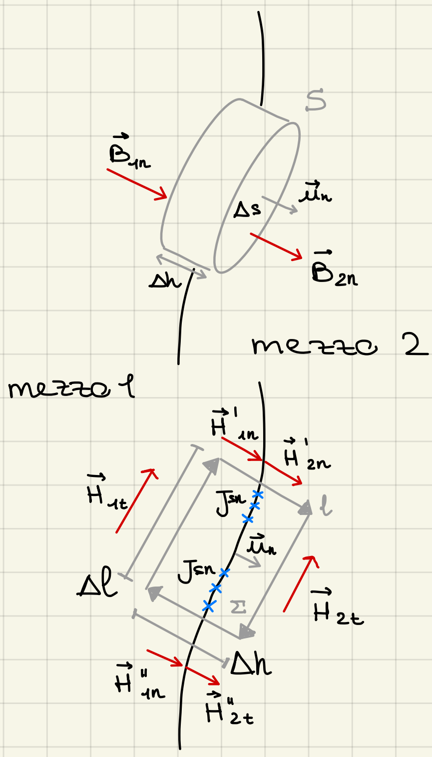
Classificazione dei materiali:

- Diamagnetici ($\chi_m \simeq -10^{-5}$)
- Paramagnetici ($\chi_m \simeq 10^{-3}$)
- Ferromagnetici



ciclo di isteresi

Condizione al contorno per $\vec H$ e $\vec B$



$$\oint_S \vec B \cdot d\vec S = B_{2n}\Delta S - B_{1n}\Delta S = 0 \implies [B_{2n} = B_{1n}]$$

$$[H_{2n} = \frac{\mu_1}{\mu_2} H_{1n}]$$ mezzi lineari e isotropi

$$\vec u_n \cdot (\vec B_2 - \vec B_1) = 0$$

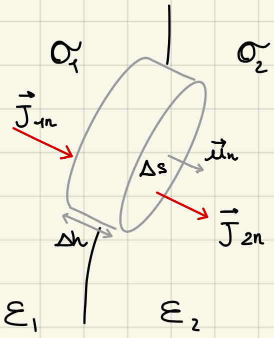
$$\oint_l \vec H \cdot d\vec l = \int_\Sigma \vec J \cdot d\Sigma$$

$$\oint_l \vec H \cdot d\vec l \underset{\Sigma\to 0}{\overset{\Delta h\to 0}{=}} (H_{2t} - H_{1t})\Delta l = J_{sn}\Delta l \implies [H_{2t} - H_{1t} = J_{sn}]$$

J_{sn}: densità di corrente superficiale $[\frac{A}{m}]$

$$\vec u_n \times (\vec H_2 - \vec H_1) = \vec J_s$$

Condizioni al contorno per $\vec{J}$



→ conducibilità

$$E_{1t} = E_{2t} \qquad \vec{J} = \sigma \vec{E} \qquad \frac{J_{2t}}{\sigma_2} = \frac{J_{1t}}{\sigma_1}$$

$$\oint_S \vec{J} \cdot d\vec{s} = 0 \implies J_{2n} = J_{1n}$$

$$\uparrow$$

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

$$\vec{\nabla} \cdot \vec{J} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0$$

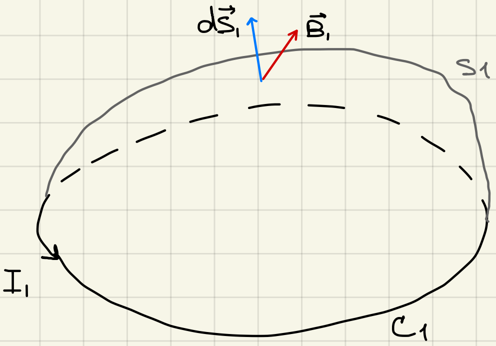
$$\left[\sigma_2 E_{2n} = \sigma_1 E_{1n}\right]$$

$$\downarrow$$

$$\varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \rho_s \longrightarrow \rho_s = \left(\varepsilon_2 - \varepsilon_1 \frac{\sigma_2}{\sigma_1}\right) E_{2n} = \left(\varepsilon_2 \frac{\sigma_1}{\sigma_2} - \varepsilon_1\right) E_{1n}$$

densità superficiale di carica $\left[\frac{C}{m^2}\right]$

Autoinduttanza Magnetica



$$L_{11} = \frac{\Phi_{m,11}}{I_1} = \frac{\int_{S_1} \vec{B}_1 \cdot d\vec{s}_1}{I_1}$$

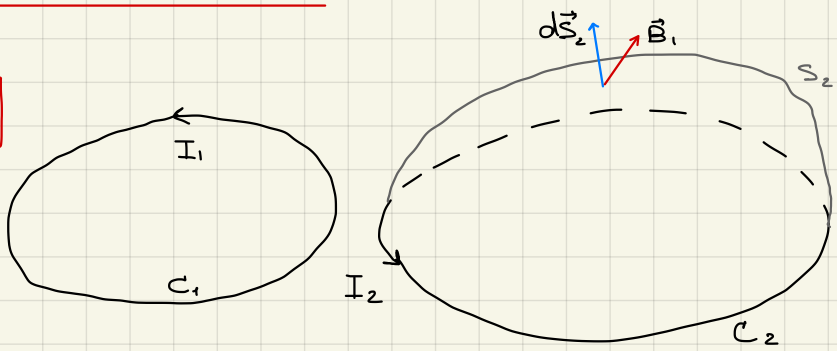
$$\vec{B}_1 = \frac{\mu I_1}{4\pi} \oint_{C_1} \frac{d\vec{l} \times \vec{u}_r}{r^2}$$

$$\Phi_{m,11} = \frac{\mu I_1}{4\pi} \int_{S_1} \left(\oint_{C_1} \frac{d\vec{l} \times \vec{u}_r}{r^2} \right) \cdot d\vec{s}$$

$$\left[L_{11} = \frac{\mu}{4\pi} \int_{S_1} \left(\oint_{C_1} \frac{d\vec{l} \times \vec{u}_r}{r^2} \right) \cdot d\vec{s} \right]$$

Mutuainduttanza

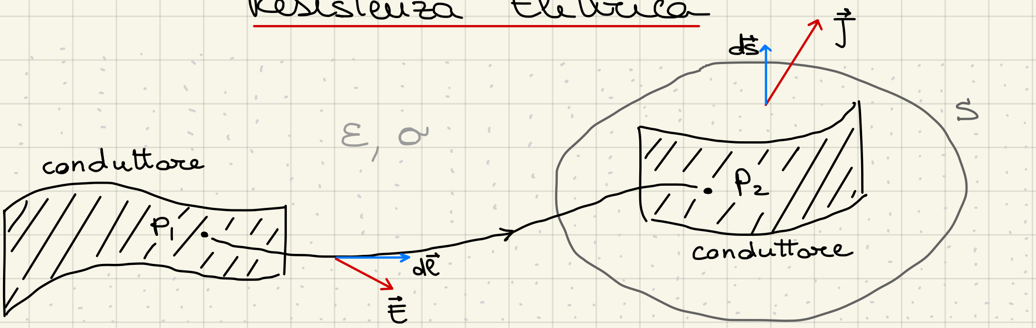
$$\left[L_{21} = \frac{\mu}{4\pi} \int_{S_2} \left(\oint_{C_1} \frac{d\vec{l} \times \vec{u}_r}{r^2} \right) \cdot d\vec{s} \right]$$



Energia del campo magnetostatico

$$w_m = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{1}{2} \mu |\vec{H}|^2 \qquad W_m = \frac{1}{2} L_{11} I^2$$

Resistenza Elettrica



$$\boxed{R = \frac{V}{I} = -\frac{\int_{P_1}^{P_2} \vec{E}\cdot d\vec{l}}{\oint_S \vec{J}\cdot d\vec{s}} = \frac{1}{\sigma}\,\frac{\int_{P_2}^{P_1} \vec{E}\cdot d\vec{l}}{\underbrace{\oint_S \vec{E}\cdot d\vec{s}}_{q_{int}/\varepsilon}}}$$

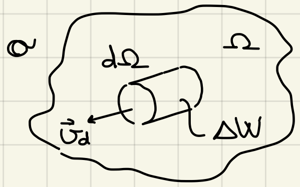
→ formula generale della resistenza elettrica (valida per qualunque geometria)

$$C = \frac{Q}{V} = \frac{\overset{q_{int}}{\oint_S \vec{D}\cdot d\vec{s}}}{-\int_{P_1}^{P_2} \vec{E}\cdot d\vec{l}}$$

$$\boxed{R\cdot C = \frac{\varepsilon}{\sigma}}$$

→ costante di tempo del materiale

Legge di Joule



N per m^3 $(N \cdot m^{-3})$

$N \cdot d\Omega$ $\quad \vec{J} = N q \vec{v}_d \quad \Delta W = N\, d\Omega\, \vec{F}\cdot d\vec{l}$

$\vec{F} = q\vec{E} \quad dP = \frac{\Delta W}{\Delta t} = N\, d\Omega\, q\vec{E}\cdot \frac{\Delta \vec{l}}{\Delta t} \rightarrow \vec{v}_d$

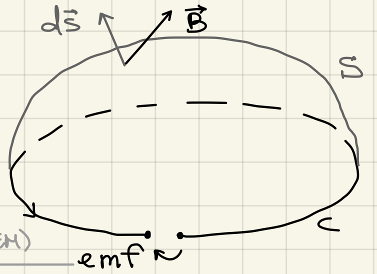
$$\boxed{\frac{dP}{d\Omega} = N q \vec{E}\cdot\vec{v}_d = \vec{E}\cdot\vec{J}} \left[\frac{W}{m^3}\right] \qquad \boxed{P = \int_\Omega \vec{E}\cdot\vec{J}\, d\Omega}$$

Regime Dinamico

STATICO: $\vec{E}$ e $\vec{H}$ indipendenti $\quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon} \quad \vec{\nabla}\times\vec{H} = \vec{J}$

DINAMICO: $\vec{E}$ e $\vec{H}$ legati fra loro (eq. di Maxwell)

→ Equazione sulla circuitazione di $\vec{E}$: Legge di Faraday



$\rightarrow \text{emf} = -\frac{d\Phi_m}{dt}$

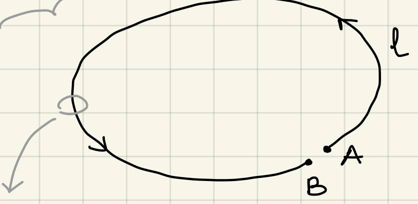
$\rightarrow \text{emf} = \oint_C \vec{E}\cdot d\vec{l}$ [V]

Forza ElettroMotrice (FEM)

→ $\phi_m = \int_S \vec{B}\cdot d\vec{s}$ $\Longrightarrow$ $\left[\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}\right]$

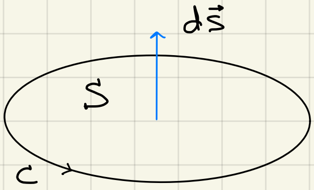
$$\oint_\ell \vec{E}\cdot d\vec{l} = \underbrace{\int_A^B \vec{E}\cdot d\vec{l}}_{\sim 0} + \boxed{\int_B^A \vec{E}\cdot d\vec{l} = emf}$$

$$\oint_\ell \vec{E}\cdot d\vec{l} = emf \neq 0$$



$E_t \equiv 0$

$\vec{E}\cdot d\vec{l} = E_t$



$$\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} =$$
$$= -\int_S \frac{\partial}{\partial t}(\vec{B}\cdot d\vec{s}) = -\int_S\left(\frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} + \vec{B}\cdot\frac{\partial \vec{s}}{\partial t}\right) =$$
$$= -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

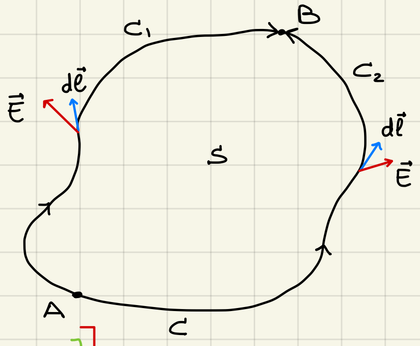
→ se consideriamo una superficie S che non varia nel tempo

$$\oint_c \vec{E}\cdot d\vec{l} = \int_{c_1} \vec{E}\cdot dl - \int_{c_2} \vec{E}\cdot dl = -\frac{d\phi_m}{dt}$$

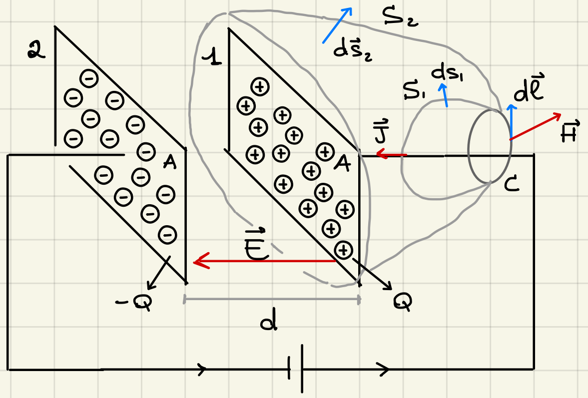
$\int_{c_1} \vec{E}\cdot dl \neq \int_{c_2} \vec{E}\cdot dl$ (se ϕ_m varia nel tempo)

$$\lim_{S\to 0} \oint_c \vec{E}\cdot d\vec{l} = \lim_{S\to 0}\left[-\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}\right] = -\frac{\partial \vec{B}}{\partial t}\cdot\vec{S}$$

$$\lim_{S\to 0} \oint_c \vec{E}\cdot d\vec{l} = (\vec{\nabla}\times\vec{E})\cdot\vec{S} = -\frac{\partial \vec{B}}{\partial t}\cdot\vec{S}$$ ⟶ $\left[\left[\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}\right]\right]$



→ Equazioni sulla circuitazione di $\vec{H}$ e corrente di spostamento



$\oint_c \vec{H}\cdot d\vec{l} = \ ?$ <

$\int_{S_1} \vec{J}\cdot d\vec{s}_1 \neq 0$

$\int_{S_2} \vec{J}\cdot d\vec{s}_2 = 0$

$$\oint_c \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{s} + \int_S \frac{\partial \vec{D}}{\partial t}\cdot d\vec{s}$$

termine stazionario e dinamico

corrente di conduzione e spostamento

$$\oint_c \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{s} + \frac{d}{dt}\int_S \vec{D}\cdot d\vec{s}$$ → se S non varia nel tempo

$\left[I_c = \int_S \vec{J}\cdot d\vec{s}\right]$ $\left[I_s = \frac{d}{dt}\int_S \vec{D}\cdot d\vec{s}\right]$

$$\lim_{S\to 0}\oint_c \vec H\cdot d\vec\ell = \lim_{S\to 0}\left[\int_S \vec J\cdot d\vec s + \int_S \frac{\partial \vec D}{\partial t}\cdot d\vec s\right] = \vec J\cdot\vec s + \frac{\partial \vec D}{\partial t}\cdot\vec s$$

$$\lim_{S\to 0}\oint_c \vec H\cdot d\vec\ell = \vec\nabla\times\vec H\cdot\vec s \Longrightarrow \left[\left[\vec\nabla\times\vec H = \vec J + \frac{\partial \vec D}{\partial t}\right]\right]$$

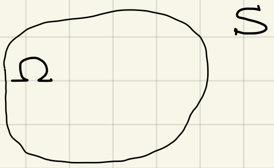
→ Equazioni sui flussi di $\vec D$ e $\vec B$: Leggi di Gauss

$$\oint_S \vec D\cdot d\vec s = \int_\Omega \rho_\Omega \, d\Omega \longrightarrow \left[\vec\nabla\cdot\vec D = \rho_\Omega\right]$$

$$\oint_{\vec B} \vec B\cdot d\vec s = 0 \longrightarrow \left[\vec\nabla\cdot\vec B = 0\right]$$

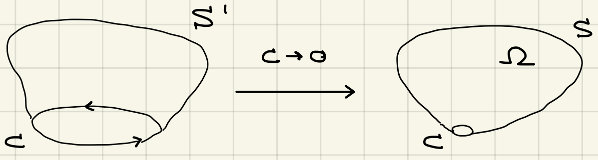
(Sono ricavabili dalle altre equazioni di Maxwell)

Legge di conservazione della carica



$$\oint_S \vec J\cdot d\vec s = -\frac{d}{dt}\int_\Omega \rho_\Omega \, d\Omega$$

$$\oint_c \vec H\cdot d\ell = \int_{S'} \vec J\cdot d\vec s + \frac{d}{dt}\int_{S'} \vec D\cdot d\vec s = 0$$



$$\Longrightarrow \oint_S \vec J\cdot d\vec s = -\frac{d}{dt}\oint_S \vec D\cdot d\vec s$$

$$= -\frac{d}{dt}\int_\Omega \vec\nabla\cdot\vec D \, d\Omega$$

$$= -\frac{d}{dt}\int_\Omega \rho_\Omega \, d\Omega$$

$$\oint \vec J\cdot d\vec s = \int_\Omega \vec\nabla\cdot\vec J \, d\Omega = -\frac{d}{dt}\int_\Omega \rho_\Omega \, d\Omega$$

$$\left[\vec\nabla\cdot\vec J = -\frac{\partial \rho_\Omega}{\partial t}\right] \qquad \left(\vec\nabla\cdot\vec J = 0 \text{ stazionario}\right)$$

$$\vec J = \sigma\vec E$$

Equazioni di Maxwell (integrali):

$$\oint_c \vec E\cdot d\vec\ell = -\frac{d}{dt}\int_S \vec B\cdot d\vec s$$

$$\oint_c \vec H\cdot d\vec\ell = \int_S \vec J\cdot d\vec s + \frac{d}{dt}\int_S \vec D\cdot d\vec s$$

$$\oint_S \vec D\cdot d\vec s = \int_\Omega \rho_\Omega \, d\Omega$$

$$\oint_S \vec B\cdot d\vec s = 0$$

$$\oint_S \vec J\cdot d\vec s = -\frac{d}{dt}\int_\Omega \rho_\Omega \, d\Omega$$

Equazioni di Maxwell (differenziali)

$$\left(\begin{aligned} \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{aligned} \right)$$

$$\vec{\nabla} \cdot \vec{D} = \rho_v$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\left(\vec{\nabla} \cdot \vec{J} = -\frac{d\rho_v}{dt} \right)$$

Relazioni costitutive dei materiali

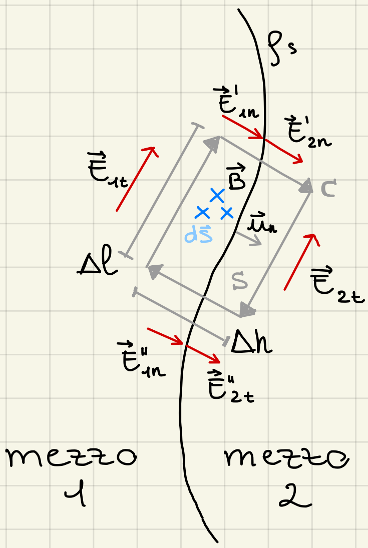
- Ci sono 15 incognite (nello spazio 3D):
- Abbiamo 6 equazioni scalari (rotore)
- Le altre 9 equazioni dalle 3 relazioni costitutive:

$$\vec{D} = f_D(\vec{E}, \vec{H}) \qquad \vec{D} = \varepsilon \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon_0 (1 + \chi_e) \vec{E}$$

$$\vec{B} = f_B(\vec{H}, \vec{E}) \qquad \vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H} = \mu_0 (1 + \chi_m) \vec{H}$$

$$\vec{J}_c = f_J(\vec{E}, \vec{H}) \qquad \vec{J}_c = \sigma \vec{E}$$

Condizioni al contorno per $\vec{E}$ (regime dinamico)



$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{s}$$

$$\Delta h \longrightarrow 0 \qquad (S \rightarrow 0)$$

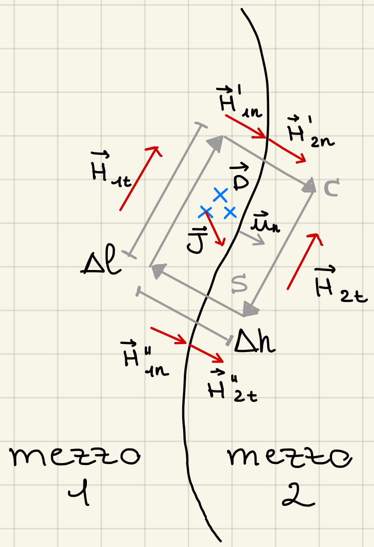
$$\oint_C \vec{E} \cdot d\vec{l} = 0 \implies E_{2t} = E_{1t}$$

→ uguale al regime stazionario

$$\vec{u}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\vec{u}_n \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s$$

Condizioni al contorno per $\vec{H}$ (regime dinamico)



$$\oint_c \vec{H} \cdot d\vec{\ell} = \int_s \vec{J} \cdot d\vec{s} + \frac{d}{dt} \int_s \vec{D} \cdot d\vec{s}$$

$$\Delta h \longrightarrow 0 \quad (S \rightarrow 0)$$

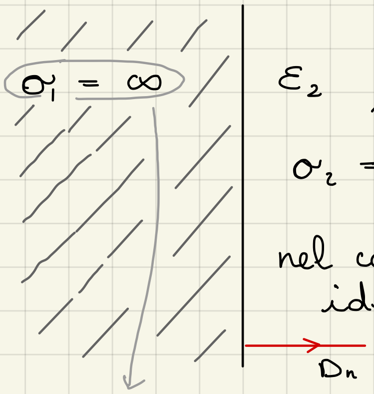
$$\Longrightarrow \left[H_{2t} - H_{1t} = J_{sn} \right]$$

→ componente della densità di corrente perpendicolare alla superficie di discontinuità

come nel caso statico

$$\vec{u}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$$

$$\vec{u}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0$$



I	II	
$E_{1t} = 0$	$E_{2t} = 0$ ↔	$\vec{u}_n \times \vec{E}_2 = 0$
$H_{1t} = 0$	$H_{2t} = J_s$	$\vec{u}_n \times \vec{H}_2 = \vec{J}_s$
$H_{1n} = 0$	$D_{2n} = \rho_s$	$\vec{u}_n \cdot \vec{D}_2 = \rho_s$
	$B_{2n} = 0 = \mu_2 H_{2n}$	$\vec{u}_n \cdot \vec{B}_2 = 0$

nel conduttore ideale

significa che il conduttore ha tempo di rilassamento nullo

Se il conduttore non è ideale ($\sigma < \infty$) il tempo di rilassamento non è più nullo e il materiale ci impiega del tempo a raggiungere questi valori

↳ in regime dinamico queste uguaglianze non sono più vere

Onde Elettromagnetiche

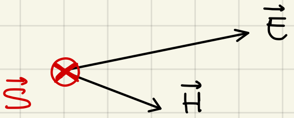
1) Teorema (e vettore) di Poynting (dominio del tempo)
2) Equazione di Helmholtz (dominio del tempo)
3) Caso particolare: onde piane in mezzi senza perdite
4) Passaggio al dominio dei fasori (regime sinusoidale)

Teorema di Poynting

Si definisce densità di potenza e si misura in $\frac{W}{m^2}$, la grandezza vettoriale $\vec{S}$

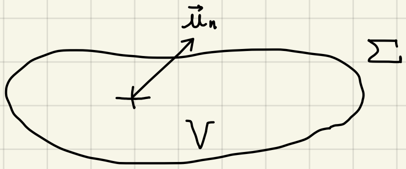
$$\boxed{\vec{S} = \vec{E} \times \vec{H}}$$

vettore di Poynting



$$\boxed{- \int_S \vec{S} \cdot d\vec{s} = \int_\Omega \vec{E} \cdot \vec{J} \, d\Omega + \int_\Omega \left(\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \right) d\Omega} \quad [W]$$

Dimostrazione:



$$\vec{S} = \vec{E} \times \vec{H} \quad \left[\frac{W}{m^2} \right]$$

$$\oint_\Sigma \vec{S} \cdot d\vec{\Sigma} = -(\text{variazione istantanea dell' energia in } V)$$

$$\oint_\Sigma \vec{S} \cdot d\vec{\Sigma} = \int_V \vec{\nabla} \cdot \vec{S} \, dV = \int_V \vec{\nabla} \cdot (\vec{E} \times \vec{H}) \, dV$$

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{H}) =$$

mezzo lin. isotropo

$$= \vec{H} \cdot \left(-\frac{\partial \vec{B}}{\partial t} \right) - \vec{E} \cdot \left(\frac{\partial \vec{D}}{\partial t} + \vec{J} \right) = \mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} - \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} - \sigma |\vec{E}|^2 =$$

$$= \frac{\mu}{2} \frac{\partial |\vec{H}|^2}{\partial t} - \frac{\varepsilon}{2} \frac{\partial |\vec{E}|^2}{\partial t} - \sigma |\vec{E}|^2$$

$$\vec{E} \cdot \vec{J}$$
$$\shortparallel$$

$$\int_V \vec{\nabla} \cdot (\vec{E} \times \vec{H}) \, dV = -\frac{\partial}{\partial t} \left[\underbrace{\int_V \left(\frac{\mu |\vec{H}|^2}{2} + \frac{\varepsilon |\vec{E}|^2}{2} \right) dV}_{\substack{\text{ENERGIA E.M.} \\ \text{IMMAGAZZINATA} \\ \text{(IN V)}}} - \underbrace{\int_V \sigma |\vec{E}|^2 dV}_{\substack{\text{POTENZA} \\ \text{DISSIPATA}}} \right.$$

$$= \oint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$

Equazione di Helmholtz (delle onde)

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad ①$$

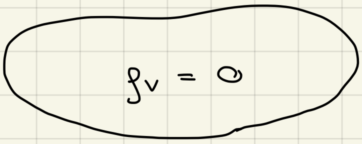
$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad ②$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

in assenza di cariche

$\nexists$ sorgenti



$$\vec{J} = \vec{J}_c + \cancel{\vec{J}_i}$$
$$\vec{J} = \sigma \vec{E}$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \quad ①$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H}) \qquad \text{ma} \qquad \vec{\nabla}\times\vec{\nabla}\times\vec{F} = \vec{\nabla}(\vec{\nabla}\cdot\vec{F}) = -\nabla^2 F$$

$$\vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{E}}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left(\varepsilon\frac{\partial\vec{E}}{\partial t} + \sigma\vec{E}\right)$$

$\emptyset =$

$$\Longrightarrow \boxed{\nabla^2 E = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}}$$

e analogam. $\boxed{\nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}}$ con la ②.

Queste 2 equazioni differenziali vettoriali (6 equazioni differenziali scalari) non sono di solito risolvibili in forma chiusa, tranne che in alcuni casi particolari

Onde piane in mezzi ideali (senza perdite)

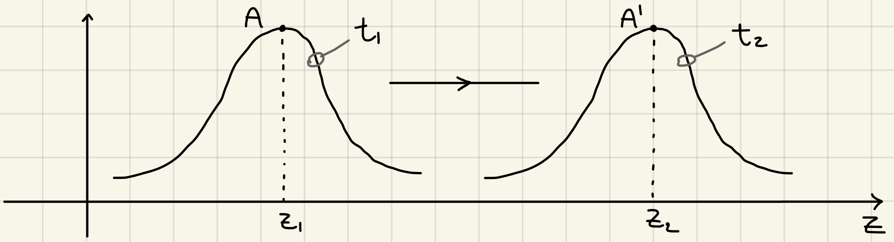
$$\nabla^2\vec{E} = \cancel{\mu\sigma\frac{\partial\vec{E}}{\partial t}} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

Dimostreremo che l'onda piana uniforme è una soluzione dell'equazione delle onde

Onda piana: i piani in cui giacciono i campi $\vec{E}$ ed $\vec{H}$ sono paralleli tra loro in ogni punto dello spazio

Onda piana uniforme: i campi $\vec{E}$ ed $\vec{H}$ non variano su tali piani (in un dato istante)

$f(z) \rightsquigarrow f\left(t \mp \frac{z}{v}\right) \longleftarrow$ perturbazione che viaggia con velocità v nel verso delle z positive / negative (onda progressiva / regressiva)



$$f\Big(\underbrace{t - \frac{z}{v}}_{x}\Big)$$

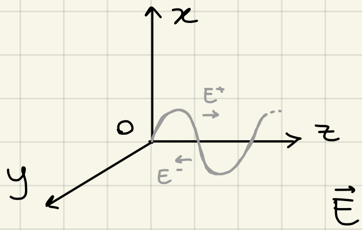
$x_1 = t_1 - \frac{z_1}{v}$

$x_2 = t_2 - \frac{z_2}{v}$

$x_1 = x_2 \qquad t_1 - \frac{z_1}{v} = t_2 - \frac{z_2}{v}$

$vt_1 - z_1 = vt_2 - z_2 \qquad \underbrace{z_2 - z_1}_{\Delta z} = v\underbrace{(t_2 - t_1)}_{\Delta t} \qquad \frac{\Delta z}{\Delta t} = v$ (velocità)

$$\vec{E}(x,y,z,t) = E_x(x,y,z,t)\,\vec{u}_x + E_y(\dots)\,\vec{u}_y + E_z(\dots)\,\vec{u}_z$$



xy piano "trasverso"

su xy $\vec{E}$ ed $\vec{H}$ costanti

indipendenti
(non interferiscono)

$$\vec{E}(z,t) = \vec{E}^+\left(t - \frac{z}{v}\right) + \vec{E}^-\left(t + \frac{z}{v}\right)$$ onda piana unif.

$$\boxed{\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0}$$ equazione di Helmoltz in mezzo senza perdite

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ operatore laplaciano

$$\longrightarrow \begin{cases} \cancel{\frac{\partial^2 E_x(z,t)}{\partial x^2}} + \cancel{\frac{\partial^2 E_x(z,t)}{\partial y^2}} + \frac{\partial^2 E_x(z,t)}{\partial z^2} - \mu\varepsilon\frac{\partial^2 E_x(z,t)}{\partial t^2} = 0 \\ \cancel{\frac{\partial^2 E_y(z,t)}{\partial x^2}} + \cancel{\cdot\;\cdot} \;\cdot + \frac{\partial^2 E_y(z,t)}{\partial z^2} - \mu\varepsilon\frac{\partial^2 E_y(z,t)}{\partial t^2} = 0 \\ \cdot\;\cancel{\cdot\;\cdot}\;\cdot\;\cdot\;\cancel{\cdot\;\cdot}\;\cdot\;\cdot\;\frac{\partial^2 E_z(z,t)}{\partial z^2} - \mu\varepsilon\frac{\partial^2 E_z(z,t)}{\partial t^2} = 0 \end{cases}$$

$$\Longrightarrow \frac{\partial^2\vec{E}(z,t)}{\partial z^2} - \mu\varepsilon\frac{\partial^2\vec{E}(z,t)}{\partial t^2} = 0$$

$\vec{E}^+\left(t - \frac{z}{v}\right)$, $\boxed{\frac{1}{v^2}\vec{E}^{+''}(z,t) = \mu\varepsilon\,\vec{E}^{+''}(z,t)}$ sse. $\frac{1}{v^2} = \mu\varepsilon \rightarrow v = \frac{1}{\sqrt{\mu\varepsilon}}$

Nello spazio vuoto

$\varepsilon = \varepsilon_0 = 8{,}854 \cdot 10^{-12}\,F/m$ $\quad \mu = \mu_0 = 4 \cdot \pi \cdot 10^{-7}\,H/m$

$$\Longrightarrow \boxed{v \simeq 3 \cdot 10^8\,m/s}$$

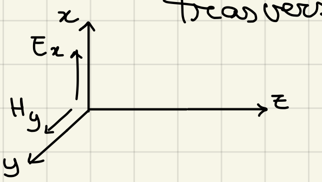
Proprietà dell'onda piana (dal ROTORE)

$$\vec{\nabla} \times \vec{F} = \left(\cancel{\frac{\partial F_z}{\partial y}} - \frac{\partial F_y}{\partial z}\right)\vec{u}_x + \left(\frac{\partial F_x}{\partial z} - \cancel{\frac{\partial F_z}{\partial x}}\right)\vec{u}_y + \left(\cancel{\frac{\partial F_y}{\partial x}} - \cancel{\frac{\partial F_x}{\partial y}}\right)\vec{u}_z$$

$$\vec{\nabla} \times \vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \qquad \vec{\nabla} \times \vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t}$$

$$\begin{cases} -\frac{\partial E_y}{\partial z} = -\mu\frac{\partial H_x}{\partial t} \\ \frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t} \\ 0 = -\mu\frac{\partial H_z}{\partial t} \end{cases} \qquad \begin{cases} -\frac{\partial H_y}{\partial z} = \varepsilon\frac{\partial E_x}{\partial t} \\ \frac{\partial H_x}{\partial z} = \varepsilon\frac{\partial E_y}{\partial t} \\ 0 = \varepsilon\frac{\partial E_z}{\partial t} \end{cases}$$

- E_z e H_z costanti (tempo) → ∄ E_z e H_z (ovvero l'onda piana uniforme non ammette componenti nella direzione perpendicolare al campo trasverso)
- $E_y \overset{\text{dipend.}}{\overline{\qquad}} H_x$ e $E_x \overset{\text{dipend.}}{\overline{\qquad}} H_y$



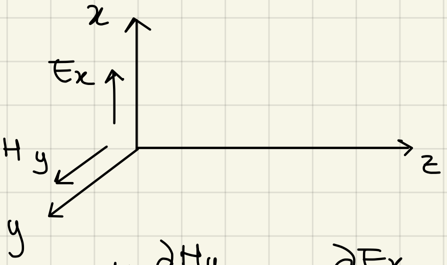
$$E_x = E_x^+\left(t-\frac{z}{v}\right) \rightarrow -\frac{1}{v}(E_x^+)' = -\mu\frac{\partial H_y}{\partial t}$$

$$H_y(z,t) = \frac{1}{v\mu}E_x^+\left(t-\frac{z}{v}\right) + \cancel{c} = H_y^+\left(t-\frac{z}{v}\right)$$

$$\frac{E_x^+(t-\frac{z}{v})}{H_y^+(t-\frac{z}{v})} = v\mu = \frac{\mu}{\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}$$

nel vuoto $\boxed{\sqrt{\frac{\mu}{\varepsilon}} = 377\,\Omega}$

Caso sinusoidale



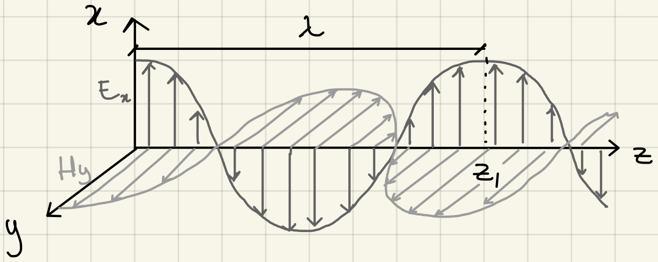
$$E_z^+(t,z) = A\cos\left[\omega\left(t-\frac{z}{c}\right)\right] =$$
$$= A\cos(\omega t-\beta z) \quad \text{con} \quad \beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\mu\frac{\partial H_y}{\partial t} = -\frac{\partial E_x}{\partial z} = A\,\text{sen}(\omega t-\beta z)(-\beta)$$

$$H_y^+(t,z) = \frac{A\beta}{\mu\omega}\cos(\omega t-\beta z) = \frac{A}{\eta_0}\cos(\omega t-\beta z)$$

$$\eta_0 = \frac{\mu\omega}{\beta} = \sqrt{\frac{\mu}{\varepsilon}} \qquad \frac{E_x^+}{H_y^+} = \eta_0 \; [\Omega]$$ impedenza intrinseca

siamo nel vuoto



$t = 0$

$$E_x^+(0,z) = A\cos(-\beta z)$$

$\beta z' = 2\pi \qquad \beta z_n = 2n\pi$

$n = 0 \qquad \beta z_0 = 0$
$n = 1 \qquad \beta z_1 = 2\pi$
. . . . . .

$$\lambda = z_n - z_{n-1} = \frac{2\pi}{\beta} = \frac{2\pi c}{\omega} = \frac{c}{f}$$

λ: lunghezza d'onda f: frequenza c: velocità dell'onda elettromagnetica

Vettore di Poynting

$$\vec{S}_{ist} = \vec{E}\times\vec{H} = A\cos(\omega t)\frac{A}{\eta_0}\cos(\omega t)\vec{u}_z = \frac{A^2}{\eta_0}\cos^2(\omega t)\vec{u}_z$$

$$\vec{S}_{med} = \frac{1}{\frac{2\pi}{\omega}} \int_0^{\frac{2\pi}{\omega}} \frac{A^2}{\eta_0}\cos^2(\omega t)\,dt \cdot \vec{u}_z$$

(z = 0 per semplicità di conti)

(da $t = 0$ a $t = T = \frac{1}{f} = \frac{2\pi}{\omega}$)   posto $\omega t = \xi$

$$\vec{S}_{med} = \frac{A^2 \cancel{\omega}}{2\pi\eta_0\,\omega} \underbrace{\int_0^{2\pi}\cos^2\xi\,d\xi}_{\pi} \cdot \vec{u}_z = \frac{A^2}{2\eta_0}\vec{u}_z \quad \left[\frac{W}{m^2}\right]$$

Si poteva ricavare lo stesso risultato osservando che:

$$\vec{S}_{(ist)} = \frac{A^2}{\eta_0}\left[\frac{1}{2} + \underbrace{\frac{1}{2}\cos 2\omega t}_{0 \text{ su un periodo di sinusoide}}\right]$$

componente in continua

Equazioni di Maxwell (fasori)

val. di picco $|E_i|$

$$E_i(x,y,z,t) = |E_i(x,y,z)|\cos(\omega t + \vartheta_{E_i}(x,y,z)) \qquad i = x, y, z$$

$|E_i(x,y,z)|$ e $|\vartheta_{E_i}(x,y,z)|$ sono rispettivamente modulo e fase di E_i

$$E_i(x,y,z,t) = Re[E_i\, e^{j\omega t}] = Re[|E_i|\, e^{j\vartheta_{E_i}}\, e^{j\omega t}]$$

$\vec{E} = Re[(E_x\vec{u}_x + E_y\vec{u}_y + E_z\vec{u}_z)\, e^{j\omega t}]$ → dipende da t

$\bar{E} = E_x\vec{u}_x + E_y\vec{u}_y + E_z\vec{u}_z$ → NON dipende da t (fasore)

$$\vec{E} = Re[\bar{E}\, e^{j\omega t}]$$
$$\vec{H} = Re[\bar{H}\, e^{j\omega t}]$$
$$\vec{D} = Re[\bar{D}\, e^{j\omega t}]$$
$$\vec{B} = Re[\bar{B}\, e^{j\omega t}]$$

$$\frac{\partial}{\partial t}(\bar{E}\, e^{j\omega t}) = j\omega \bar{E}\, e^{j\omega t}$$

$$\oint_C \bar{E}\cdot d\bar{l} = -j\omega\int_S \bar{B}\cdot d\bar{s} \qquad \bar{\nabla}\times\bar{E} = -j\omega\bar{B}$$

$$\oint_C \bar{H}\cdot d\bar{l} = \int_S \bar{J}\cdot d\bar{s} + j\omega\int_S \bar{D}\cdot d\bar{s} \qquad \bar{\nabla}\times\bar{H} = \bar{J} + j\omega\bar{D}$$

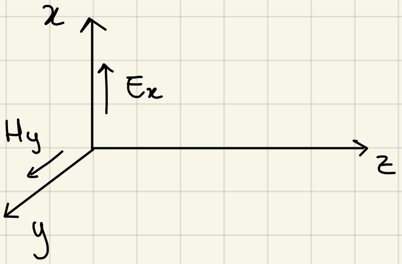
$$\oint_S \bar{D}\cdot d\bar{s} = \int_\Omega \rho_\Omega\, d\Omega \qquad \bar{\nabla}\cdot\bar{D} = \rho_\Omega$$

$$\oint_S \bar{B}\cdot d\bar{s} = 0 \qquad \bar{\nabla}\cdot\bar{B} = 0$$

$$\nabla^2\bar{E} = \mu\sigma(j\omega\bar{E}) + \mu\varepsilon(j\omega)^2\bar{E} = j\omega\mu(\sigma + j\omega\varepsilon)\bar{E} = \gamma^2\bar{E}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Onde piane uniformi (fasori)



$\bar{E}$ costante su xy

$\bar{E} = \bar{E}(z)$

$$\boxed{\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0}$$

$$\underbrace{\bar{E}}_{\text{vettore di fasori}} = \underbrace{E_x \vec{u}_x + E_y \vec{u}_y + E_z \vec{u}_z}_{\text{fasori}}$$

$$E_x = E_x(z) = E_x^+(0)\,e^{-\gamma z} + E_x^-(0)\,e^{+\gamma z}$$

$$E_x^+(0) = |E_x^+(0)|\,e^{j\varphi_0}$$

$$\frac{\partial^2 \bar{E}}{\partial z^2} - \gamma^2 \bar{E} = 0 \longrightarrow \bar{E}^+(0)\,e^{-\gamma z} + \bar{E}^-(0)\,e^{+\gamma z}$$

$$\bar{H} \longrightarrow \bar{\nabla} \times \bar{E} = -j\omega\mu \bar{H} = \frac{\partial E_x}{\partial z}\vec{u}_y - \frac{\partial E_y}{\partial z}\vec{u}_x$$

$$\boxed{H_z = 0} \quad e \quad \boxed{E_z = 0}$$

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z} \quad e \quad H_y = -\frac{1}{j\omega\mu}\frac{\partial E_x}{\partial z}$$

$$E_x(z) = E_x^+(0)\,e^{-\gamma z} + E_x^-(0)\,e^{+\gamma z}; \quad \text{il campo } \vec{H}\ ?$$

$$H_y(z) = -\frac{1}{j\omega\mu}\left[E_x^+(0)\,e^{-\gamma z}(-\gamma) + E_x^-(0)\,e^{+\gamma z}\cdot\gamma\right]$$

$$\longrightarrow H_y(z) = \frac{\gamma}{j\omega\mu}E_x^+(0)\,e^{-\gamma z} - \frac{\gamma}{j\omega\mu}E_x^-(0)\,e^{\gamma z}$$

$$\frac{E_x^+(z)}{H_y^+(z)} = \frac{j\omega\mu}{\gamma} = \eta \qquad \frac{E_x^-(z)}{H_y^-(z)} = -\frac{j\omega\mu}{\gamma} = -\eta$$

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon} = \alpha + j\beta$$

α: costante di attenuazione $\left[\frac{Np}{m}\right]$ — "Neper" adimensionale

β: costante di fase $\left[\frac{rad}{m}\right]$

γ: costante di propagazione $[m^{-1}]$

$$E_x^+(z) = E_x^+(0)\,e^{-\gamma z} = E_x^+(0)\,e^{-\alpha z}e^{-j\beta z} \quad \text{(FASORI)}$$

$$\downarrow$$

$$E_x^+(z,t) = Re\left\{E_x^+(0)\,e^{-\alpha z}\,e^{-j\beta z}\,e^{j\omega t}\right\} = \quad \text{(TEMPO)}$$

$$= |E_x^+(0)|\,e^{-\alpha z}\cos(\omega t - \beta z + \varphi_0) \quad \text{con } \varphi_0 = \measuredangle E_x^+(0)$$

$$\beta = \frac{2\pi}{\lambda} \qquad \boxed{\alpha > 0 \quad e \quad \beta > 0}$$ (altrimenti non hanno significato fisico)

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} \quad [\Omega]$$

→ se $\sigma \neq 0$ allora $\eta \in \mathbb{C}$

→ Mezzo ideale (senza perdite)

$\sigma = 0$ $\quad$ μ ed ε REALI

$\gamma^2 = -\omega^2\mu\varepsilon$ $\quad$ $\gamma = j\omega\sqrt{\mu\varepsilon} = j\beta \left[\frac{rad}{m}\right]$

$$\boxed{\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v} = \omega\sqrt{\mu\varepsilon}} \qquad \alpha = 0$$

$E_x^+(z) = E_x^+(0)\, e^{-j\beta z}$ $\quad$ $\eta = \sqrt{\frac{j\omega\mu}{(\cancel{\sigma} + j\omega\varepsilon)}} = \sqrt{\frac{\mu}{\varepsilon}}$ (η é REALE)

↳ l'onda ritarda ma non s'attenua

(stesso risultato ricavato nel dominio del tempo)

→ Buon conduttore

$\sigma > 0$ $\quad$ $\gamma^2 = -\omega^2\mu\varepsilon + j\omega\mu\sigma$ $\quad$ $\cancel{\omega}\cancel{\mu}\sigma \gg \omega^{\cancel{2}}\cancel{\mu}\varepsilon$

$\boxed{\sigma \gg \omega\varepsilon}$ affinché il conduttore sia buono

la condizione non dipende solo dal materiale, ma anche dalla frequenza della perturbazione elettromagnetica

$$↳ \gamma \simeq \sqrt{j\omega\mu\sigma} = \sqrt{j}\,\sqrt{\omega\mu\sigma}$$

$\pm\frac{1+j}{\sqrt{2}}$ α e β devono essere positivi

$$= \frac{1+j}{\sqrt{2}}\sqrt{\omega\mu\sigma} = (1+j)\sqrt{\pi f\mu\sigma} = \alpha + j\beta$$

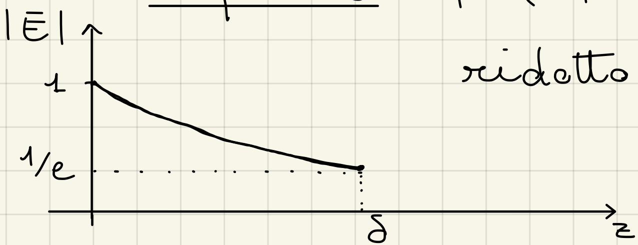
$(\omega = 2\pi f)$ $\qquad$ $\alpha = \beta = \sqrt{\pi f \mu\sigma}$

$$\eta = j\frac{\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j}\,\sqrt{\omega\mu\sigma}} = \frac{1+j}{\sqrt{2}}\sqrt{\frac{\omega\mu}{\sigma}} = (1+j)\sqrt{\frac{\pi f\mu}{\sigma}} = \frac{E_x^+(z)}{H_y^+(z)}$$

$\Longrightarrow$ campo elettrico e magnetico sono sfasati di 45°

Ampiezza

$|\bar{E}(z)| = |\bar{E}(0)|\, e^{-\alpha z}$



ridotto e^{-1} per $z = \frac{1}{\alpha} =$

$= \frac{1}{\sqrt{\pi f\mu\sigma}} = \delta$ spessore pelle

Lunghezza d'onda

$$\overset{\alpha}{\overset{\shortparallel}{\beta}} = \frac{2\pi}{\lambda} = \frac{1}{\delta} \Longrightarrow \boxed{\lambda = 2\pi\delta} \qquad \lambda = \frac{v}{f}$$

Se nel vuoto: In un buon conduttore

$$\lambda_0 = \frac{c}{f} \qquad \lambda \ll \lambda_0 \qquad \boxed{v = \frac{\omega}{\beta} = \omega\delta}$$

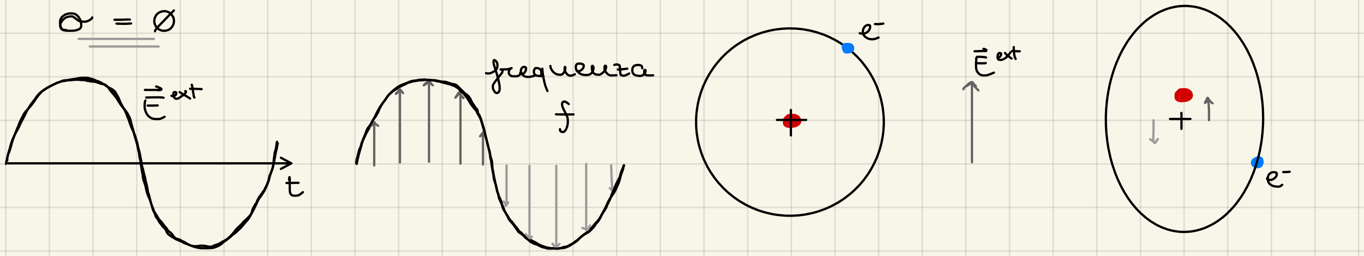
In un buon conduttore, la velocitá di propagazione dell'onda elettromagnetica é di molti ordini di grandezza inferiore che nel vuoto.

Se $\sigma \longrightarrow \infty$, $\delta \to 0$

Perdite nei dielettrici

come una molla

$\sigma = 0$



$$\varepsilon = \varepsilon' - \underset{\uparrow}{j}\,\varepsilon'' = \varepsilon_0\varepsilon_r \qquad \bar{D} = \varepsilon\bar{E} \qquad \varepsilon, \varepsilon_r \in \mathbb{C} \qquad \varepsilon', \varepsilon'' \in \mathbb{R}$$

RITARDO

Per tener conto dell'effetto "molla" che le onde elettromagnetiche provocano in materiali dielettrici, la costante dielettrica non é piú reale ma complessa

$$\gamma = \sqrt{-\omega^2\mu\varepsilon} = \sqrt{-\omega^2\mu\varepsilon' + j\omega^2\mu\varepsilon''} \qquad (\text{anche } \mu = \mu' - j\mu'')$$

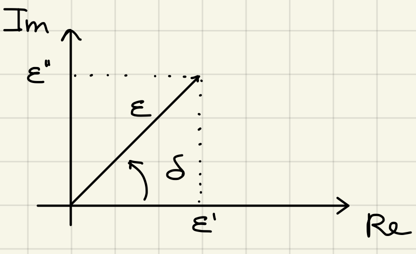
$$= \sqrt{-\omega^2\mu\varepsilon' + j\omega\mu(\omega\varepsilon'')} \quad \text{se} \quad \sigma = 0$$

$$\gamma = \sqrt{-\omega^2\mu\varepsilon + j\omega\mu\sigma} \quad \text{se} \quad \sigma \neq 0$$

⟹ La parte complessa della costante dielettrica si comporta come la costante di conducibilitá in un conduttore ($\omega\varepsilon'' = \sigma_{eq}$) tuttavia se $\omega \to 0$ $\sigma_{eq} \to 0$ mentre σ non varia

$\frac{\varepsilon''}{\varepsilon} = \tan\delta$ tangente di perdita

↖ non é lo spessore pelle



$\sigma > 0$ $\varepsilon = \varepsilon' - j\varepsilon''$ $\mu = \mu' - j\mu''$

$$\left.\begin{array}{l}\gamma = \sqrt{-\omega^2\mu\varepsilon + j\omega\mu\sigma} = \alpha + j\beta \\ \eta = j\dfrac{\omega\mu}{\gamma}\end{array}\right\}$$ sempre valide

Teorema di Poynting (fasori)

Nel dominio del tempo: $$\underset{\begin{cases}f_1 = 0 \\ f_2 = 2f_0\end{cases}}{\underset{\uparrow}{\vec{S}}} = \underset{f_0}{\underset{\uparrow}{\vec{E}}} \times \underset{f_0}{\underset{\uparrow}{\vec{H}}}$$

1) Nuova definizione di $\left[\bar{S} = \frac{1}{2}\bar{E}\times\bar{H}^*\right]$

Stesso approccio della potenza in elettrotecnica

fasori V I $P = \frac{1}{2}\bar{V}\cdot\bar{I}$

tempo $v(t)$ $i(t)$ $p(t) = v(t)\,i(t)$

$$P_{med} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}\mathrm{Re}\{\bar{V}\cdot\bar{I}\}$$

2) Dimostreremo che $\vec{S}_{med} = \frac{1}{T}\int_0^T \vec{S}(t)\,dt = \frac{1}{2}\mathrm{Re}\{\bar{E}\times\bar{H}^*\}$

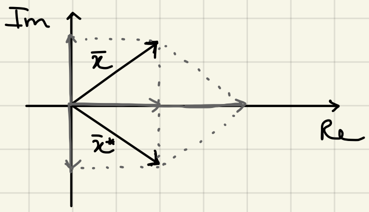
3) Calcoleremo il flusso di potenza trasportato da un'onda TEM piana in un mezzo generico

Nel dominio del tempo:

$$\vec{E} = \mathrm{Re}\{\bar{E}e^{j\omega t}\} = \frac{1}{2}(\bar{E}e^{j\omega t} + \bar{E}^* e^{-j\omega t})$$

$$\vec{H} = \mathrm{Re}\{\bar{H}e^{j\omega t}\} = \frac{1}{2}(\bar{H}e^{j\omega t} + \bar{H}^* e^{-j\omega t})$$

$$\vec{S} = \vec{E}\times\vec{H}$$



$$\mathrm{Re}\{\bar{x}\} = \frac{\bar{x}+\bar{x}^*}{2}$$

Nel dominio dei fasori:

$$\bar{S} = \frac{1}{4}(\bar{E}\times\bar{H}^* + \bar{E}^*\times\bar{H}) + \frac{1}{4}(\bar{E}\times\bar{H}e^{2j\omega t} + \bar{E}^*\times\bar{H}^*e^{-2j\omega t})$$

chiamiamo $\bar{A} = \bar{E} \times \bar{H}^*$ e $\bar{B} = \bar{E} \times \bar{H}$

$$\vec{S} = \frac{1}{4}\left(\bar{A} + \bar{A}^*\right) + \frac{1}{4}\left(\bar{B} e^{j2\omega t} + \bar{B}^* e^{-j2\omega t}\right) =$$

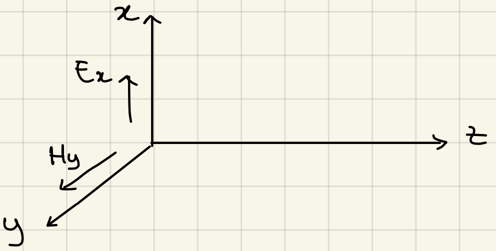
$$= \frac{1}{2} Re\left\{\bar{A}\right\} + \frac{1}{2} Re\left\{\bar{B} e^{j2\omega t}\right\} =$$

$$= \frac{1}{2} Re\left\{\bar{E} \times \bar{H}^*\right\} + \frac{1}{2} Re\left\{\left(\bar{E} \times \bar{H}\right) e^{-j2\omega t}\right\}$$

$\cos 2\omega t$, integrata sul periodo T è pari a $\equiv 0$

$$\vec{S}_{med} = \frac{1}{T}\int_0^T \vec{S}(t)\,dt = \frac{1}{2} Re\left\{\bar{E} \times \bar{H}^*\right\} \quad \text{c.v.d.} \quad ②$$

Onde piane: flusso di densità di potenza (fasori)



$$E_x(z) = E_x^+(0)\, e^{-\alpha z} e^{-j\beta z} + E_x^-(0)\, e^{+\alpha z} e^{+j\beta z}$$

$$H_y(z) = \frac{E_x^+(0)}{\eta} e^{-\alpha z} e^{-j\beta z} - \frac{E_x^-(0)}{\eta} e^{+\alpha z} e^{+j\beta z}$$

$$\vec{S}_m = \frac{1}{2} Re\left\{\bar{E} \times \bar{H}^*\right\} = \frac{1}{2} Re\left\{E_x(z) \cdot H_y^*(z)\right\} \vec{u}_z =$$

$$H_y^*(z) = \frac{E_x^{+*}(0)}{\eta^*} e^{-\alpha z} e^{+j\beta z} - \frac{E_x^{-*}(0)}{\eta^*} e^{+\alpha z} e^{-j\beta z}$$

③ $$= \frac{1}{2}\left[\frac{|E_x^+(0)|^2}{|\eta|} e^{-2\alpha z} \cos\varphi_\eta - \frac{|E_x^-(0)|^2}{|\eta|} e^{2\alpha z} \cos\varphi_\eta - \frac{2\,|E_x^+(0)|\,|E_y^-(0)|}{|\eta|} \text{sen}\left(2\beta z + \varphi_{E^-(0)} - \varphi_{E^+(0)}\right) \cdot \text{sen}\,\varphi_\eta\right] \vec{u}_z$$

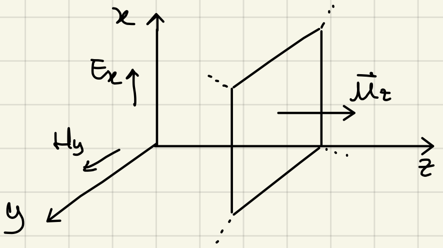
$\vec{S}_m^+ = \frac{1}{2}\frac{|E_x^+(0)|^2}{|\eta|} e^{-2\alpha z} \cos\varphi_\eta \, \vec{u}_z$ densità di potenza trasportata dall'onda progressiva

$\vec{S}_m^- = -\frac{1}{2}\frac{|E_x^-(0)|^2}{|\eta|} e^{+2\alpha z} \cos\varphi_\eta \, \vec{u}_z$ densità di potenza trasportata dall'onda regressiva

$\vec{S}_m^a = \frac{|E_x^+(0)|\,|E_y^-(0)|}{|\eta|} \text{sen}\left(2\beta z + \varphi_{E^-(0)} - \varphi_{E^+(0)}\right) \cdot \text{sen}\,\varphi_\eta \, \vec{u}_z$ termine di accoppiamento

$\hookrightarrow$ ∄ se c'è solo un'onda
∄ se il mezzo non ha perdite: η reale → $\varphi_\eta = 0$

Come era già stato detto, onda progressiva e regressiva di solito non si "parlano". Questo non è più vero in mezzi con perdite.



$$E_x(z) = E_x^+(0)\, e^{-\alpha z}\, e^{-j\beta z}$$

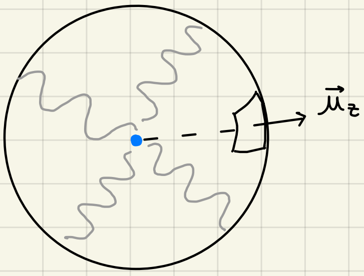
$$H_y(z) = \frac{E_x^+(0)}{\eta}\, e^{-\alpha z}\, e^{-j\beta z}$$

$\vec{S}_m^+ = \frac{1}{2} \mathrm{Re}\{ E_x \cdot H_y^* \} \vec{u}_z \left[\frac{W}{m^2} \right]^2$ densità di potenza

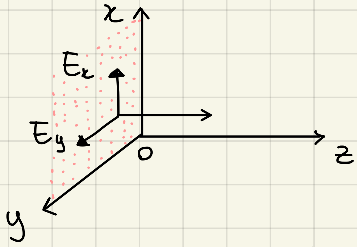
costante sul piano $z = \text{cost.}$

$\Longrightarrow$ Se l'onda piana uniforme esistesse davvero trasporterebbe potenza infinita! (l'onda è estesa su tutto il piano xy).
Perciò nella realtà fisica non esiste!

Tuttavia è comunque utile studiarla in quanto le onde sferiche sono approssimabili, su un elemento infinitesimo del loro fronte, a delle onde piane uniformi.



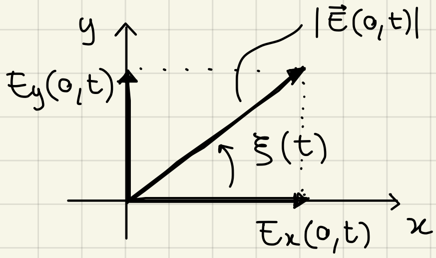
Polarizzazione (tempo)



$$\begin{cases} E_x(z,t) = E_x \cos(\omega t - \beta z) \\ E_y(z,t) = E_y \cos(\omega t - \beta z + \varphi_0) \end{cases}$$

Sul piano trasverso $(z = 0)$

$$\vec{E}(0,t) = E_x \cos \omega t\, \vec{u}_x + E_y \cos(\omega t + \varphi_0) \vec{u}_y$$



Due casi notevoli:

- $\varphi_0 = 0$ (o π) E_x, E_y qualsiasi ⟶ POLARIZZAZIONE LINEARE

$$\xi(t) = \text{arctg}\,\frac{E_y(0,t)}{E_x(0,t)} = \text{arctg}\,\frac{E_y \cancel{\cos\omega t}}{E_x \cancel{\cos\omega t}} = \xi_0 \text{ costante}$$

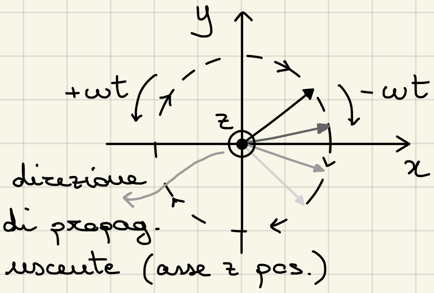
$$|\vec{E}(0,t)|^2 = E_x^2\cos^2\omega t + E_y^2\cos^2\omega t = (E_x^2 + E_y^2)\cos^2\omega t$$

↳ $|\vec{E}(z,t)|$ varia con il $\cos\omega t$

- $\varphi_0 = \pm\frac{\pi}{2}$ $\quad E_x = E_y = E \longrightarrow$ POLARIZZAZIONE CIRCOLARE

$$\xi(t) = \text{arctg}\,\frac{E_y\cos(\omega t \pm \pi/2)}{E_x\cos\omega t} = \text{arctg}\left(\frac{\mp \text{sen}\,\omega t}{\cos\omega t}\right) = \mp\omega t$$

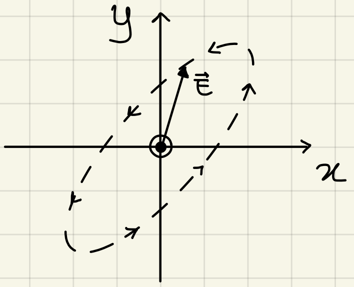
$$|\vec{E}(0,t)|^2 = E^2\cos^2\omega t + E^2\cos^2\left(\omega t \pm \frac{\pi}{2}\right) = E^2\cos^2\omega t + E^2\,\text{sen}^2\omega t = E^2$$



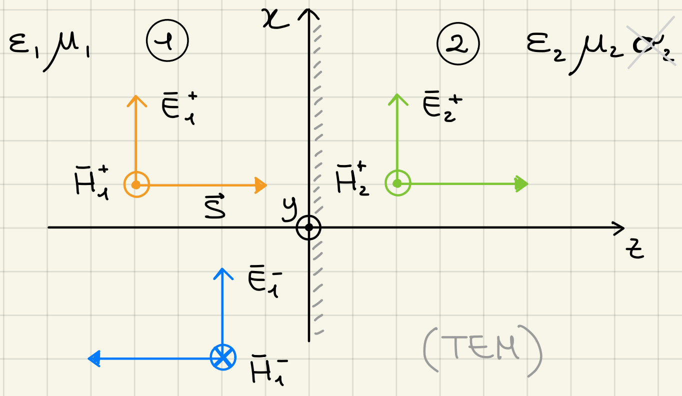
$\varphi_0 = \frac{\pi}{2}$ $\quad \xi(t) = -\omega t \longrightarrow$ CIRC. SINISTRA

$\varphi_0 = -\frac{\pi}{2}$ $\quad \xi(t) = \omega t \longrightarrow$ CIRC. DESTRA

Invece φ_0 ∀ e E_x, E_y ∀ ? ⟶ POLARIZZAZIONE ELLITTICA



Incidenza normale su discontinuità piana



Cond. al contorno $(z=0)$:

$$\begin{cases} E_{1t} = E_{2t} \\ H_{1t} = H_{2t} \end{cases}$$

Onda incidente (nota) ⟵ ci dobbiamo ricondurre a questa che è nota

$$\vec{E}_1^+(z) = E_1^+(0)\,e^{-\gamma_1 z}\,\vec{u}_x \quad \text{e} \quad \vec{H}_1^+(z) = \frac{E_1^+(0)}{\eta_1}\,e^{-\gamma_1 z}\,\vec{u}_y$$

Onda riflessa

$$\bar{E}_1^-(z) = E_1^-(0)\, e^{+\gamma_1 z}\, \vec{u}_x \quad e \quad \bar{H}_1^-(z) = -\frac{E_1^-(0)}{\eta_1}\, e^{+\gamma_1 z}\, \vec{u}_y$$

Onda trasmessa

$$\bar{E}_2^+(z) = E_2^+(0)\, e^{-\gamma_2 z}\, \vec{u}_x \quad e \quad \bar{H}_2^+ = \frac{E_2^+(0)}{\eta_2}\, e^{-\gamma_2 z}\, \vec{u}_y$$

conservazione dei campi (cond. al cont.)

$$\begin{cases} E_1^+(0) + E_1^-(0) = E_2^+(0) \\ H_1^+(0) + H_1^-(0) = H_2^+(0) \end{cases} \longrightarrow \begin{cases} E_1^+(0) + E_1^-(0) = E_2^+(0) \\ \dfrac{E_1^+(0)}{\eta_1} - \dfrac{E_1^-(0)}{\eta_1} = \dfrac{E_2^+(0)}{\eta_2} \end{cases} \longrightarrow$$

$$\longrightarrow \begin{cases} E_1^-(0) = E_1^+(0) \cdot \Gamma(0) \quad \text{con} \quad \boxed{\Gamma(0) = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \qquad |\Gamma(0)| \leq 1 \\ E_2^+(0) = E_1^+(0) \cdot T(0) \quad \text{con} \quad \boxed{T(0) = \dfrac{2\eta_2}{\eta_2 + \eta_1}} = 1 + \Gamma(0) \qquad |T(0)| \leq 2 \end{cases}$$

Γ COEFFICIENTE DI RIFLESSIONE

T COEFFICIENTE DI TRASMISSIONE

Campi totali nel mezzo ①

$$\bar{E}_1(z) = \bar{E}_1^+(z) + \bar{E}_1^-(z) = \left(E_1^+(0)\, e^{-\gamma_1 z} + E_1^-(0)\, e^{+\gamma_1 z}\right) \vec{u}_x =$$

$$= E_1^+(0)\left(e^{-\gamma_1 z} + \Gamma(0)\, e^{+\gamma_1 z}\right) \vec{u}_x$$

$$\bar{H}_1(z) = \bar{H}_1^+(z) + \bar{H}_1^-(z) =$$

$$= \frac{E_1^+(0)}{\eta_1}\left(e^{-\gamma_1 z} - \Gamma(0)\, e^{+\gamma_1 z}\right) \vec{u}_y$$

Campi totali nel mezzo ②

$$\bar{E}_2(z) = \bar{E}_2^+(z) = E_2^+(0)\, e^{-\gamma_2 z}\, \vec{u}_x = E_1^+(0) \cdot T(0)\, e^{-\gamma_2 z}\, \vec{u}_x$$

$$\bar{H}_2(z) = \bar{H}_2^+(z) = \frac{E_2^+(0)}{\eta_2}\, e^{-\gamma_2 z}\, \vec{u}_y = \frac{E_1^+(0)}{\eta_2} \cdot T(0)\, e^{-\gamma_2 z}\, \vec{u}_y$$

Ipotesi: mezzo ① e mezzo ② IDEALI $\begin{cases} \sigma = 0 \\ \varepsilon \text{ e } \mu \text{ REALI} \end{cases}$

$$\gamma_1 = j\beta_1 \qquad \gamma_2 = j\beta_2 \qquad (\alpha_1 = 0, \quad \alpha_2 = 0)$$

COEFFICIENTE DI RIFLESSIONE per qualsiasi z

$$\left[\Gamma(z) = \frac{E_1^+(z)}{E_2^+(z)} = \frac{E_1^-(0)\, e^{+j\beta_1 z}}{E_1^+(0)\, e^{-j\beta z}} = \Gamma(0)\, e^{2j\beta z}\right]$$

Definiamo IMPEDENZA D'ONDA (nella sezione z)

$$Z(z) = \frac{E_1(z)}{H_1(z)} = \frac{E_1^+(z) + E_1^-(z)}{H_1^+(z) + H_1^-(z)} = \eta_1 \cdot \frac{e^{-j\beta z} + \Gamma(0) e^{+j\beta z}}{e^{-j\beta z} - \Gamma(0) e^{+j\beta z}}$$

$$\left[Z(z) = \eta_1 \left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)}\right)\right]$$

(da non confondere con l' IMPEDENZA INTRINSECA

$$\eta_1 = \frac{E_1^+(z)}{H_1^+(z)} = -\frac{E_1^-(z)}{H_1^-(z)}$$)

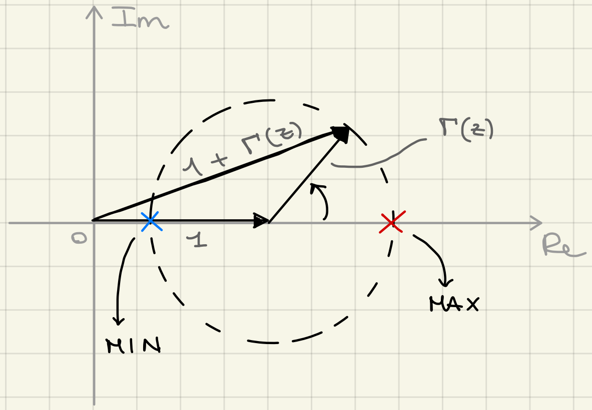
$$\Longrightarrow \left[E_1(z) = E_1^+(z)\left(1 + \underbrace{\Gamma(0)\, e^{+2j\beta z}}\right)\right]$$

costante ?

$$|E_1(z)| = |E_1^+(z)| \cdot |1 + \Gamma(z)| = |E_1^+(0)| \cdot |1 + \Gamma(z)|$$

se il mezzo è ideale (senza perdite)

Piano dei fasori



Distanza fasoriale fra due minimi (o due massimi):

$$2\beta \Delta z = 2\pi$$

$$2 \cdot \frac{2\pi}{\lambda} \cdot \Delta z = 2\pi \rightarrow \left[\Delta z = \frac{\lambda}{2}\right]$$

$$|E_1|_{MAX} = |E_1^+(0)| \cdot (1 + |\Gamma(0)|)$$

$$|E_1|_{MIN} = |E_1^+(0)| \cdot (1 - |\Gamma(0)|)$$

$$|H_1(z)| = \frac{|E_1^+(0)|}{\eta_1} \cdot |1 - \Gamma(0) e^{2j\beta z}|$$

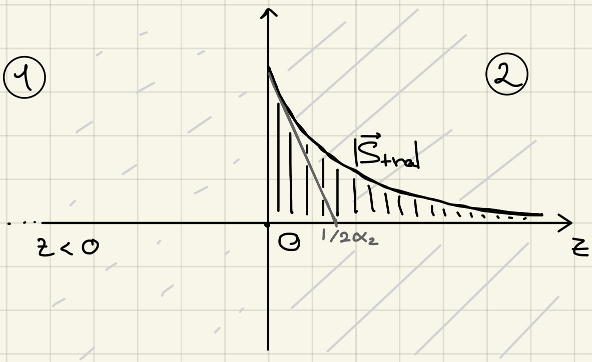
$$\vec{S}_m^+ = \vec{S}_{inc} = \frac{1}{2} \frac{|E_1^+(0)|^2}{\eta_1} \vec{u}_z \qquad \vec{S}_m^- = \vec{S}_{rif} = -\frac{1}{2} \frac{|E_1^-(0)|^2}{\eta_1} \vec{u}_z \quad \Big\} \text{ mezzo } ①$$

$$\vec{S}_{tra} = \frac{1}{2} \frac{|E_2^+(0)|^2}{\eta_2} \vec{u}_z = \frac{1}{2} \frac{|E_1^+(0)|^2 |T|^2}{\eta_2} \vec{u}_z \quad \Big\} \text{ mezzo } ②$$

$$\vec{S}_{rif} = -\frac{1}{2}\frac{|E_1^+(0)|^2}{\eta_1}|\Gamma(0)|^2\vec{u}_z = -\vec{S}_{inc}|\Gamma(0)|^2 \quad \Big\} \text{ mezzo ①}$$

$\Longrightarrow$ Risulta $\vec{S}_{inc} + \vec{S}_{rif} = \vec{S}_{tra} \longrightarrow \boxed{\vec{S}_{tra} = \vec{S}_{inc}(1-|\Gamma(0)|^2)}$

Perdite (mezzo ②) :



$$\begin{cases} \varepsilon_2 = \varepsilon_2' - j\varepsilon_2'' \\ \mu_2 = \mu_2' - j\mu_2'' \\ \sigma_2 \end{cases}$$

$$E_1^+(z) = E_1^+(0)e^{-\gamma_1 z} = E_1^+(0)e^{-\alpha_1 z}e^{-j\beta_1 z}$$

$$\eta_2 = \sqrt{\frac{j\omega(\mu_2'-j\mu_2'')}{\sigma_2 + j\omega(\varepsilon_2'-j\varepsilon_2'')}} \in \mathbb{C}$$

$$\gamma_2 = \sqrt{j\omega(\mu_2'-j\mu_2'')[\sigma_2 + j\omega(\varepsilon_2'-j\varepsilon_2'')]} = \alpha_2 + j\beta_2$$

$$\Gamma(0) = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \in \mathbb{C} \qquad T(0) = 1+\Gamma(0)$$

$$\begin{cases} \vec{E}_2(z) = E_1^+(0)\cdot T\cdot e^{-\alpha_2 z}e^{-j\beta_2 z}\vec{u}_x \\ \vec{H}_2(z) = \frac{E_1^+(0)}{\eta_2}\cdot T\, e^{-\alpha_2 z}e^{-j\beta_2 z}\vec{u}_y \end{cases}$$

$$\vec{S}_{tra} = \frac{1}{2}\mathrm{Re}\{\bar{E}\times\bar{H}^*\}\vec{u}_z = \text{nel mezzo ②}$$

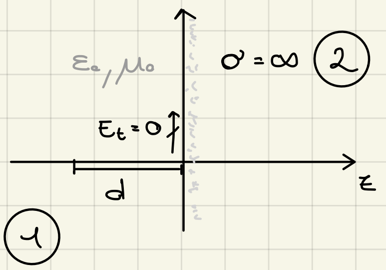
$$= \frac{1}{2}\mathrm{Re}\left\{T\cdot E_1^+(0)e^{-\alpha_2 z}e^{-j\beta_2 z}\cdot T^*\frac{E_1^+(0)^*}{\eta_2^*}e^{-\alpha z}e^{+j\beta_2 z}\right\}\vec{u}_z =$$

$$= \frac{1}{2}\frac{|T|^2}{|\eta_2|}\cdot|E_1^+(0)|^2e^{-2\alpha_2 z}\cos\varphi_\eta\,\vec{u}_z \qquad (\text{con } \varphi_\eta = \angle\eta_2)$$

$$\boxed{\vec{S}_{tra}(z) = S_{tra}(0)e^{-2\alpha_2 z}\vec{u}_z}$$

Mezzo ② : conduttore perfetto ($\sigma = \infty$)

$$\eta_2 = 0\,\Omega \qquad \boxed{\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = -1}$$



($\eta_1 = 377\,\Omega$ mezzo ① = vuoto)

$$T = 1 + \Gamma = 0 \longrightarrow \vec{S}_{tra} = 0 \qquad \vec{S}_{rif} = -\vec{S}_{inc}$$

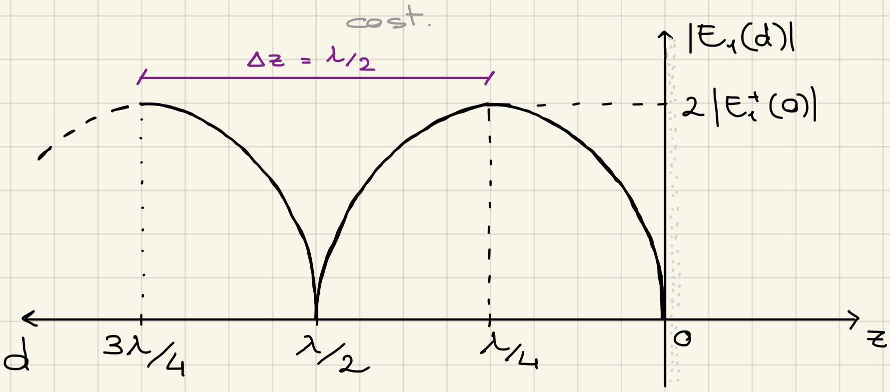
$$\begin{cases} E_1^+(z) = E_1^+(0)\, e^{-j\beta_1 z} \\ E_1^-(z) = -E_1^+(0)\, e^{+j\beta_1 z} \end{cases} \qquad E_1^-(z) = E_1^-(0)\, e^{+j\beta_1 z} = \Gamma(0)\, E_1^+(0)\, e^{+j\beta_1 z}$$

$$E_1(z) = E_1^+(z) + E_1^-(z) = E_1^+(0)\left(e^{-j\beta_1 z} - e^{+j\beta z}\right)$$

$\boxed{d = -z}$ distanza dall'interfaccia

$$E_1(d) = E_1^+(0)\cdot\left[e^{+j\beta_1 d} - e^{-j\beta_1 d}\right] = E_1^+(0)\cdot 2j\,\text{sen}(\beta d)$$

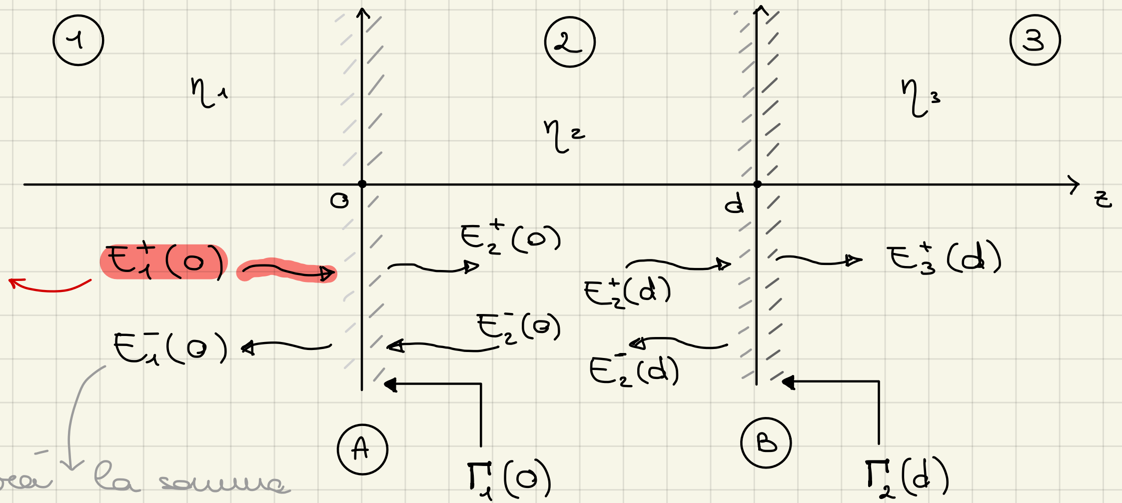
$$|E_1(d)| = 2\,\underbrace{|E_1^+(0)|}_{\text{cost.}}\,|\text{sen}(\beta d)|$$



ONDA PURAMENTE STAZIONARIA

MAX: $\beta d = \frac{\pi}{2} \qquad \frac{2\pi}{\lambda} d = \frac{\pi}{2} \qquad \boxed{d = \frac{\lambda}{4}} \qquad |E_1(d)| = 2\,|E_1^+(0)|$

Incidenza normale su multistrato (piana)



forzante (è ciò che conosco)

sarà la somma delle infinite onde riflesse e trasmesse tra le due interfacce

All' interfaccia Ⓑ : $(z = d)$

$$\begin{cases} E_2^-(d) = E_2^+(d)\,\Gamma_2(d) \\ E_3^+(d) = E_2^+(d)\,T_2(d) \end{cases} \quad \text{con} \quad \begin{cases} \Gamma_2(d) = \dfrac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \\ T_2(d) = 1 + \Gamma_2(d) = \dfrac{2\eta_3}{\eta_3 + \eta_2} \end{cases}$$

(é come se fosse il caso giá studiato di una superficie singola piana)

All' interfaccia Ⓐ : $(z = 0)$

$$\begin{cases} E_1^+(0) + E_1^-(0) = E_2^+(0) + E_2^-(0) \\ H_1^+(0) + H_1^-(0) = H_2^+(0) + H_2^-(0) \end{cases}$$

A dx : $E_2^+(0) = E_2^+(d)\,e^{\gamma_2 d} \leftarrow E_2^+(d) = E_2^+(0)\,e^{-\gamma_2 d}$

$(z = d)$ $E_2^-(0) = E_2^-(d)\,e^{-\gamma_2 d} = E_2^+(d)\,\Gamma_2(d)\,e^{-\gamma_2 d}$
$= E_2^+(0)\,e^{-\gamma_2 d}\,\Gamma_2(d)\,e^{-\gamma_2 d}$

A sx : $E_1^+(0)$, $E_1^-(0) = E_1^+(0)\,\Gamma_1(0)$ con $\Gamma_1(0) \neq \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

conservazione dei campi ←

$$\begin{cases} \overset{E_1^+}{E_1^+(0)}\left(1 + \overset{E_1^-}{\Gamma_1(0)}\right) = \overset{E_2^+}{E_2^+(0)} + \overset{E_2^-}{E_2^+(0)}\,\underbrace{e^{-\gamma_2 d}\,\Gamma_2(d)\,e^{-\gamma_2 d}}_{\Gamma_2(0)} \\ \overset{H_1^+}{\dfrac{E_1^+(0)}{\eta_1}}\left(1 - \overset{H_1^-}{\Gamma_1(0)}\right) = \overset{H_2^+}{\dfrac{E_2^+(0)}{\eta_2}}\left(1 - \overset{H_2^-}{\Gamma_2(0)}\right) \end{cases}$$

$$\rightarrow \eta_1 \frac{1 + \Gamma_1(0)}{1 - \Gamma_1(0)} = \underbrace{\frac{1 + \Gamma_2(0)}{1 - \Gamma_2(0)}\,\eta_2} \longrightarrow \boxed{\Gamma_1(0) = \frac{Z_{2,A} - \eta_1}{Z_{2,A} + \eta_1}}$$

$Z_{2,A}$ impedenza d'onda nel mezzo ② in $z = 0$

$$E_1^-(0) = E_1^+(0)\,\Gamma_1(0) \quad \text{e} \quad E_2^+(0) = E_1^+(0) \cdot \frac{1 + \Gamma_1(0)}{1 + \Gamma_2(0)}$$

Nel mezzo ① : $(z < 0)$

$$\begin{cases} E_1(z) = E_1^+(0)\,e^{-j\beta_1 z} + E_1^+(0)\,\Gamma_1(0)\,e^{+j\beta_1 z} \\ H_1(z) = \dfrac{E_1^+(0)}{\eta_1}\,e^{-j\beta_1 z} - \dfrac{E_1^+(0)}{\eta_1}\,\Gamma_1(0)\,e^{+j\beta_1 z} \end{cases}$$

Nel mezzo ② : $(0 < z < d)$

$$\begin{cases} E_2(z) = E_2^+(0)\, e^{-\gamma_2 z} + E_2^-(0)\, e^{+\gamma_2 z} \\ H_2(z) = \dfrac{E_2^+(0)}{\eta_2} e^{-\gamma_2 z} - \dfrac{E_2^-(0)}{\eta_2} e^{+\gamma_2 z} \end{cases}$$

$\eta = \frac{E^+}{H^+} = -\frac{E^-}{H^-}$ impedenza intrinseca

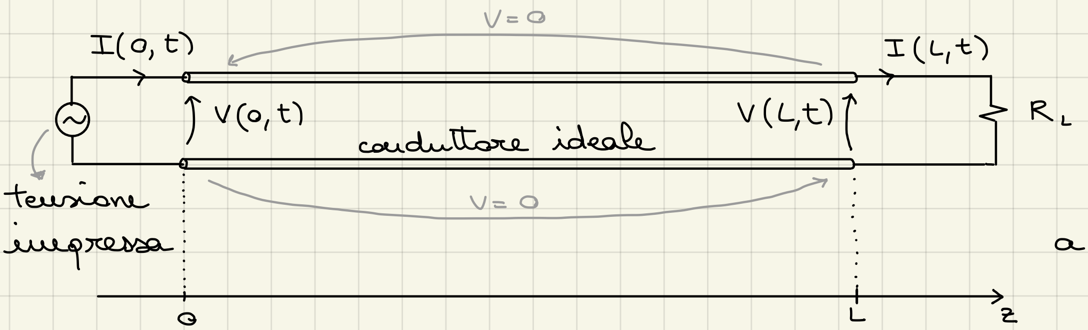
$Z = \frac{E}{H} = \frac{E^+ + E^-}{H^+ + H^-}$ impedenza d'onda

in particolare nel mezzo ③ : $\eta_3 = Z_{3,0}$

Nel mezzo ③ : $(z > d)$

$$\begin{cases} E_3(z) = E_3^+(d)\, e^{-\gamma_3 (z-d)} \\ H_3(z) = \dfrac{E_3^+(d)}{\eta_3} e^{-\gamma_3 (z-d)} \end{cases}$$

Linee di Trasmissione $\boxed{\text{TEM}}$



$$\begin{cases} V(0,t) \neq V(L,t) \\ I(0,t) \neq I(L,t) \end{cases}$$

se $L \geq \lambda$

a causa dell'induzione elettromagnetica

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \neq 0$$

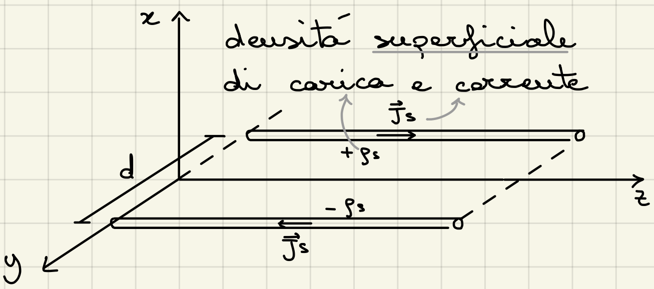
Come procederemo :

1) Dimostreremo che $\vec{H}$ e $\vec{E}$ sono "quasi-stazionari" sul piano trasverso $(\perp z)$ $\longrightarrow$ definizione di V e I

2) Utilizzando Maxwell otterremo due equazioni per V e I $\longrightarrow$ eq. delle onde

3) Analisi nel dominio del tempo

4) " " " dei fasori



Hp : conduttori ideali

dielettrico ideale

$\boxed{d \ll \lambda}$

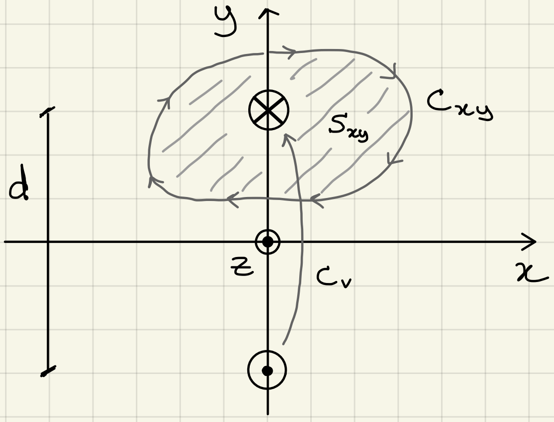
In statica e se ρ_s e $\vec{J}_s$ fossero costanti con z:

$\vec{E}$ e $\vec{H}$ ⊥ asse z $\longrightarrow$ ∄ E_z, H_z

(TEM)

ρ_s e $\vec{J}_s$ variano "lentamente" $\longrightarrow$ E_z, H_z sono trascurabili

(x le hp.)



$$\oint_{C_{xy}} \vec{E}\cdot d\vec{\ell} = -\frac{\partial}{\partial t}\int_{S_{xy}} \vec{B}\cdot d\vec{S}$$

diretto come $\vec{u}_z$

$$\oint_{C_{xy}} \vec{H}\cdot d\vec{\ell} = \frac{\partial}{\partial t}\int_{S_{xy}} \vec{D}\cdot d\vec{S} + \int_{S_{xy}} \vec{J}\cdot d\vec{S}$$

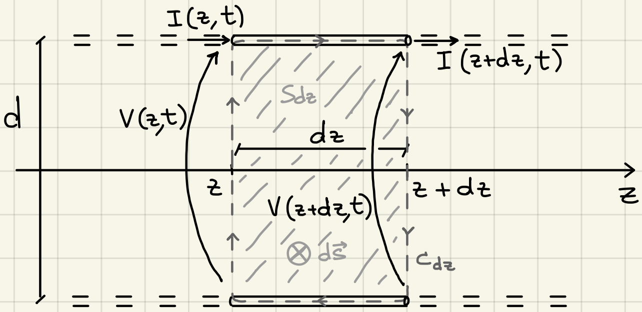
ma: $\vec{B}\cdot d\vec{S} = B_z \simeq 0$

$\vec{D}\cdot d\vec{S} = D_z \simeq 0$

$$\Longrightarrow \oint_{C_{xy}} \vec{E}\cdot d\vec{\ell} = 0 \qquad \boxed{\oint_{C_{xy}} \vec{H}\cdot d\vec{\ell} = \int_{S_{xy}} \vec{J}\cdot d\vec{S} = I(z,t)}$$

↓

$$\boxed{V(z,t) = -\int_{C_v} \vec{E}\cdot d\vec{\ell}}$$



dz "piccolo"

$$V(z+dz,t) \simeq V(z,t) + \frac{\partial V(z,t)}{\partial z}dz + \dots$$

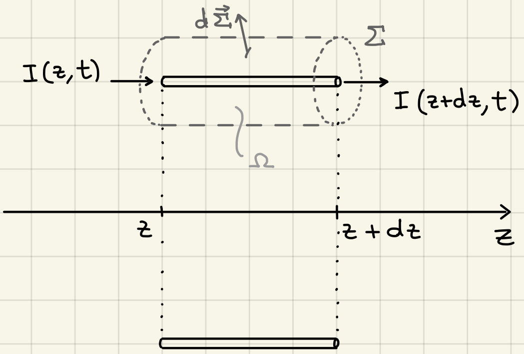
$$\oint_{C_{dz}} \vec{E}\cdot d\vec{\ell} = -V(z,t) + 0 + V(z+dz,t) + 0 =$$

$$= \frac{\partial V(z,t)}{\partial z}\cdot dz$$

ma $\oint_{C_{dz}} \vec{E}\cdot d\vec{\ell} = -\frac{\partial}{\partial t}\int_{S_{dz}} \vec{B}\cdot d\vec{S} = -\frac{\partial}{\partial t}\phi_M(z,t)$

ma $\phi_M(z,t) \simeq I(z,t)\,\mathcal{L}\,dz$ ($\mathcal{L}$: induttanza specifica del circuito $[\frac{H}{m}]$)

$$\longrightarrow \boxed{\frac{\partial V(z,t)}{\partial z} = -\mathcal{L}\frac{\partial}{\partial t}I(z,t)} \quad (1)$$



$$\oint_{\Sigma} \vec{J}\cdot d\vec{\Sigma} = -\frac{\partial}{\partial t}\int_{\Omega} \rho_\Omega\, d\Omega$$

$$I(z+dz,t) = I(z,t) + \frac{\partial I(z,t)}{\partial z}\cdot dz + \dots$$

$$\oint_{\Sigma} \vec{J}\cdot d\vec{\Sigma} = I(z+dz,t) - I(z,t) = \frac{\partial I(z,t)}{\partial z}\cdot dz = -\frac{\partial}{\partial t}\left[q(z,t)\,dz\right]$$

Ricordando che: $q(z,t) = \mathcal{C}\cdot V(z,t)$ carica per u.l. $\left[\frac{C}{m}\right]$

$$\longrightarrow \quad \boxed{\frac{\partial I(z,t)}{\partial z} = -\mathcal{C}\frac{\partial V(z,t)}{\partial t}} \ (2)$$

($\mathcal{C}$: capacitá specifica $\left[\frac{F}{m}\right]$)

Derivando (1) rispetto a z e (2) rispetto a t:

$$\frac{\partial^2 V(z,t)}{\partial z^2} = -\mathcal{L}\frac{\partial}{\partial z}\frac{\partial}{\partial t} I(z,t) \qquad \frac{\partial}{\partial t}\frac{\partial I(z,t)}{\partial z} = -\mathcal{C}\frac{\partial^2 V(z,t)}{\partial t^2}$$

Sostituendo:

$$\boxed{\frac{\partial^2 V(z,t)}{\partial z^2} = \mathcal{L}\mathcal{C}\frac{\partial^2 V(z,t)}{\partial t^2}} \qquad \boxed{\frac{\partial^2 I(z,t)}{\partial z^2} = \mathcal{L}\mathcal{C}\frac{\partial V(z,t)}{\partial t^2}}$$

⟶ Ricordano le equazioni delle onde di $\vec{E}$ e $\vec{H}$!

$$\begin{cases} V(z,t) = V^{+}\left(t-\frac{z}{v}\right) + V^{-}\left(t+\frac{z}{v}\right) \\ I(z,t) = I^{+}\left(t-\frac{z}{v}\right) + I^{-}\left(t+\frac{z}{v}\right) \end{cases} \quad \text{con} \quad v = \frac{1}{\sqrt{LC}} \left[\frac{m}{s}\right]$$

$$LC = \frac{1}{v^2} \longrightarrow \frac{H}{m}\cdot\frac{F}{m} = \frac{\cancel{V}\cdot s}{\cancel{A}}\,\frac{1}{m}\cdot\frac{\cancel{A}s}{\cancel{V}}\,\frac{1}{m} = \frac{s^2}{m^2} \ ✓$$

IMPEDENZA CARATTERISTICA Z_c

$$Z_c = \frac{V^{+}(z,t)}{I^{+}(z,t)} = -\frac{V^{-}(z,t)}{I^{-}(z,t)} \quad [\Omega]$$

$$\frac{\partial}{\partial z}\left[V^{+}\left(t-\frac{z}{v}\right) + V^{-}\left(t+\frac{z}{v}\right)\right] = -\mathcal{L}\frac{\partial}{\partial t}\left[I^{+}\left(t-\frac{z}{v}\right) + I^{-}\left(t+\frac{z}{v}\right)\right]$$

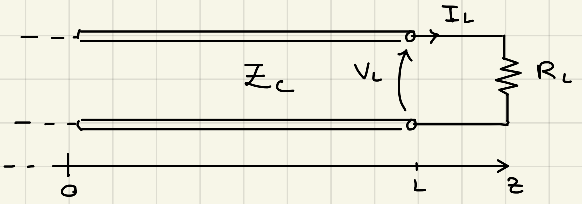
$$-\frac{1}{v}V^{+'}\left(t-\frac{z}{v}\right) + \frac{1}{v}V^{-'}\left(t+\frac{z}{v}\right) = \quad " \quad " \quad " \quad "$$

$$\frac{1}{v\mathcal{L}}V^{+}\left(t-\frac{z}{v}\right) - \frac{1}{v\mathcal{L}}V^{-}\left(t+\frac{z}{v}\right) = I^{+}\left(t-\frac{z}{v}\right) + I^{-}\left(t+\frac{z}{v}\right) + \cancel{c}$$

statica, non ci interessa

$$Z_c = \frac{V^{+}}{I^{+}} = -\frac{V^{-}}{I^{-}} = v\cdot\mathcal{L} = \frac{\mathcal{L}}{\sqrt{\mathcal{L}\mathcal{C}}} = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}}$$

(reale, V^{+} e I^{+} in fase)



$V_L = V(L,t)$

$I_L = I(L,t)$

$$\begin{cases} V(L,t) = V^+(L,t) + V^-(L,t) \\ I(L,t) = I^+(L,t) + I^-(L,t) \end{cases} \quad \underline{\text{ma}} \quad \boxed{\begin{aligned} &\frac{V^+}{I^+} = Z_c = -\frac{V^-}{I^-} \\ &V(L,t) = R_L \cdot I(L,t) \end{aligned}}$$

$$\longrightarrow \left.\begin{cases} V^+(L,t) + V^-(L,t) = Z_c\left(I^+(L,t) - I^-(L,t)\right) \\ V^+(L,t) + V^-(L,t) = R_L\left(I^+(L,t) + I^-(L,t)\right) \end{cases}\right\} \text{ simultaneamente valide}$$

- $R_L = Z_c \rightarrow \underbrace{I^-(L,t) = 0}_{\hookrightarrow \text{ NO onda riflessa}} \rightarrow \underbrace{V^-(L,t) = 0}_{\hookleftarrow}$ (carico adattato)

- $R_L \neq Z_c \rightarrow V^+(L,t) + V^-(L,t) = R_L\left[\dfrac{V^+(L,t)}{Z_c} - \dfrac{V^-(L,t)}{Z_c}\right]$

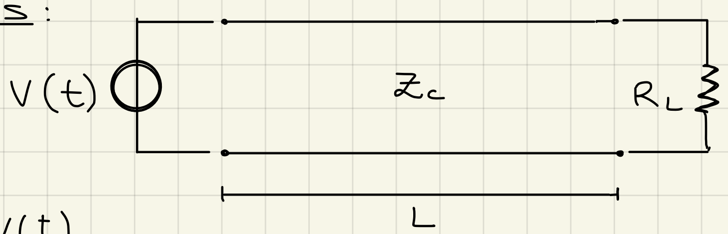
Si definisce COEFFICIENTE DI RIFLESSIONE:

$$\Gamma_L = \frac{V^-(L,t)}{V^+(L,t)}$$

$$\longrightarrow \quad V^+(L,t)\left(\frac{R_L}{Z_c} - 1\right) = V^-(L,t)\left(1 + \frac{R_L}{Z_c}\right)$$

$$\boxed{\Gamma_L = \frac{V^-(L,t)}{V^+(L,t)} = \frac{R_L - Z_c}{R_L + Z_c}} \qquad \left(\begin{aligned} &-1 \leq \Gamma_L \leq 1 \\ &\Gamma_L = 0 \text{ se } R_L = Z_c \end{aligned}\right)$$

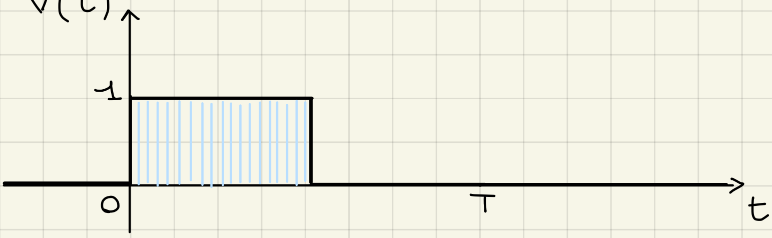
Es:



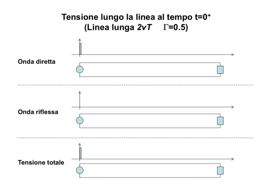
$Z_c = 50\,\Omega$

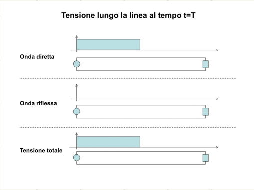
$R_L = 150\,\Omega$

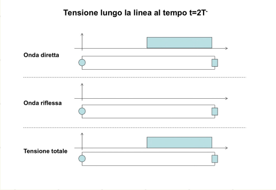
$L = 2v \cdot T$

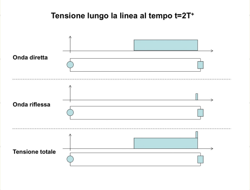


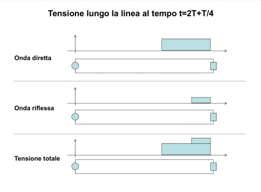
$$\Gamma_L = \frac{V^-(L,t)}{V^+(L,t)} = \frac{150 - 50}{150 + 50} = \frac{1}{2}$$

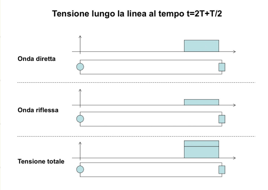

Tensione lungo la linea al tempo $t=0^+$
(Linea lunga $2vT$ $\Gamma=0.5$)

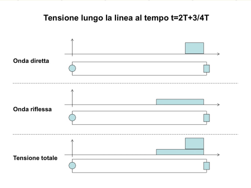

Tensione lungo la linea al tempo t=T

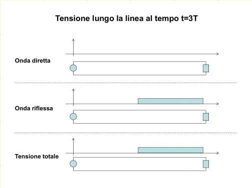

Tensione lungo la linea al tempo $t=2T^-$

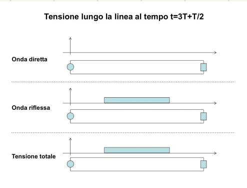

Tensione lungo la linea al tempo $t=2T^+$


Tensione lungo la linea al tempo t=2T+T/4

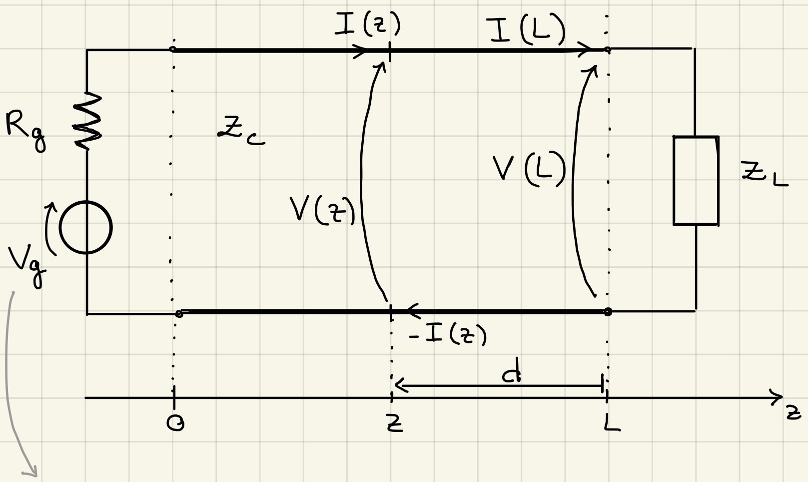

Tensione lungo la linea al tempo t=2T+T/2


Tensione lungo la linea al tempo t=2T+3/4T


Tensione lungo la linea al tempo t=3T


Tensione lungo la linea al tempo t=3T+T/2

Regime sinusoidale stazionario (fasori)



$$\frac{d^2V(z)}{dz^2} = -\underbrace{\omega^2 LC}_{\beta^2}\,V(z)$$

$$\frac{d^2I(z)}{dz^2} = -\overbrace{\omega^2 LC}\,I(z)$$

con $LC = \frac{1}{v^2}$ e $\frac{\omega^2}{v^2} = \beta^2$

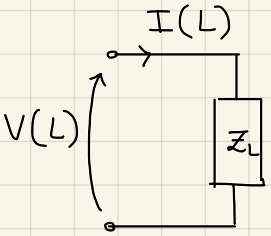
($\beta = \frac{2\pi}{\lambda}$ costante di fase)

fasore: $V_g(t) = |V_g|\cos(\omega t + \varphi) = \text{Re}\{\underbrace{|V_g|e^{j\varphi}}_{V_g}e^{j\omega t}\}$

$$\begin{cases} V(z) = V^+(0)e^{-j\beta z} + V^-(0)e^{+j\beta z} \\ I(z) = I^+(0)e^{-j\beta z} + I^-(0)e^{+j\beta z} \end{cases}$$ (fasori)

$$\begin{cases} V(z,t) = |V^+(0)|\cdot\cos(\omega t - \beta z + \varphi_{V^+}) + |V^-(0)|\cos(\omega t + \beta z + \varphi_{V^-}) \\ I(z,t) = |I^+(0)|\cos(\omega t - \beta z + \varphi_{I^+}) + |I^-(0)|\cos(\omega t + \beta z + \varphi_{I^-}) \end{cases}$$ (tempo)

Usare i fasori ci permette di studiare carichi diversi dalla sola resistenza, come induttori e capacità



$$\begin{cases} V(L) = Z_L\, I(L), \quad Z_L \in \mathbb{C} \\ \frac{V^+(L)}{I^+(L)} = -\frac{V^-(L)}{I^-(L)} = Z_c \in \mathbb{R} \end{cases}$$ di solito

$$V^+(L) + V^-(L) = Z_L[I^+(L) + I^-(L)]$$

e poi $V^+(L) + V^-(L) = \frac{Z_L}{Z_c}[V^+(L) - V^-(L)]$

Definiamo $\Gamma_L = \frac{V^-(L)}{V^+(L)} = \frac{Z_L - Z_c}{Z_L + Z_c}$ (complesso)

$|\Gamma_L| \leq 1$ (con carichi Z_L passivi)

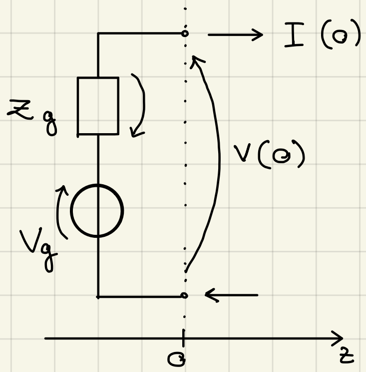
$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \frac{V^-(0)e^{+j\beta z}}{V^+(0)e^{-j\beta z}} = \Gamma(0)\,e^{2j\beta z} \underset{\uparrow}{=} \Gamma_L\, e^{-2j\beta(L-z)}$$

(essendo $\Gamma_L = \frac{V^-(L)}{V^+(L)} = \frac{V^-(0)}{V^+(0)}e^{j2\beta L}$)

→ $\Gamma(d) = \Gamma_L e^{-2j\beta d}$ $(d = L - z)$

Tensione (e corrente) lungo la linea:

$$V(z) = V^+(z) + V^-(z) = V^+(z) + V^+(z)\Gamma(z) = V^+(z)\left[1+\Gamma(z)\right]$$
$$= V^+(0)\, e^{-j\beta z}\left[1+\Gamma(z)\right]$$



$$\begin{cases} V(0) = V_g - Z_g I(0) \\ V(z) = V^+(0)\, e^{-j\beta z}\left[1 + \Gamma_L e^{-2j\beta(L-z)}\right] \\ I(z) = \underbrace{\frac{V^+(0)}{Z_c}}_{I^+(0)} e^{-j\beta z}\left[1 - \Gamma_L e^{-2j\beta(L-z)}\right] \end{cases}$$

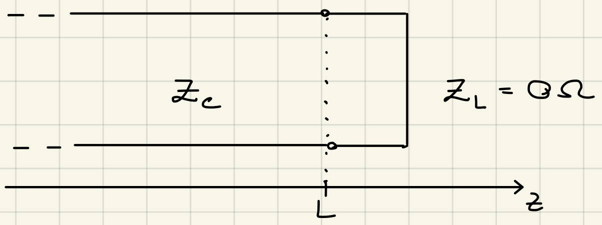
$$\Longrightarrow \quad V^+(0)\left(1 + \Gamma_L e^{-2j\beta L}\right) = V_g - \cancel{Z_g}\,\frac{V^+(0)}{\cancel{Z_c}}\left(1 - \Gamma_L e^{-2j\beta L}\right)$$

Hp: $Z_g = Z_c$ (generatore adattato)

$$\boxed{V^+(0) = \frac{V_g}{2}}$$

$$V(z) = \frac{V_g}{2} e^{-j\beta z}\left[1 + \Gamma_L e^{-2j\beta(L-z)}\right] =$$
$$= \underbrace{\frac{V_g}{2}}_{\text{ONDA DIRETTA}} e^{-j\beta z} + \underbrace{\left(\frac{V_g}{2}\Gamma_L e^{-2j\beta L}\right)}_{\text{ONDA RIFLESSA}} e^{+j\beta z}$$
$$= V^+(0)\, e^{-j\beta z} + V^-(0)\, e^{+j\beta z}$$

Inviluppo



Cortocircuito

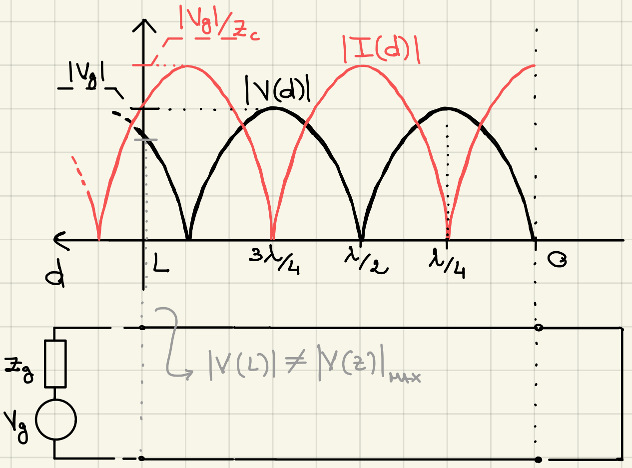
$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = -1$$

$$V(z) = \frac{V_g}{2} e^{-j\beta z}\left[1 - e^{-j2\beta(L-z)}\right]$$

$$V(z) = \frac{V_g}{2} e^{-j\beta z}\left[e^{j\beta(L-z)} - e^{-j\beta(L-z)}\right]$$

$$|V(z)| = \frac{|V_g|}{2} \cdot \left|j2\,\mathrm{sen}\left[\beta(L-z)\right]\right| = |V_g| \cdot \left|\mathrm{sen}\left[\beta(L-z)\right]\right|$$

→ $|V(d)| = |V_g| \cdot |\mathrm{sen}\, \beta d|$ in $z = L$ $(d = 0) \rightarrow |V(L)| = 0$
(come ci si aspetta da un cortocircuito)



$$MAX: \ \beta d_n = (2n+1)\frac{\pi}{2} = \frac{2\pi}{\lambda} d_n \rightarrow d_0 = \frac{\lambda}{4}$$

$$MIN: \ \beta d_n = n\pi \rightarrow d_1 = \frac{\lambda}{2}$$

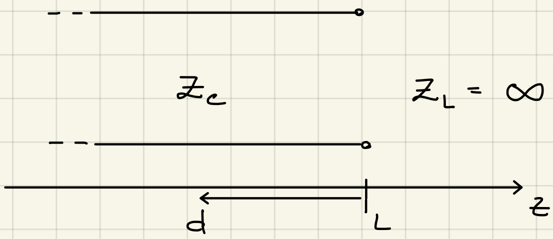
$$|I(z)| = \frac{|V_g|}{Z_c} \cdot |\cos[\beta(L-z)]|$$

→ $|I(d)| = \frac{|V_g|}{Z_c} |\cos \beta d|$

In $z = L \rightarrow I(L) = \frac{|V_g|}{Z_c}$

Circuito aperto

$$\Gamma_L = 1$$



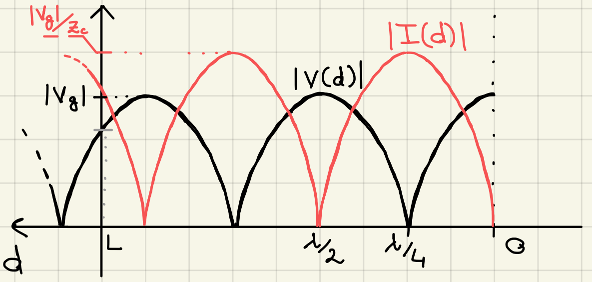
$$V(z) = \frac{V_g}{2} e^{-j\beta z}\left[1 + e^{-j2\beta(L-z)}\right] =$$

$$= \frac{V_g}{2} e^{-j\beta L}\left[e^{+j\beta(L-z)} + e^{-j\beta(L-z)}\right] =$$

$$= \frac{V_g}{2} e^{-j\beta L} \cdot 2\cos[\beta(L-z)]$$

$|V(z)| = |V_g| \cdot |\cos[\beta(L-z)]|$ → $|V(d)| = |V_g|\, |\cos \beta d|$

$$|I(d)| = \frac{|V_g|}{Z_c} |\mathrm{sen}\, \beta d|$$



$$\text{in } d = \frac{\lambda}{4}: \begin{cases} |V(d)| = 0 \\ |I(d)| = \frac{|V_g|}{Z_c} \end{cases}$$

Carico adattato

$Z_L = Z_c$ $\Gamma_L = 0$ $V(z) = \frac{V_g}{2} e^{-j\beta z}$ $|V(z)| = \frac{|V_g|}{2}$

tensione e corrente costanti

Carichi puramente reattivi

$Z_L = jX_L$ $\quad X_L = \omega L$ induttori

$X_L = \frac{1}{\omega C}$ capacità

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L - Z_c} = \frac{jX_L - Z_c}{jX_L + Z_c} \longrightarrow |\Gamma_L| = 1$$

→ onda puramente stazionaria

Rapporto di Onda Stazionaria (ROS)
(Standing Wave Ratio – SWR)

$$ROS = \frac{|V(d)|_{MAX}}{|V(d)|_{min}} = \frac{\cancel{|V^+(0)|}\,[\,1 + |\Gamma(z)|\,]}{\cancel{|V^+(0)|}\,[\,1 - |\Gamma(z)|\,]} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \geqslant 1$$

carico adattato: ROS = 1 $\qquad$ c.c., c.a. o reattivi: ROS = ∞

Rimuovo l'ipotesi precedente: $Z_g \neq Z_c$

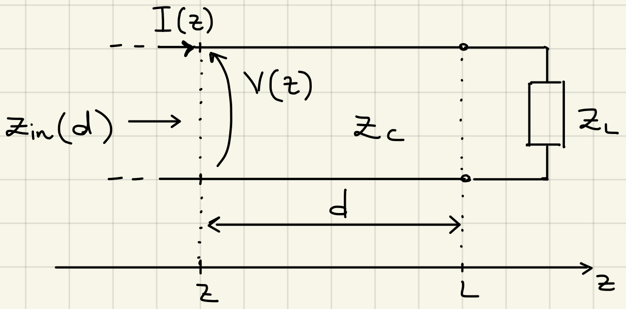
$$V^+(0)\left(1 + \Gamma_L e^{-2j\beta L}\right) = V_g - Z_g \cdot \frac{V^+(0)}{Z_c}\left(1 - \Gamma_L e^{-2j\beta L}\right)$$

$$\boxed{V^+(0) = \frac{V_g}{\left[1 + \Gamma_L e^{-2j\beta L} + \frac{Z_g}{Z_c}\left(1 - \Gamma_L e^{-2j\beta L}\right)\right]}}$$

$V^+(0)$ complessa (V_g e $V^+(0)$ non in fase)

$V^+(0)$ dipende dal carico Z_L (vs. Z_c)

Impedenza lungo la linea



Impedenza d'ingresso: $Z_{in}(d) = \frac{V(d)}{I(d)}$

$$Z_{in}(z) = \frac{V(z)}{I(z)} = \frac{V^+(0)e^{-j\beta z}(1 + \Gamma(z))}{I^+(0)e^{-j\beta z}(1 - \Gamma(z))} = $$

$$= Z_c \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

$$Z_{in}(z) = Z_c \frac{1 + \Gamma_L e^{-2j\beta(L-z)}}{1 - \Gamma_L e^{-2j\beta(L-z)}} \Longrightarrow Z_{in}(d) = Z_c \frac{1 + \Gamma_L e^{-2j\beta d}}{1 - \Gamma_L e^{-2j\beta d}}$$

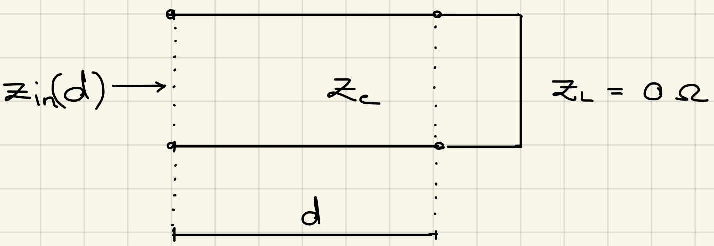
$$= Z_c \frac{Z_L + jZ_c \operatorname{tg}(\beta d)}{Z_c + jZ_L \operatorname{tg}(\beta d)}$$

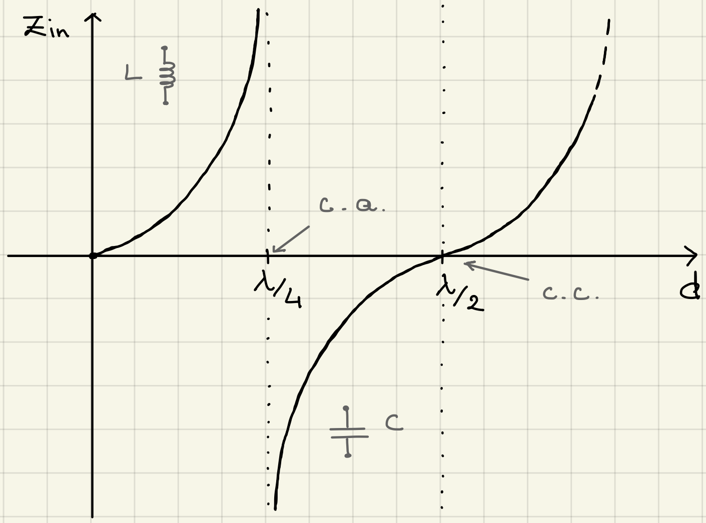
(sapendo che $e^{-j2\beta d} = \cos(2\beta d) - \operatorname{sen}(2\beta d)$)

Nel caso di un cortocircuito:

$$Z_{in}(d) = jZ_c tg(\beta d) = jZ_c tg\left(\frac{2\pi}{\lambda} d\right)$$

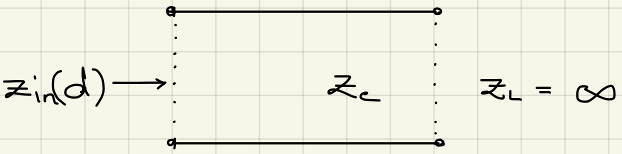
↑ é totalmente reattiva!

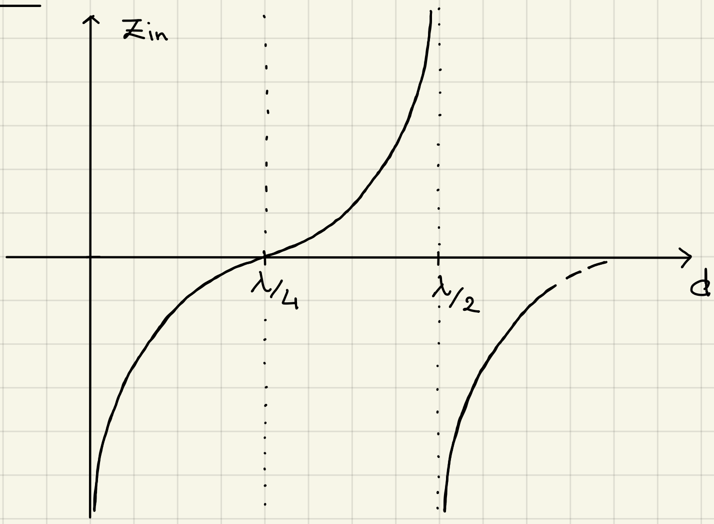




Nel caso di un circuito aperto:

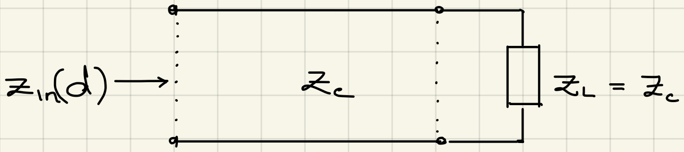
$$Z_{in}(d) = -j\frac{Z_c}{tg(\beta d)} =$$
$$= -jZ_c\, cotg(\beta d)$$





Nel caso di carico adattato:

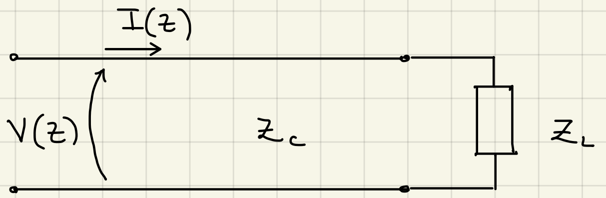
$\Gamma_L = 0$ $\quad Z_{in}(d) = Z_c$



Trasformatore $\frac{\lambda}{4}$ o Invertitore di impedenza

$$d = \frac{\lambda}{4} \qquad Z_L\ \forall \quad \longrightarrow \quad \boxed{Z_{in}(d) = \frac{Z_c^2}{Z_L}}$$

Flusso di potenza lungo la linea



Poiché conosco tensione e corrente di una linea TEM, posso usarli per calcolare la densità di potenza (invece che usare il teorema di Poynting)

$$P_m = \frac{1}{2}\,\text{Re}\left\{\bar{V}\cdot\bar{I}^*\right\}$$

Ricordando che:

$$V(z) = V^+(0)\,e^{-j\beta z}\left[1+\Gamma(z)\right] \quad e \quad I(z) = \frac{V^+(0)}{Z_c}\,e^{-j\beta z}\left[1-\Gamma(z)\right]$$

$$\longrightarrow \quad P_m(z) = \frac{1}{2}\,\text{Re}\left\{\frac{V^+(0)\cdot V^+(0)^*}{Z_c}\left[1+\Gamma(z)\right]\left[1-\Gamma(z)^*\right]\right\} =$$

$$= \frac{1}{2}\,\frac{|V^+(0)|^2}{Z_c}\,\text{Re}\left\{1 + \underbrace{\Gamma(z)+\Gamma(z)^*}_{\text{immag.}} - |\Gamma(z)|^2\right\} =$$

$$= \frac{1}{2}\,\frac{|V^+(0)|^2}{Z_c}\left(1-|\Gamma_L|^2\right) \quad (\text{essendo } |\Gamma(z)| = |\Gamma_L|)$$

$$P_m^+(z) = \frac{1}{2}\,\text{Re}\left\{V^+(z)\,I^+(z)^*\right\} = \frac{1}{2}\,\text{Re}\left\{V^+(0)\,e^{-j\beta z}\,\frac{V^+(0)^*}{Z_c}\,e^{+j\beta z}\right\}$$

$$= \frac{1}{2}\,\frac{|V^+(0)|^2}{Z_c}$$

$$P_m^-(z) = \frac{1}{2}\,\text{Re}\left\{V^-(z)\,I^-(z)^*\right\} = -\frac{1}{2}\,\frac{|V^+(0)|^2}{Z_c}\cdot|\Gamma_L|^2$$

$$\boxed{P_m(z) = P_m^+(z) + P_m^-(z)}$$

Con dei carichi reattivi (C, L, c.c., c.a.):

$$|\Gamma_L| = 1 \quad \longrightarrow \quad P_m(z) = 0$$

non posso trasferire potenza.

Carta di Smith

$$Z_{in}(d) = Z_c\,\frac{1+\Gamma(d)}{1-\Gamma(d)} \qquad \text{normalizzata } \bar{z}_{in} = \frac{Z_{in}}{Z_c}$$

assume $\bar{Z}_c = 1$ $\longrightarrow$ $\bar{Z}_{in}(d) = \dfrac{1+\Gamma(d)}{1-\Gamma(d)} = r + jx$

$$\Gamma(d) = p + jq$$

$$r + jx = \frac{1 + p + jq}{1 - p - jq}$$ r costante, x costante

→ $$\left(p - \frac{r}{r+1}\right)^2 + q^2 = \frac{1}{(r+1)^2}$$ (eq. parte reale (1))

→ $$(p-1)^2 + \left(q - \frac{1}{x}\right)^2 = \frac{1}{x^2}$$ (eq. parte immaginaria (2))

→ r costante → la (1) é una circonferenza di raggio $\frac{1}{r+1}$ e centro $\left(\frac{r}{r+1}, 0\right)$

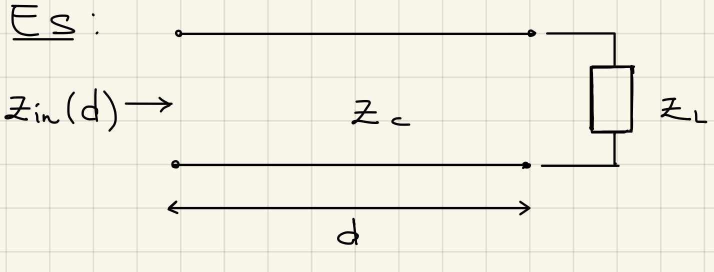
→ x costante → la (2) é una circonferenza di raggio $\frac{1}{x}$ e centro $\left(1, \frac{1}{x}\right)$

→ Casi notevoli:

$r = 0$ $\bar{z} = jx$ $|\Gamma| = 1$ → circonferenza esterna

$r = 1$ $x = 0$ → centro della carta (carico adattato)

Es:



$Z_c = 50\,\Omega$

$Z_L = 30 - j40\ \Omega$

$d = 0,165\lambda$ $Z_{in}(d) = ?$

1) Normalizzare a Z_c

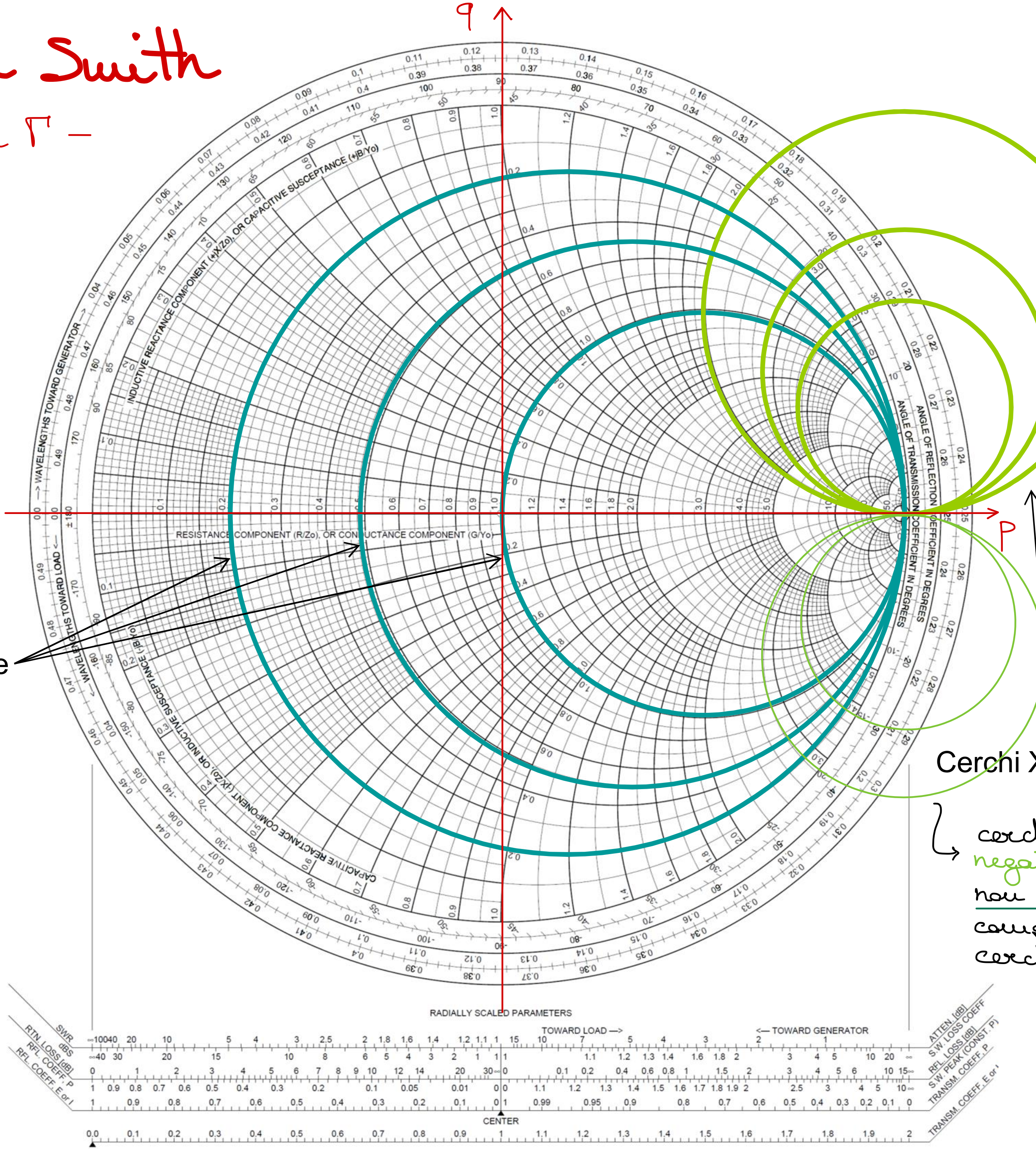
$$\bar{z}_L = \frac{Z_L}{Z_c} = \frac{30 - 40j}{50} = 0,6 - 0,8j$$

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = -j0,5$$

$$\Gamma(d) = \Gamma_L e^{-2j\beta d}$$ ruota in senso orario di $\underline{\underline{2\beta d}} = 2 \cdot \frac{2\pi}{\lambda} \cdot d = 0,66\pi$

Carta di Smith

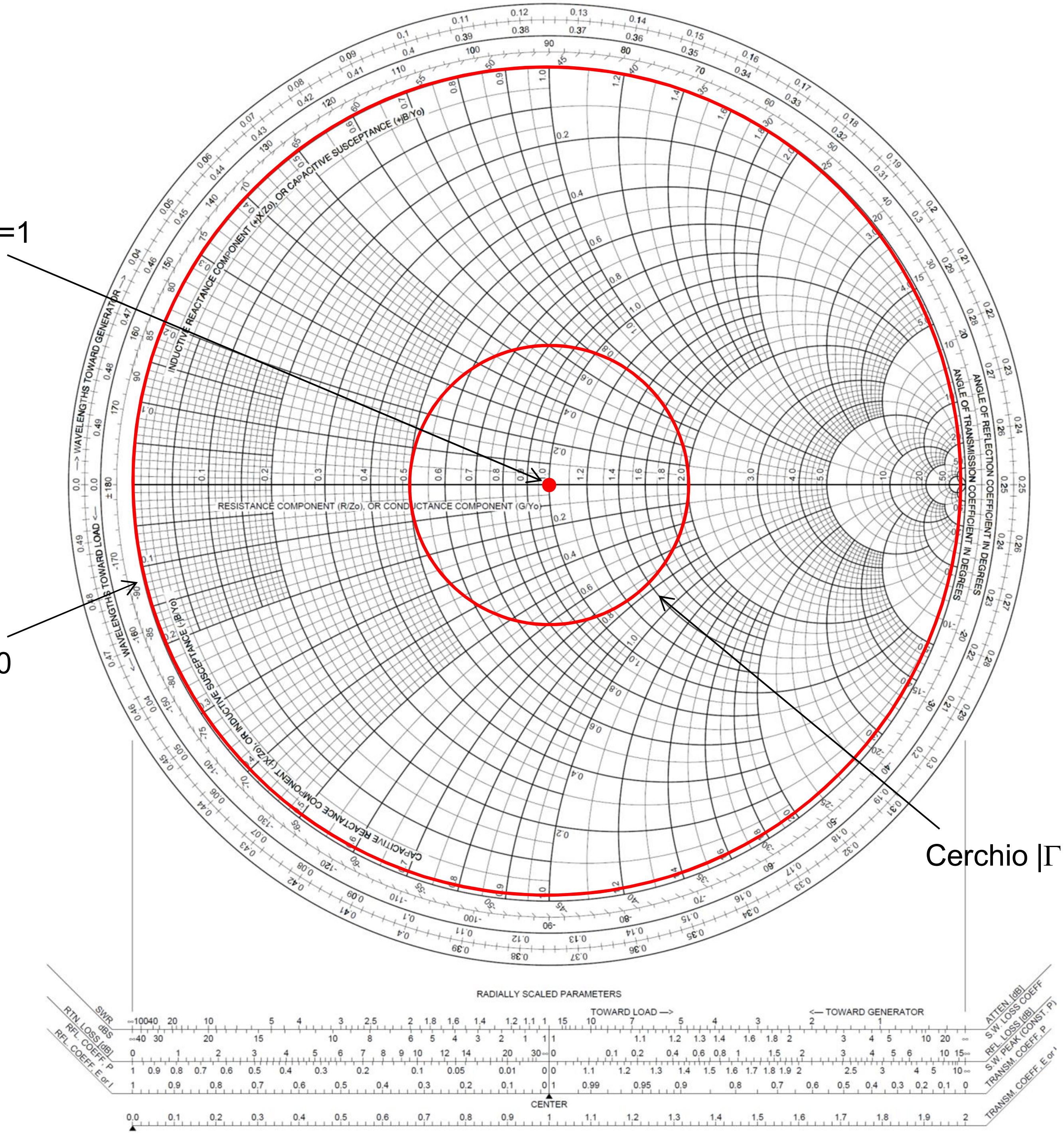
— Mappa di Γ —



Cerchi R costante

Cerchi X costante

↳ cerchi a reattanza negativa; invece non esistono (per componenti passive) cerchi a resistenza negativa



|Γ|=0   Z=1

Cerchio R=0
|Γ|=1

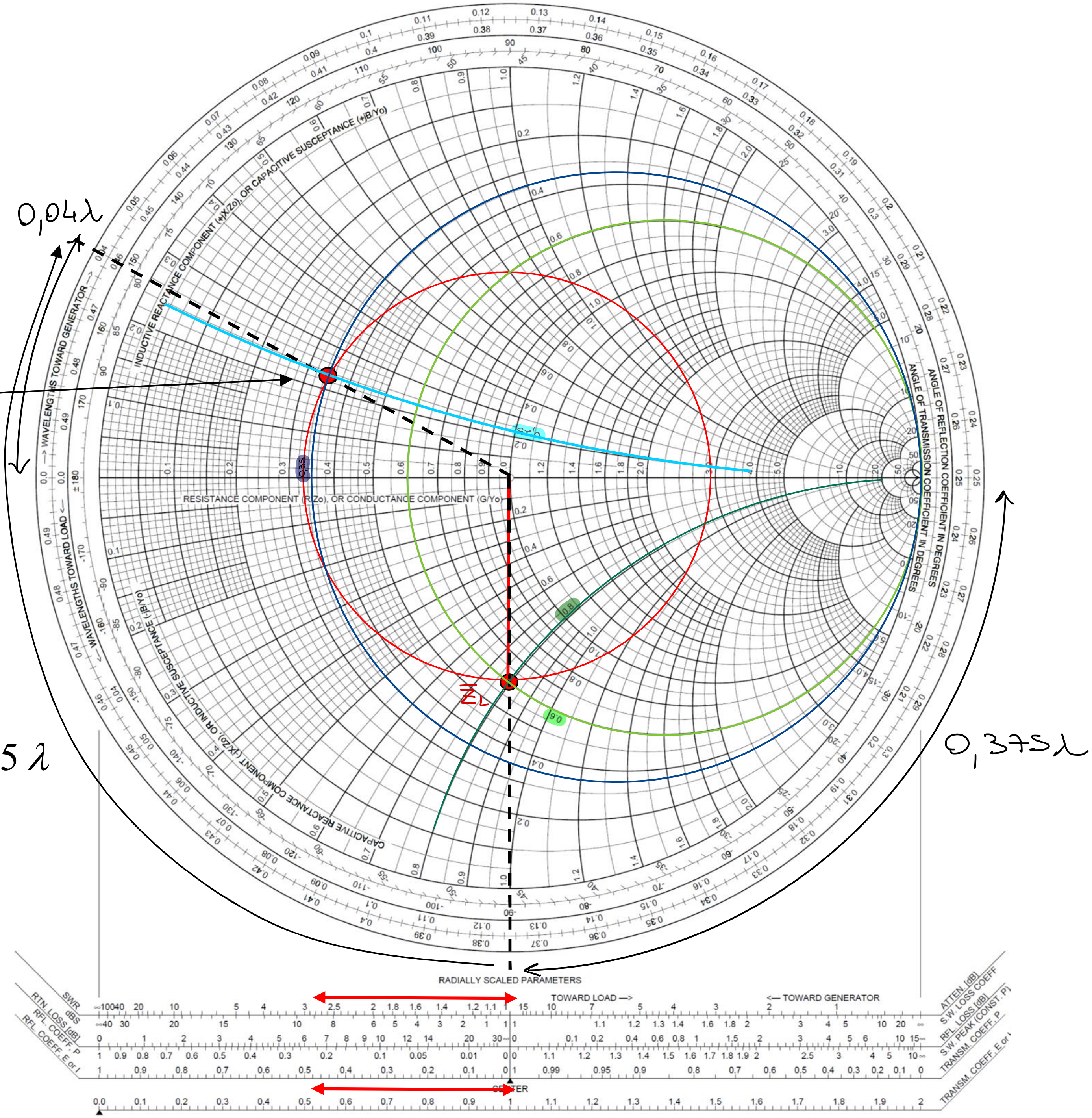
Cerchio |Γ| costante

$Z_c = 50\ \Omega$

$$Z_L = (30 - j40)\ \Omega$$

$$\overline{Z}_L = 0.6 - j0.8$$

$$\boxed{\overline{Z}_{IN}} = 0.35 + j0.23$$



$$d = 0.165\ \lambda$$

$$\downarrow$$

$$\sim \frac{2}{3}\pi$$

Partiamo da $0,375\lambda$

Finiamo in $0,375\lambda + \overset{d}{0,165\lambda} = 0,54\lambda \longrightarrow 0,04\lambda$

$\bar{Z}_{in} = 0,35 + j\,0,23 \implies Z_{in} = \bar{Z}_{in} \cdot Z_c = 17,6 + j\,11,3\ \Omega$

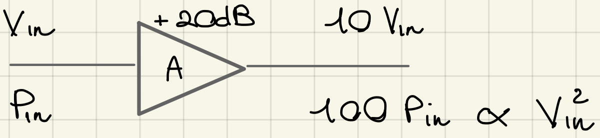
Approfondimento: $\boxed{\text{dB e Np}}$

Sono numeri puri - adimensionali - usati per esprimere rapporti.

Tensioni, correnti, campi: $20 \log_{10} \frac{V}{V_0}$ $\quad 20 \log_{10} \frac{E}{E_0}$

Potenza, densità di potenza: $10 \log_{10} \frac{P}{P_0}$ $\quad 10 \log_{10} \frac{S}{S_0}$

riferimento



$$\boxed{1\ \text{Np} \longleftrightarrow 8,686\ \text{dB}}$$

$$20 \log_{10} \frac{V}{V_0} = \alpha_{dB} \qquad \& \qquad \frac{V}{V_0} = e^{\alpha_{Np}}$$

$$\implies \frac{V}{V_0} = 10^{\frac{\alpha_{dB}}{20}} = e^{\alpha_{Np}} \longrightarrow \ln 10^{\frac{\alpha dB}{20}} = \alpha_{Np} = \frac{\alpha_{dB}}{20} \overset{2,302}{\ln 10}$$

$$\longrightarrow \alpha_{Np} = 8,686\,\alpha_{dB}$$

Potenza

$dB_W \qquad 30\,dB_W \longrightarrow 1000\,mW$

$dB_m \qquad 10\,dBm \longrightarrow 10\,mW$

$\qquad\quad -10\,dBm \longrightarrow 0,1\,mW$

Tensione

$dB_V \qquad 20\,dB_V \longrightarrow 10\,V$

$dB_{\mu V} \qquad 0\,dB_{\mu V} \longrightarrow 1\,\mu V \ (10^{-6}\,V)$

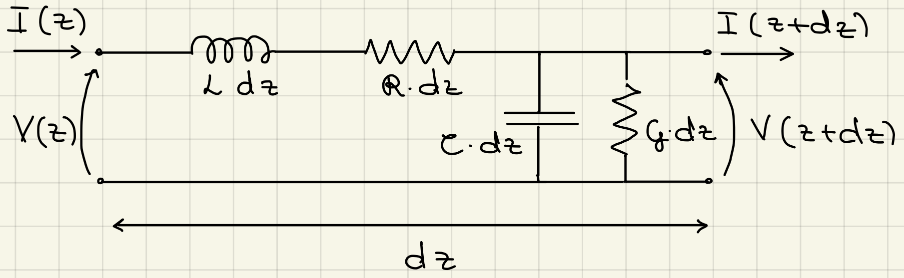
Perdite nelle linee

correnti

- Perdite nei conduttori ($\sigma \neq \infty$)
- Perdite nel dielettrico ($\varepsilon = \varepsilon' - j\varepsilon''$ o $\mu = \mu' - j\mu''$)

campo elettrico

Modello delle perdite:



L, R, C e G sono grandezze fisiche per unità di lunghezza

$$\begin{cases} V(z+dz) + I(z)\,(R + j\omega L)\,dz = V(z) \\ I(z) = V(z)\,(G + j\omega C)\,dz + I(z+dz) \end{cases}$$

chiamando $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$\begin{cases} V(z) = V^+(0)\,e^{-\gamma z} + V^-(0)\,e^{+\gamma z} \\ I(z) = I^+(0)\,e^{-\gamma z} + I^-(0)\,e^{+\gamma z} \end{cases} \qquad (\gamma = \alpha + j\beta)$$

$$Z_c = \frac{V^+(z)}{I^+(z)} = \frac{V^+(0)}{I^+(0)} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (\text{complessa})$$

Hp: piccole perdite $\longrightarrow$ $R \ll j\omega L$ e $G \ll j\omega C$

(buoni conduttori, buon dielettrico)

$$Z_c \simeq \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \qquad (\text{reale})$$

trascurabile

$$\gamma = \sqrt{-\omega^2 LC + j\omega RC + j\omega LG + \cancel{RG}}$$

$$\simeq \sqrt{-\omega^2 LC + j\omega(RC + LG)} = j\omega\sqrt{LC}\cdot\underbrace{\sqrt{1 - j\frac{(RC+LG)}{\omega LC}}}$$

$$\sqrt{1+x} \quad (\text{con } x \ll 1) \approx 1 + \frac{x}{2}$$

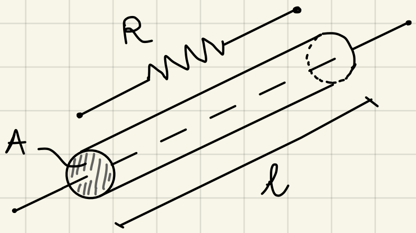
$$\simeq j\omega\sqrt{LC}\left(1 - j\frac{RC + LG}{2\omega LC}\right) = \underbrace{j\omega\sqrt{LC}}_{j\beta} + \underbrace{\frac{CR + LG}{2\sqrt{LC}}}_{\alpha}$$

$$j\beta \quad + \quad \alpha$$

$$\longrightarrow \begin{cases} \beta = \omega\sqrt{LC} \\ \alpha = \underbrace{\frac{R}{2Z_c}}_{\alpha_c} + \underbrace{\frac{G Z_c}{2}}_{\alpha_D} \end{cases} \quad \left(\text{dove } Z_c = \sqrt{\frac{L}{C}} \right)$$

attenuazione legata alle perdite nel conduttore ← α_c

α_D → attenuazione legata alle perdite nel dielettrico

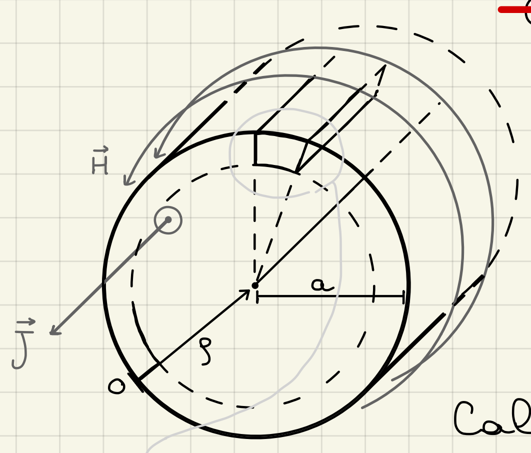


$\sigma = \frac{1}{r}$

In corrente continua ($f = 0$)

$$\boxed{R = \frac{l}{\sigma \cdot A}} \longleftarrow \text{non è applicabile se } f \neq 0$$

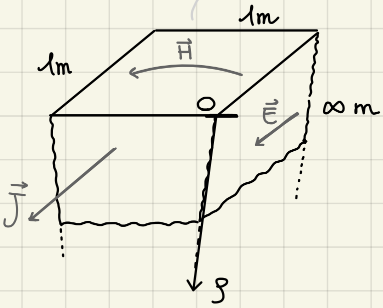
Effetto pelle



$$\boxed{\sigma \vec{E} = \vec{J}} \qquad \vec{J}(\rho) = \sigma \vec{E}(\rho)$$

$$E(\rho) = E(0)\, e^{-\gamma\rho} = E(0)\, e^{-\alpha\rho}\, e^{-j\beta\rho}$$

con $\alpha = \beta = \frac{1}{\delta}$ (nel metallo)



Calcoliamo la $P_{diss} = \frac{1}{2}|I|^2 R$ (blocco di $\boxed{1m \times 1m \times \infty m}$)

$$P_{diss} = \frac{1}{2}\int_\Omega \sigma |\vec{E}|^2 d\Omega = \frac{1}{2}\int_\Omega |\vec{J}| d\Omega \quad (\text{con } J(\rho) = J(0)e^{-\gamma\rho})$$

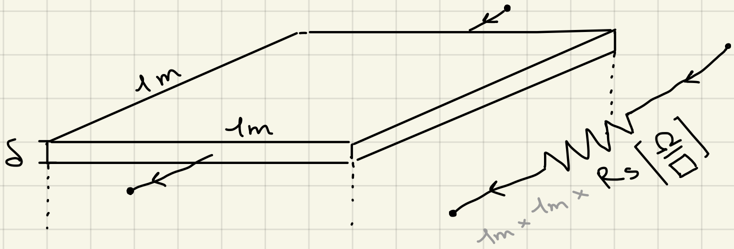
$$= \frac{1}{2\sigma}\int_0^{+\infty} |J(0)|^2 e^{-2\alpha\rho} d\rho = \frac{|J(0)|^2}{2\sigma}\int_0^{+\infty} e^{-2\alpha\rho} d\rho =$$

$$= \frac{|J(0)|^2}{4\alpha\sigma} = \frac{|J(0)|^2 \delta}{4\sigma}$$

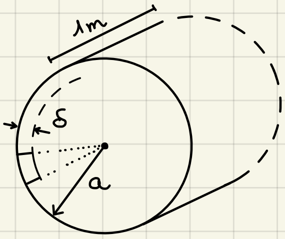
$$I = \int_\Sigma J(\rho)\, d\Sigma = \left[\int_0^{+\infty} J(\rho) d\rho\right] \cdot (1m) = \int_0^{+\infty} J(0)\, e^{-\gamma\rho} d\rho = \frac{J(0)}{\gamma} = \frac{J(0)\cdot\delta}{(1+j)}$$

$$\longrightarrow \boxed{R = \frac{2P_{diss}}{|I|^2} = \frac{\cancel{2}\, |J(\cancel{0})|^2 \cancel{\delta}}{\cancel{4}\sigma} \cdot \frac{\cancel{2}}{|J(\cancel{0})|^2 \delta^{\cancel{2}}} = \frac{1}{\sigma \cdot \delta}} \overset{\times 1m \times 1m}{[\Omega]}$$

R è anche la resistenza in corrente continua di un conduttore $\boxed{1m \times 1m \times \delta m}$



Resistenza superficiale (di un quadro) $\frac{\Omega}{\square}$



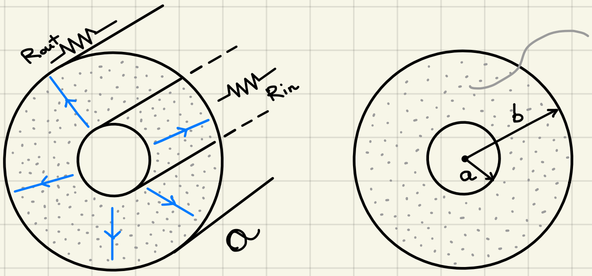
$\delta << a$ in un normale conduttore

p é il perimetro $(2\pi a)$

$$R = \frac{1}{\sigma \delta p} = R_s \cdot \frac{1}{p} \quad \left[\frac{\Omega}{m}\right]$$

Linee TEM - parametri fisici $(\mathcal{R}, \mathcal{L}, \mathcal{C}, \mathcal{G})$

Cavo coassiale



$\varepsilon = \varepsilon' - j\varepsilon''$ (piccole perdite $\varepsilon'' << \varepsilon'$)

$$\mathcal{C} = \frac{2\pi\varepsilon'}{\ln\left(\frac{b}{a}\right)} \qquad \mathcal{L} = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \sqrt{\frac{\omega \mu}{2\sigma}} = \frac{1}{\sigma \delta} \quad \left[\frac{\Omega}{\square}\right]$$

$$\mathcal{R} = \mathcal{R}_{out} + \mathcal{R}_{in} = \frac{\mathcal{R}_s}{2\pi b} + \frac{\mathcal{R}_s}{2\pi a} = \frac{R_s}{2\pi}\left(\frac{1}{b} + \frac{1}{a}\right)$$

Ricordando che $\frac{G}{C} = \frac{\sigma}{\varepsilon'} \longrightarrow \mathcal{G} = \mathcal{C} \frac{\omega\varepsilon''}{\varepsilon'} = \frac{2\pi\omega\varepsilon''}{\ln\left(\frac{b}{a}\right)}$

$$Z_c = \sqrt{\frac{L}{C}} = \frac{\eta}{2\pi} \log\left(\frac{b}{a}\right) \qquad \alpha_c = \frac{\mathcal{R}}{2 Z_c} \qquad \boxed{\alpha_D = \frac{\mathcal{G} Z_c}{2} = \frac{\pi}{\lambda} \cdot \frac{\varepsilon''}{\varepsilon'}}$$

vera x tutte le linee TEM

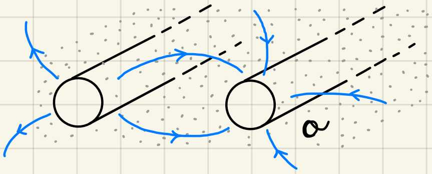
$\mathcal{L} \cdot \mathcal{C} = \mu \cdot \varepsilon$ per linee TEM

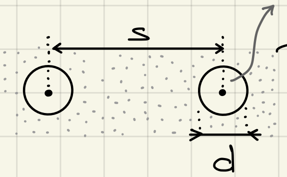
$$\mathcal{L} = \frac{\mu_0 \varepsilon'}{\mathcal{C}} = \frac{\mu_0 \cancel{\varepsilon'}}{2\pi \cancel{\varepsilon'}} \ln\left(\frac{b}{a}\right) = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$Z_c = \sqrt{\frac{\mu_0 \varepsilon'}{\mathcal{C}^2}} = \frac{\sqrt{\mu_0 \varepsilon'}}{\mathcal{C}}$$

Linea bifilare

l'interno dei conduttori é solitamente vuoto in quanto non parteciperebbe alla conduzione a causa dell'effetto pelle





$\varepsilon = \varepsilon' - j\varepsilon''$

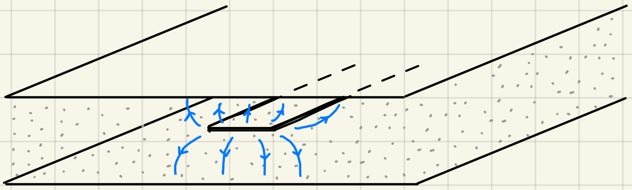
$$\mathcal{C} = \frac{\pi \varepsilon'}{\operatorname{arccosh}\left(\frac{s}{d}\right)} \qquad \mathcal{L} = \frac{\mu_0 \varepsilon'}{\mathcal{C}} = \frac{\mu_0}{\pi} \operatorname{arccosh}\left(\frac{s}{d}\right)$$

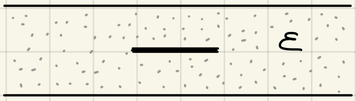
$$G = \frac{\pi \omega \varepsilon''}{\operatorname{arccosh}\left(\frac{s}{d}\right)} \qquad R = \frac{2R_s}{\pi d}\left[\frac{s/d}{\sqrt{(s/d)^2-1}}\right]$$

$$Z_c = \frac{\eta}{\pi}\operatorname{arccosh}\left(\frac{s}{d}\right) \qquad \alpha_c = \frac{R}{2Z_c} \qquad \alpha_D = \frac{\pi}{\lambda}\cdot\frac{\varepsilon''}{\varepsilon'} = \frac{G Z_c}{2}$$

La linea bifilare è poco utilizzata perché le sue linee di campo elettrico e magnetico non sono contenute e interagiscono con i corpi circostanti.

Stripline

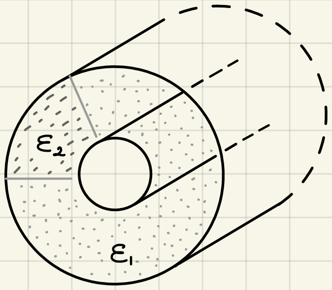




Formule dei parametri complicate, non esatte ma approssimanti

Linee quasi-TEM - parametri fisici

Realizzate con dielettrico non omogeneo



$v = \frac{1}{\sqrt{\mu\varepsilon}} = ?$ → il moto dell'onda non può essere TEM puro perché la v sarebbe diversa nei due dielettrici

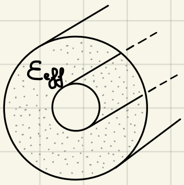
1) Calcolo C

2) Calcolo $L = L_0$ (nel vuoto $\varepsilon = \varepsilon_0$)

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L_0 C}}$$

non dipende dal dielettrico

$$\varepsilon_{eff} = \frac{LC}{\mu_0}$$



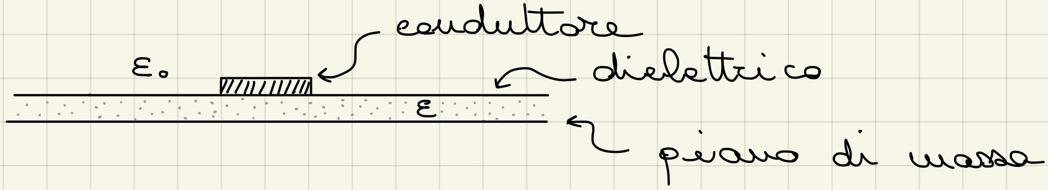
$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \varepsilon_{eff}}}$$

$LC \neq \mu\varepsilon$ 1 o 2?

ma

$LC = \mu\varepsilon_{eff}$

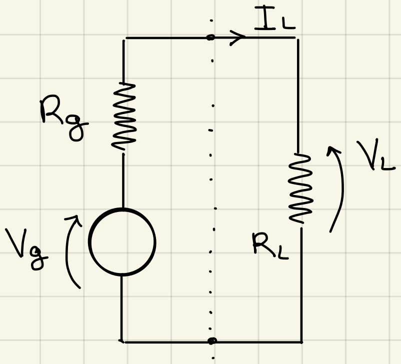
Microstriscia



Anche per la microstrip le formule dei parametri sono complicate e non esatte ma approssimanti.

Adattamento (di impedenza) → per evitare che ci sia onda riflessa

Trasferimento di potenza ad un carico



- Potenza massima al carico?
- In che condizioni si verifica?

$$P_L = \frac{1}{2}\,\mathrm{Re}\left\{V_L I_L^*\right\} = \frac{1}{2}\,|I_L|^2 R_L$$

$$I_L = \frac{V_g}{R_g+R_L} \longrightarrow P_L = \frac{1}{2}\,|V_g|^2 \frac{R_L}{(R_g+R_L)^2} \qquad \begin{cases} R_L \to 0 & P_L \to 0 \\ R_L \to \infty & P_L \to 0 \end{cases}$$

↓ ci deve essere almeno un massimo di P_L

Ricerca del massimo:

$$\frac{dP_L}{dR_L} = 0 \longrightarrow \frac{1}{(R_g+R_L)^2} - \frac{2R_L}{(R_g+R_L)^3} = 0 \longrightarrow \boxed{R_g = R_L}$$

adattamento

$$P_{L,MAX} = \frac{|V_g|^2}{8R_g} = P_D \text{ (potenza disponibile)}$$

↓ dipende solo dai parametri del generatore

Ricordando che: $P_m^+(z) = \frac{1}{2}\frac{|V^+(0)|^2}{Z_c}$ con $V^+(0) = V_g/2$ (se $R_g = Z_c$) carico adatt.

$$P_m^+(z) = \frac{|V_g|}{8R_g}$$

Quindi deve anche valere: $P_L = P_D(1-|\Gamma_L|^2)$. Verifichiamolo:

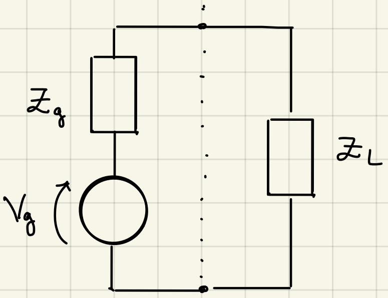
$$V_L = V_g\,\frac{R_L}{R_L+R_g} \qquad P_L = \frac{1}{2}\frac{|V_L|^2}{R_L} = \frac{1}{2}\,|V_g|^2 \frac{R_L^{\cancel{2}}}{(R_L+R_g)^2}\cdot\frac{1}{\cancel{R_L}} =$$

$$= \frac{1}{2}\,|V_g|^2 \frac{R_L}{(R_L+R_g)^2}$$

Linee di trasmiss.:

$$\Gamma_L = \frac{R_L-R_g}{R_L+R_g} \qquad P_D = \frac{|V_g|^2}{8R_g} \qquad P_L = P_D(1-|\Gamma_L|^2)$$

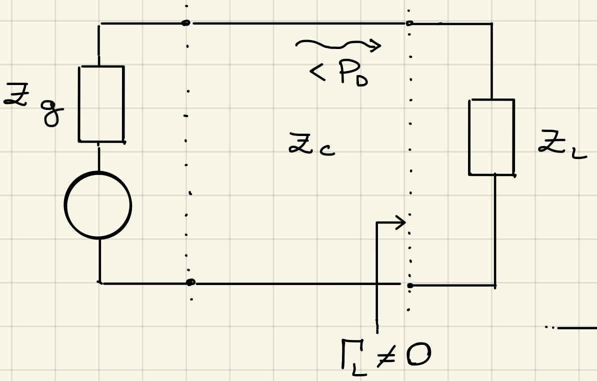
$$P_L = \frac{|V_g|^2}{8R_g}\left(1-\frac{(R_L-R_g)^2}{(R_L+R_g)^2}\right) = \frac{1}{2}\,|V_g|^2\frac{R_L}{(R_L+R_g)^2}$$



carico adattato $Z_L = Z_g^*$

(max. trasferimento di potenza)

$Z_g = R_g + j X_g$ $\quad P_D = \frac{|V_g|^2}{8 R_g}$ → la parte immag. non impatta sulla potenza disponibile

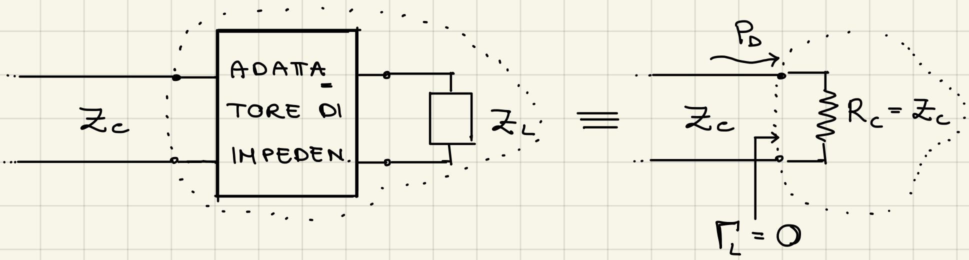


Z_c reale

Se $\boxed{Z_L \neq Z_c}$ $\longrightarrow$ $\Gamma_L \neq 0$ $\quad P_L < P_D$

(disadattamento)

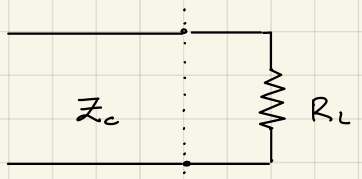
$\Downarrow$



Strutture adattanti:

- trasformatore $\lambda/4$ (solo carichi reali)
- trasformatore $\lambda/4$ con neutral. (solo carichi compl.)
- stub semplice
- doppio stub

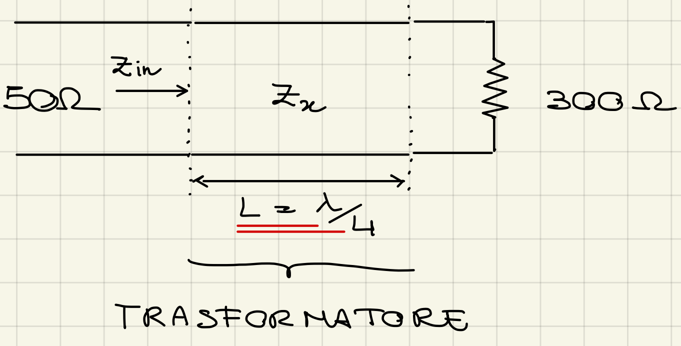
Trasformatore $\lambda/4$

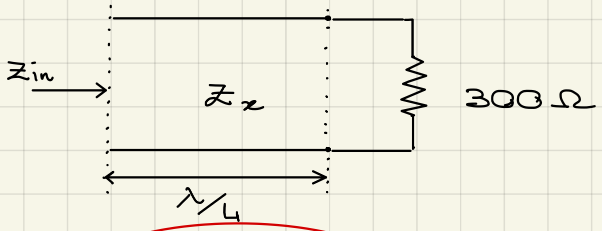


$Z_c = 50\Omega$

$R_L = 300\Omega$

Voglio che $Z_{in} = Z_L$ (carico adattato)

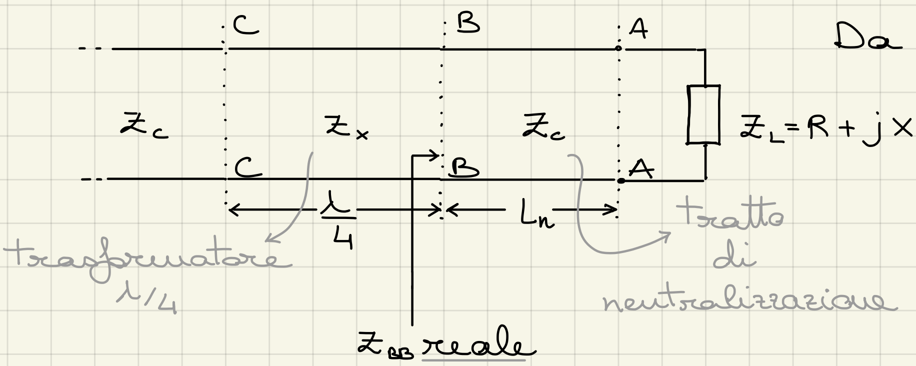




$$\boxed{Z_{in} = \frac{Z_x^2}{Z_L}}$$

$Z_x = \sqrt{300 \cdot 50} = 122{,}5\Omega$

Trasformatore $\lambda/4$ con neutralizzazione

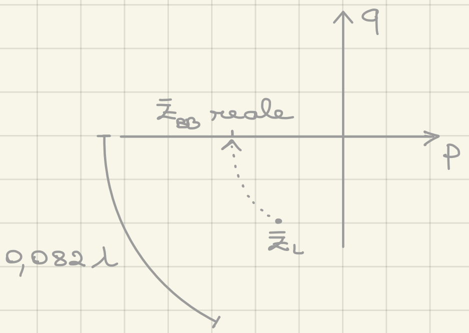


Da determinare: L_n e Z_x.

$Z_L = 30 - j20$ [Ω]

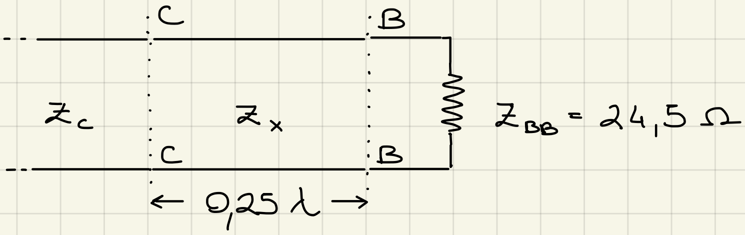
$Z_c = 50\,\Omega$

$$\bar{z}_L = \frac{Z_L}{Z_c} = \frac{30 - j20}{50} = 0,6 - j0,4$$



$\longrightarrow \bar{z}_{BB} = 0,49 \qquad L_n = 0,082\lambda$

$$Z_{BB} = \bar{Z}_{BB} \cdot Z_c = 24,5\Omega$$

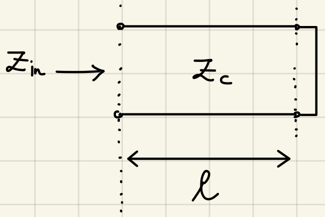


$\longrightarrow Z_x = \sqrt{Z_{BB} \cdot Z_c} = 35\Omega$

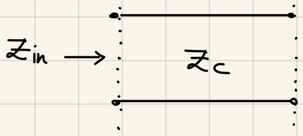
come un trasformatore $\frac{\lambda}{4}$

Stub singolo (semplice)

Stub: elemento reattivo realizzato con linea di trasmissione



$Z_{in} = jZ_c \operatorname{tg}\beta l$



$Z_{in} = \dfrac{Z_c}{j \operatorname{tg}\beta l}$

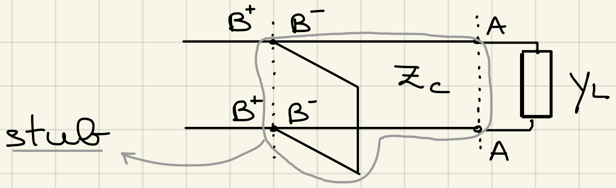
$\Big\}$ STUB

Gli stub sono collocati in un punto opportuno della linea di trasmissione in serie o in parallelo

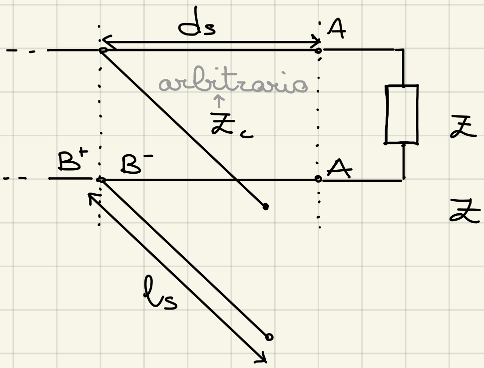
Stub parallelo (lavoriamo con le ammettenze)

posso usare la stessa carta di Smith

$$\bar{y}_L = \frac{Y_L}{Y_c} = \frac{Z_c}{Z_L}$$



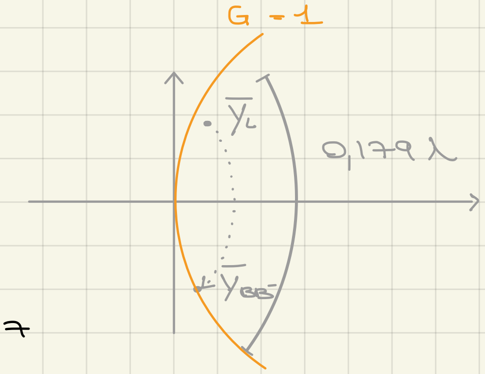
Con un tratto di linea si trasforma $\bar{y}_L = g_L + jb_L$ in $\underline{1} + jb = \bar{y}_{BB}$; connettiamo in parallelo $\bar{y}_s = -jb$; in $\overline{BB^+}$ $\boxed{\bar{y}_{BB^+} = 1}$ → adattamento



$Z_L = 30 + j20\,[\Omega]$

$Z_c = 50\,\Omega$

$\bar{y}_L = \frac{Z_c}{Z_L} = 1{,}15 + j\,0{,}77$

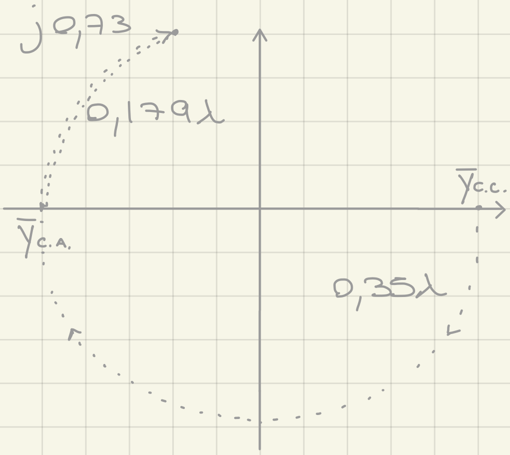


$\boxed{\text{stub parallelo in circuito aperto}}$

$\bar{y}_s = +j\,0{,}73$

$\bar{y}_{BB^-} = \underline{1} - \underline{j\,0{,}73}$ ← deve annullare

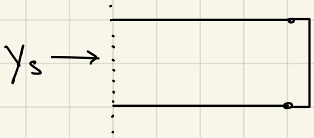
$d_s = 0{,}179\lambda$



Stub circuito aperto

C.A. $\bar{y}_{C.A} = 0\,\Omega^{-1}$ $l_s = 0{,}1\lambda$

Stub corto circuito



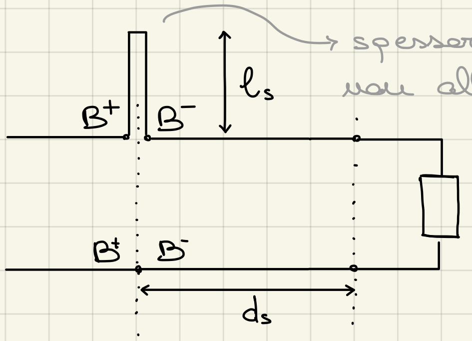
C.C. $\bar{y}_{c.c.} = \infty\,\Omega^{-1}$ $l_s = 0{,}35\lambda$

Stub serie

a) Con un tratto di linea si trasforma Z_L in $\bar{z}_{BB^-} = 1 + jx$

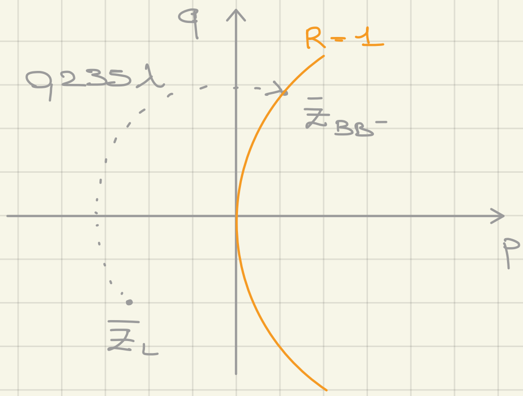
b) Si introduce in serie uno stub $x_s = -jx$

c) $\bar{z}_{BB^+} = 1$ → adattamento



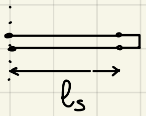
spessore infinitesimo, non allunga la linea

$\bar{Z}_L = 0,6 - j0,4$

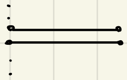


$\bar{Z}_{BB^-} = 1 + 0,73j$ $\qquad X_s = -j0,73$

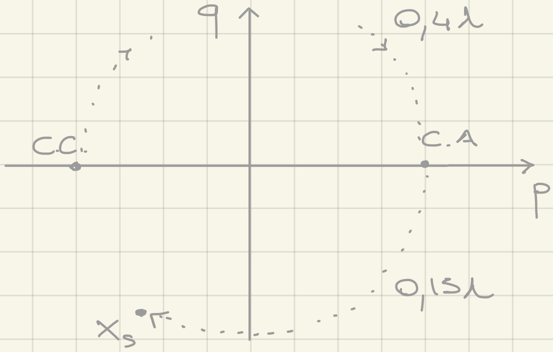
$d_s = 0,235\lambda$



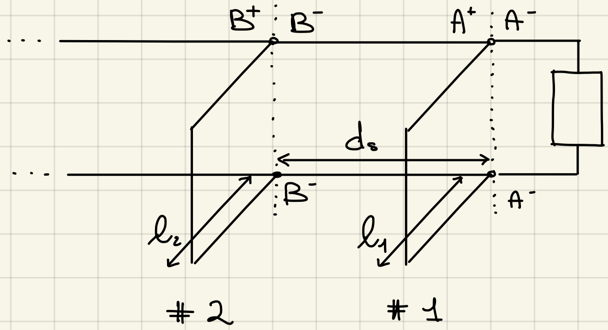
$Z_{cc} = 0\Omega$ $\qquad l_s = 0,4\lambda$



$Z_{C.A.} = \infty\Omega$ $\qquad l_s = 0,15\lambda$



Doppio Stub (parallelo)



$Z_L = 58 - j34\ [\Omega]$

$d_s = 0,3\lambda$ $\qquad Z_c = 50\ \Omega$

$\bar{y}_L = 0,64 + j0,38$

NB: gli stub sono <u>reattivi</u> ⟶ modificano solo la parte immaginaria (dell'ammettenza);
il tratto d_s servirà a modificare anche la parte reale.

In AA^-: siamo in $\bar{y}_L = 0,64 + j0,38$

stub #1

In AA^+: siamo sul cerchio a parte reale 0,64 $(0,64 + jb_{AA^+})$

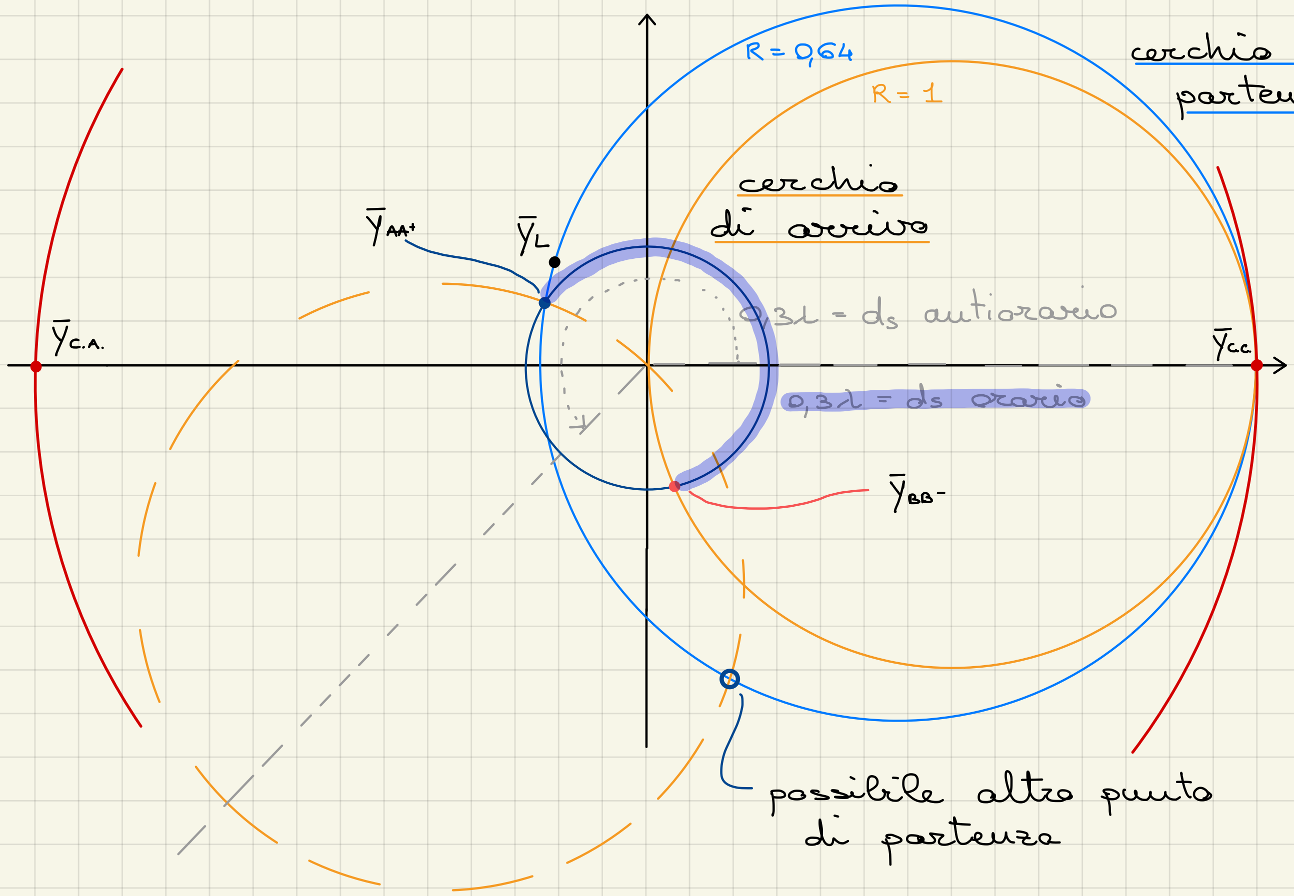
d_s

In BB^-: " " " " " " 1 $(1 + jb_{BB^-})$

stub #2

In BB^+: siamo nel centro della carta di Smith $(\bar{y}_{BB^+} = 1)$

Dobbiamo identificare il "punto di partenza" in AA$^+$ ($\bar{y}_{AA^+} = 0,64 + j b_{AA^+}$) che, ruotato in senso orario di d_s, finisce nel "punto di arrivo" in BB$^-$ ($\bar{y}_{BB^-} = 1 + j b_{BB^-}$)



$\bar{y}_{AA^+} = 0,64 + j\,0,225$

$\bar{y}_{s_1} = j(0,225 - 0,38) = j\,0,155$

in C.C. $\begin{cases} l_1 = 0,226\lambda \\ l_2 = 0,324\lambda \end{cases}$

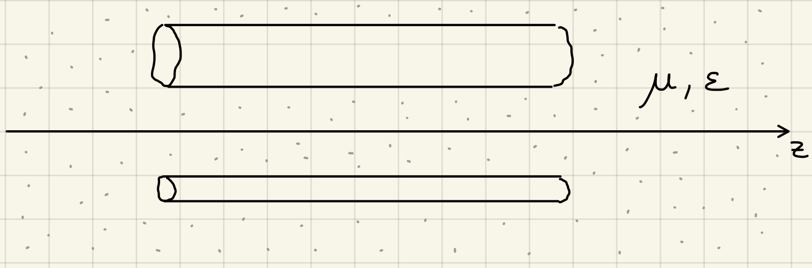
$\bar{y}_{BB^-} = 1 - j\,0,5$

$\bar{y}_{s_2} = j\,0,5$

in C.A. $\begin{cases} l_1 = 0,476\lambda \\ l_2 = 0,074\lambda \end{cases}$

Strutture guidanti non-TEM (modi)

Strutture metalliche, cilindriche (asse diretto come $\hat{u}_z$), uniformi (lungo la direzione $\hat{u}_z$)



Hp: mezzo senza perdite (conduttori ideali)

$$\bar{E}(x,y,z) = \bar{E}(x,y)\,e^{\pm\gamma z} = \bar{E}(x,y)\,e^{\pm(\alpha+j\beta)z}$$

il campo rispetto a z cambia solo in modulo e fase

Equazioni di Helmholtz:

$$\nabla^2\bar{E} = -k^2\bar{E} \qquad \nabla^2\bar{H} = -k^2\bar{H} \qquad \boxed{k^2 = \omega^2\mu\varepsilon}$$

(devono essere soddisfatte nella regione <u>esterna</u> ai conduttori)

$$\nabla^2 = \underbrace{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}}_{\nabla_t^2} + \frac{\partial^2}{\partial z^2} = \nabla_t^2 + \frac{\partial^2}{\partial z^2} \qquad \left(\text{con } \frac{\partial^2\bar{E}}{\partial z^2} = \gamma^2\bar{E}\right)$$

$$\longrightarrow \quad \boxed{\nabla_t^2\bar{E} = -(\gamma^2+k^2)\,\bar{E}} \qquad \boxed{\nabla_t^2\bar{H} = -(\gamma^2+k^2)\,\bar{H}}$$

1) Eq. del rotore $(\bar{\nabla}\times\bar{E},\ \bar{\nabla}\times\bar{H})$

↓

2) Esprimiamo <u>tutte</u> le componenti di $\bar{E}$ e $\bar{H}$ in funzione delle due componenti E_z e H_z

$$\boxed{\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}} \qquad \boxed{\bar{\nabla}\times\bar{H} = j\omega\varepsilon\bar{E}}$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \qquad \frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \qquad -\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

$$\begin{cases} E_x = -\dfrac{1}{\gamma^2+k^2}\left(\gamma\dfrac{\partial E_z}{\partial x} + j\omega\mu\dfrac{\partial H_z}{\partial y}\right) \\[2ex] E_y = \dfrac{1}{\gamma^2+k^2}\left(-\gamma\dfrac{\partial E_z}{\partial y} + j\omega\mu\dfrac{\partial H_z}{\partial x}\right) \\[2ex] H_x = \dfrac{1}{\gamma^2+k^2}\left(j\omega\varepsilon\dfrac{\partial E_z}{\partial y} - \gamma\dfrac{\partial H_z}{\partial x}\right) \\[2ex] H_y = -\dfrac{1}{\gamma^2+k^2}\left(j\omega\varepsilon\dfrac{\partial E_z}{\partial x} + \gamma\dfrac{\partial H_z}{\partial y}\right) \end{cases}$$

Si definisce $K_c^2 = \gamma^2 + K^2$ (senza perdite $\gamma = j\beta$)

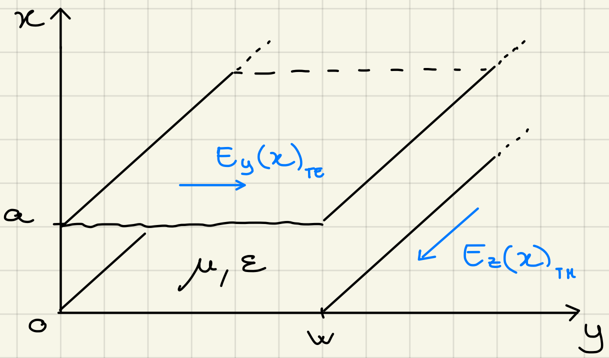
$$\left[\nabla_t^2 E_z = -K_c^2 E_z\right] \quad e \quad \left[\nabla_t^2 H_z = -K_c^2 H_z\right]$$

Classificazione (dei modi)

$E_z = H_z = 0$ TEM $H_z = 0,\ E_z \neq 0$ TM

$H_z \neq 0,\ E_z = 0$ TE $E_z \neq 0$ e $H_z \neq 0$ ibridi (TM+TE)

Linea a striscia (trascurare gli effetti di bordo):



- Modo TEM: $E_z = 0,\ H_z = 0$ $\gamma^2 = -K^2$ cioè $\gamma = \pm jK = \pm j\omega\sqrt{\mu\varepsilon}$

$$H_y = \frac{\gamma}{j\omega\mu} E_x = \pm\sqrt{\frac{\varepsilon}{\mu}}\, E_x \quad \ldots$$ (come già visto)

- Modi TM: $H_z = 0,\ E_z \neq 0$ $\nabla_t^2 E_z = -K_c^2 E_z$

$$\frac{\partial}{\partial y} = 0 \qquad \boxed{\frac{\partial^2 E_z}{\partial x^2} = -K_c^2 E_z} \qquad \text{con } K_c^2 = \gamma^2 + K^2$$

perché trascuriamo gli effetti di bordo

$$E_z(x) = A\,\text{sen}(K_c x) + B\cos(K_c x)$$

A e B vanno determinate con condizioni al contorno

→ conduttori ideali (campi tangenti nulli)

$$\begin{cases} E_z(0) = 0 \implies B = 0 \\ E_z(a) = 0 \implies A\,\text{sen}(K_c a) = 0 \qquad K_c a = m\pi \qquad m = 1, 2 \ldots \end{cases}$$

$$\begin{cases} E_z(x) = A \operatorname{sen}\left(\frac{m\pi x}{a}\right) \qquad \boxed{TM_m} \\ E_x(x) = -\frac{\gamma}{K_c^2}\frac{dE_z}{dx} = -\frac{\gamma a}{m\pi} A \cos\left(\frac{m\pi x}{a}\right) \\ H_y(x) = -\frac{j\omega\varepsilon}{K_c^2}\frac{dE_z}{dx} = -j\frac{\omega\varepsilon a}{m\pi} A \cos\left(\frac{m\pi x}{a}\right) \\ H_x = 0, \quad E_y = 0, \quad H_z = 0 \end{cases}$$

Analizziamo γ:

$$K_c^2 = K^2 + \gamma^2 = \left(\frac{m\pi}{a}\right)^2 \qquad \gamma = \sqrt{K_c^2 - K^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon}$$

affinché il modo TM_m sia "in propagazione"

$$\gamma = j\beta_z \text{ (immag. pura)} \longrightarrow \left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon < 0$$

$$\omega > \frac{1}{\sqrt{\mu\varepsilon}} \cdot \frac{m\pi}{a}$$

$$\Longrightarrow \quad \omega > \underbrace{\frac{m\pi v}{a}}_{\omega_c \text{ (pulsazione di taglio / cut-off)}}$$

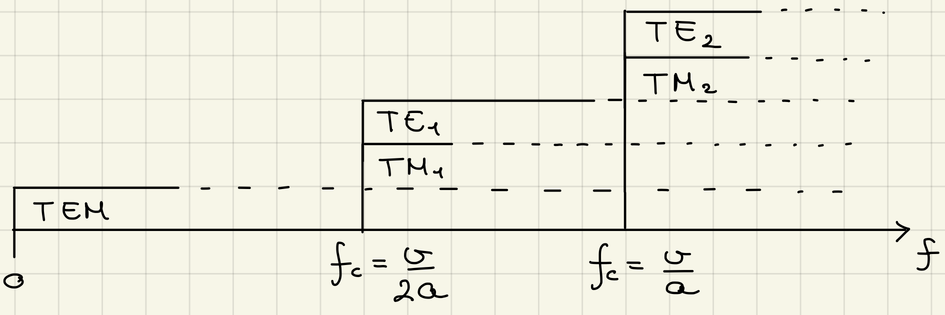
$$\left\{\gamma = j\overbrace{\underbrace{\omega\sqrt{\mu\varepsilon}}_{K(=\beta)}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}^{\beta_z}\right\} \qquad (\omega > \omega_c)$$

$$\left\{\gamma = \alpha = \frac{m\pi}{a}\sqrt{1-\left(\frac{\omega}{\omega_c}\right)^2}\right\} \qquad (\omega < \omega_c)$$

Se $\omega < \omega_c$: $e^{-j\beta_z z}$ (propagazione)

Se $\omega > \omega_c$: $e^{-\alpha_z z}$ (attenuazione)

$$\omega_c = \frac{m\pi v}{a} = 2\pi f_c \qquad \boxed{\begin{array}{l} f_c = \frac{mv}{2a} = \frac{v}{\lambda_c} \\ \lambda_c = \frac{2a}{m} \end{array}}$$



Velocità di "propagazione" v_f:

$$\beta_z = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_f} = \frac{\omega}{v_f} \longrightarrow v_f = \frac{\omega}{\beta_z} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu\varepsilon}\,\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

v dell'onda TEM

$$v_f = \frac{v}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$; nel vuoto $v = c$

$v_f > c$! è possibile perché non è una velocità reale (non è la velocità di propagazione dell'energia) ma è una velocità apparente (velocità di fase)

$v_f = v_f(\omega)$ → frequenze diverse si muovono a velocità diverse ("dispersione")

Segnale a BANDA STRETTA:

$$s(t) = f(t)\cos\omega_0 t = \mathrm{Re}\{f(t)\,e^{j\omega_0 t}\}$$

$f(t) \longleftrightarrow F(\omega)$ $\quad$ $s(t) \longleftrightarrow S(\omega)$ (trasformata di Fourier)

$$S(\omega) = F(\omega - \omega_0) \qquad S(\omega) \longrightarrow \boxed{A\,e^{-j\beta_z L}} \longrightarrow S_o(\omega) \qquad (|A| \leq 1)$$

$$S_o(\omega) = A\,S(\omega)\,e^{-j\beta_z(\omega)L} \quad \text{con} \quad \beta_z = \underbrace{\omega\sqrt{\mu\varepsilon}}_{\beta}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

L: lunghezza della guida

Sviluppo in serie di Taylor di β_z:

$$\beta_z(\omega) = \underbrace{\beta_z(\omega_o)}_{\beta_o} + \underbrace{\left.\frac{d\beta_z(\omega)}{d\omega}\right|_{\omega_o}}_{\beta'_o} \cdot \underbrace{(\omega - \omega_o)}_{\Delta\omega} + \dots$$

$$S_o(\omega) \simeq A\,S(\omega)\,e^{-j\beta_o L}\,e^{-j\beta'_o\Delta\omega L}$$

$\updownarrow$

$$s_o(t) \simeq A\,\underbrace{f(t-\beta'_o L)}_{v_g = \frac{1}{\beta'_o} = \frac{1}{\frac{d\beta_z(\omega)}{d\omega}}}\,\underbrace{\cos(\omega_o t - \beta_o L)}_{v_f = \frac{\omega}{\beta_z}}$$

informazione $\qquad$ portante

$v_g = v\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \leq c$ $\boxed{v_f \cdot v_g = v^2}$

↳ velocitá di gruppo

Nelle linee TEM:

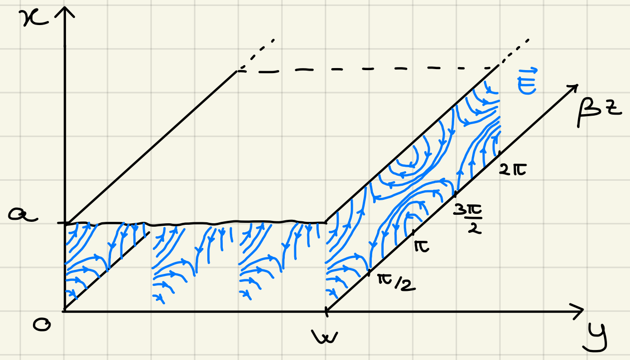
$\beta = \frac{\omega}{v}$ $v_f = \frac{\omega}{\beta} = v$ $v_g = \frac{1}{\frac{d\beta(\omega)}{d\omega}} = v$

→ v_f e v_g sono uguali nei modi TEM ←

Impedenza modale

$$\left[Z_{TM} = \frac{E_t}{H_t} = \frac{E_x}{H_y} = \eta\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} = \frac{\beta_z}{\omega\varepsilon} \right]$$

TM1 $E_z(x) = A\,\text{sen}\left(\frac{\pi x}{a}\right)$ $E_x(x) = \frac{\gamma a}{\pi}\cos\left(\frac{\pi x}{a}\right)$



$$\begin{cases} E_z(x,z) = A\,\text{sen}\left(\frac{\pi x}{a}\right)e^{-j\beta_z z} \\ E_x(x,z) = \frac{\gamma a}{\pi} A\cos\left(\frac{\pi x}{a}\right)e^{-j\beta_z z} \\ E_y = 0 \end{cases}$$

NB: le cariche indotte sui conduttori superiore ed inferiore sono dello stesso segno

↓

non é possibile definire un potenziale fra i due conduttori (campo $\vec{E}$ non irrotazionale)!

- Modi TE: $E_z = 0$, $H_z \neq 0$ $\nabla_t^2 H_z = -K_c^2 H_z$

$H_z(x) = A\,\text{sen}(K_c x) + B\cos(K_c x)$

Condizioni al contorno: $E_y \propto \frac{\partial H_z}{\partial x}$ $E_y(0) = 0$ $E_y(a) = 0$

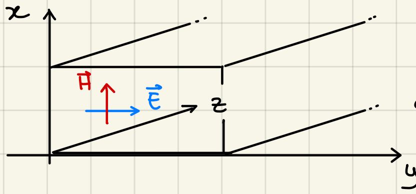
$\downarrow$ $A = 0$ $\downarrow$ $K_c = \frac{m\pi}{a}$

$$\begin{cases} H_z = B\cos\left(\frac{m\pi x}{a}\right) \\ E_y = -j\frac{\omega\mu}{m\pi}aB\,\text{sen}\left(\frac{m\pi}{a}x\right) \\ H_x = j\frac{\beta}{m\pi}aB\,\text{sen}\left(\frac{m\pi}{a}x\right) \\ E_x = 0, \quad H_y = 0, \quad E_z = 0 \end{cases} \qquad \boxed{TE_m}$$

$$\left\{\gamma = j\beta_z = j\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}\right\}$$

$$(\omega > \omega_c)$$

$$\left[Z_{TE} = \frac{E_t}{H_t} = -\frac{E_y}{H_x} = \frac{\eta}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}\right]$$



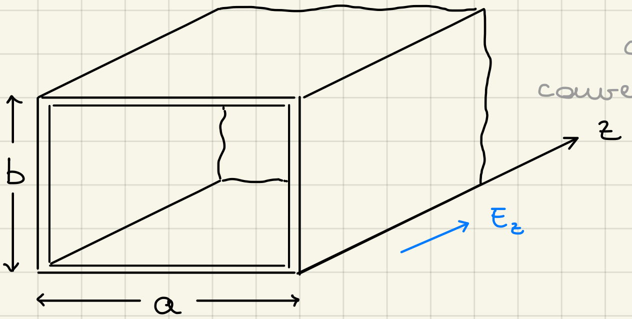
Z_{TE} è riferita al verso delle z positive

v_f e v_g sono le stesse del modo TM_m.

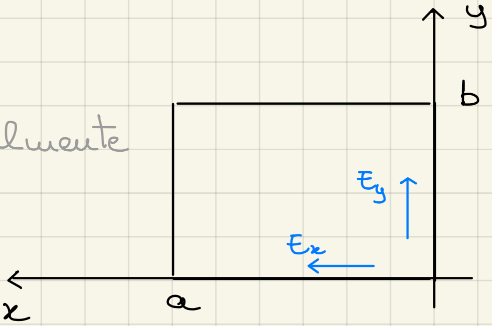
NB: non c'è carica indotta sui 2 conduttori, inoltre non ci sono componenti di campo magnetico lungo l'asse y ↓

non è possibile definire una corrente che scorre lungo i conduttori (verso z, c'è invece corrente verso y)

Guide d'onda rettangolari (metalliche)



a > b convenzionalmente



non ci sono 2 conduttori

Un solo conduttore → ∄ modo TEM (∄ soluzione statica)

Solo onde TM e/o TE.

Onde TM $\nabla_t E_z(x,y) = -K_c^2 E_z(x,y)$ hp: $E_z(x,y) = F(x)G(y)$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = -k_c^2 E_z \longrightarrow F''(x)\,G(y) + F(x)\,G''(y) =$$

$$= -k_c^2 F(x)\,G(y)$$

$$\longrightarrow \frac{F''(x)}{F(x)} + \frac{G''(y)}{G(y)} = -K_c^2 \Longrightarrow \frac{F''(x)}{F(x)} = -K_x^2, \quad \frac{G''(y)}{G(y)} = -K_y^2$$

con $K_x^2 + K_y^2 = K_c^2$

Soluzione dell' eq. differenziale:

$$E_z(x,y) = \Big[\underbrace{A' \operatorname{sen}(K_x x) + B' \cos(K_x x)}_{F(x)}\Big] \cdot \Big[\underbrace{C' \operatorname{sen}(K_y y) + D' \cos(K_y y)}_{G(y)}\Big]$$

Determiniamo A', B', C', D':

$E_z(0,y) = 0 \longrightarrow B' = 0$ $E_z(x,0) = 0 \longrightarrow D' = 0$

campo elettrico tangente al conduttore nullo

$A' \cdot C'$

$$\Longrightarrow \begin{cases} E_z(x,y) = A \operatorname{sen}(K_x x) \operatorname{sen}(K_y y) \quad (H_z = 0) \end{cases}$$

$E_z(a,y) = 0 \longrightarrow K_x a = m\pi$ $\boxed{K_x = \frac{m\pi}{a}}$ $(m = 0, 1, \dots, \infty)$

$E_z(x,b) = 0 \longrightarrow K_y b = n\pi$ $\boxed{K_y = \frac{n\pi}{b}}$ $(n = 0, 1, \dots, \infty)$

$\boxed{TM_{mn}}$ $$K_c^2 = K_x^2 + K_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \sqrt{K_c^2 - K^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon < 0 \longrightarrow \omega > \omega_c \text{ con}$$

no attenuazione

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\left\{ \gamma = j\beta_z = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \right\} \quad (\omega > \omega_c)$$

v_f e v_g hanno la stessa espressione dei modi TM_m

$$\begin{cases} E_x = -\dfrac{j\beta_z K_x}{K_c^2} A \cos(K_x x)\,\text{sen}(K_y y)\, e^{-j\beta_z z} \\ E_y = -\dfrac{j\beta_z K_y}{K_c^2} A\, \text{sen}(K_x x) \cos(K_y y)\, e^{-j\beta_z z} \\ H_x = \dfrac{j\omega\varepsilon K_y}{K_c^2} A\, \text{sen}(K_x x) \cos(K_y y)\, e^{-j\beta_z z} \\ H_y = -\dfrac{j\omega\varepsilon K_x}{K_c^2} A \cos(K_x x)\, \text{sen}(K_y y)\, e^{-j\beta_z z} \end{cases} \qquad \boxed{TM_{mn}}$$

- $E_x = 0$ in $y=0$ e $y=b$ $\qquad$ $E_y = 0$ in $x=0$ e $x=0$
- Sia m che n devono essere > 0 (altrimenti $E_z \equiv 0$)
- Il modo a frequenza più bassa è TM_{11}

Onde TE $\qquad E_z = 0$

$\nabla_t H_z(x,y) = -K_c^2 H_z(x,y)$ $\qquad$ hp: $E_z(x,y) = M(x)N(y)$

Analogamente al modo TM si ricava:

$$H_z(x,y) = \left[A''\,\text{sen}(K_x x) + B''\cos(K_x x)\right]\left[C''\,\text{sen}(K_y y) + D''\cos(K_y y)\right]$$

$$E_x = -\frac{j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial y} =$$
$$= -\frac{j\omega\mu}{K_c^2} K_y \left[A''\,\text{sen}(K_x x) + B''\cos(K_x x)\right]\left[C''\cos(K_y y) - D''\,\text{sen}(K_y y)\right]$$

$$E_y = \frac{j\omega\mu}{K_c^2} K_x \left[A''\cos(K_x x) - B''\,\text{sen}(K_x x)\right]\left[C''\,\text{sen}(K_y y) + D''\cos(K_y y)\right]$$

$E_x(x,0) = 0 \longrightarrow C'' = 0$ $\qquad$ $E_y(0,y) = 0 \longrightarrow A'' = 0$

$$\Longrightarrow \Big\{ H_z = \overset{B''\cdot D''}{B} \cos(K_x x)\cos(K_y y)$$

$E_x(0,b) = 0 \longrightarrow K_y b = n\pi$ $\qquad$ $K_y = \dfrac{n\pi}{b}$ $\quad (n = 0, 1, \dots, \infty)$

$E_y(a,0) = 0 \longrightarrow K_x a = m\pi$ $\qquad$ $K_x = \dfrac{m\pi}{a}$ $\quad (m = 0, 1, \dots, \infty)$

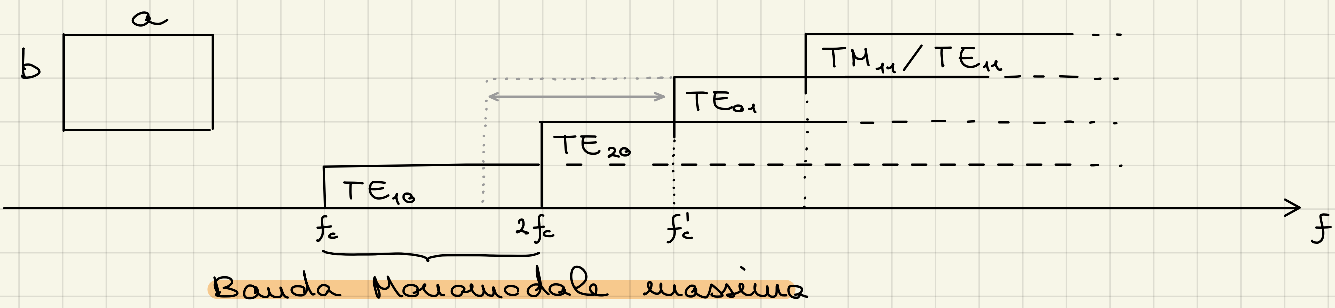
ω_c e γ (o β) sono come per il modo TM_{mn}

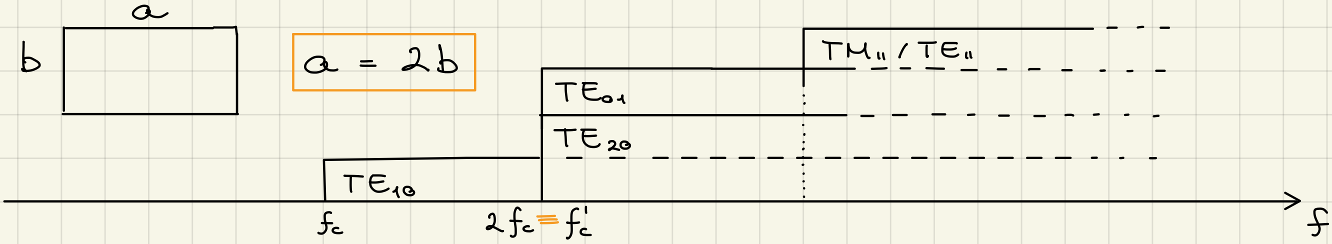
$$\begin{cases} E_x = j\dfrac{\omega\mu k_y}{k_c^2} B \cos(k_x x)\,\text{sen}(k_y y)\, e^{-j\beta_z z} \\ E_y = -j\dfrac{\omega\mu k_y}{k_c^2} B\, \text{sen}(k_x x) \cos(k_y y)\, e^{-j\beta_z z} \\ H_x = j\dfrac{\beta_z k_x}{k_c^2} B\, \text{sen}(k_x x) \cos(k_y y)\, e^{-j\beta_z z} \\ H_y = j\dfrac{\beta_z k_y}{k_c^2} B \cos(k_x x)\, \text{sen}(k_y y)\, e^{-j\beta_z z} \end{cases} \qquad \boxed{TE_{mn}}$$

- m o n possono annullarsi → TE_{10} e TE_{01}
- Modo a frequenza più bassa - modo fondamentale

TE_{10} (se $a > b$) TE_{01} (se $a < b$)

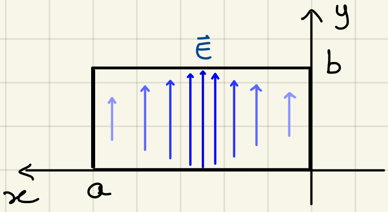
per convenzione è sempre questo





Modo fondamentale $\boxed{TE_{10}}$

$m = 1$, $n = 0$ $k_x = \dfrac{\pi}{a}$, $k_y = 0$ $\lambda_c = 2a$

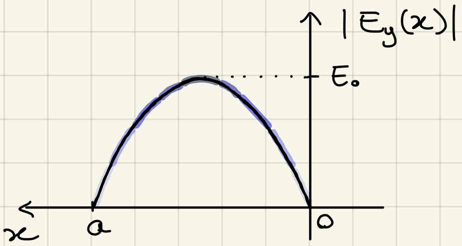


$$H_z = B \cos(k_x x)\, e^{-j\beta z}$$

$$E_y = -j\frac{\omega\mu B}{k_x}\, \text{sen}(k_x x)\, e^{-j\beta z} = $$
$$= E_0\, \text{sen}\left(\frac{\pi}{a}x\right) e^{-j\beta z}$$

$$H_x = \frac{j\beta B}{k_x}\, \text{sen}(k_x x)\, e^{-j\beta z} = -\frac{E_0}{Z_{TE}}\, \text{sen}\left(\frac{\pi}{a}x\right) e^{-j\beta z}$$

con $\left[Z_{TE} = -\dfrac{E_y}{H_x} = \dfrac{\omega\mu}{\beta} = \dfrac{\omega\mu}{k\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \right]$



$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \qquad v = \frac{1}{\sqrt{\mu\varepsilon}}$$

(di solito $\eta = \eta_0$, $v = c$)

$$Z_{TE} = -\frac{E_y}{H_x} = \frac{\eta}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$f_c = \frac{1}{2a\sqrt{\mu\varepsilon}} = \frac{v}{2a} \qquad \lambda_c = 2a$$

$$v_f = \frac{v}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad v_g = v\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \qquad \lambda_g = \frac{v_f}{f} = \frac{2\pi}{\beta}$$

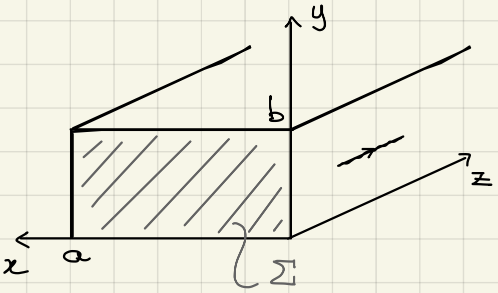
$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)}} \quad \text{con} \quad \lambda = \frac{v}{f}$$

Calcolo della potenza nella guida (lungo z)

$$P = \frac{1}{2}\,\mathrm{Re}\left\{\int_{\Sigma}\left(\bar{E}\times\bar{H}^*\right)\cdot d\bar{\Sigma}\right\} =$$

$$= \frac{1}{2}\,\mathrm{Re}\left\{\int_0^a\int_0^b\left(-E_y\cdot H_x^*\right)dy\,dx\right\} =$$

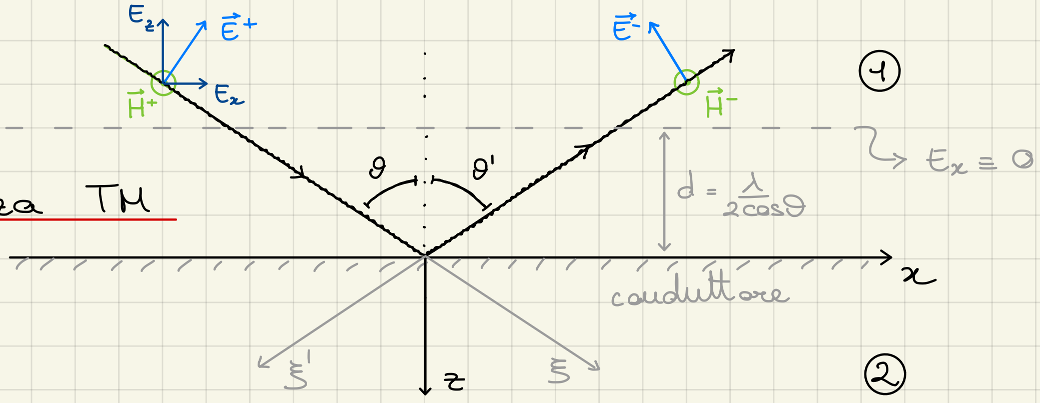
$$= \frac{1}{2}\frac{|E_0|^2 b}{Z_{TE}}\int_0^a \mathrm{sen}^2\left(\frac{\pi x}{a}\right)dx =$$



$$\left[P = \frac{|E_0|^2\, a\, b}{4\, Z_{TE}}\right]$$

Incidenza obliqua (di onde TEM piane) su superficie metallica piana (ideale)

Incidenza TM



$$\bar{E}(x,z) = \bar{E}^+ e^{-j\beta\xi} + \bar{E}^- e^{j\beta\xi'} \quad \text{ma} \quad \begin{cases} \xi = x\,\mathrm{sen}\,\vartheta + z\cos\vartheta \\ \xi' = -x\,\mathrm{sen}\,\vartheta' + z\cos\vartheta' \end{cases}$$

$$E_x(x,z) = E^+\cos\vartheta\; e^{-j\beta(x\,\text{sen}\vartheta + z\cos\vartheta)} - E^-\cos\vartheta'\, e^{j\beta(-x\,\text{sen}\vartheta' + z\cos\vartheta')}$$

$$E_z(x,z) = E^+\text{sen}\vartheta\; e^{-j\beta(x\,\text{sen}\vartheta + z\cos\vartheta)} - E^-\text{sen}\vartheta'\, e^{j\beta(-x\,\text{sen}\vartheta' + z\cos\vartheta')}$$

$$H_y(x,z) = H^+\, e^{-j\beta(x\,\text{sen}\vartheta + z\cos\vartheta)} + H^-\, e^{j\beta(-x\,\text{sen}\vartheta' + z\cos\vartheta')}$$

Condizioni al contorno:

piano conduttore $(z=0)$ $\quad E_x(x,0) = 0 \quad (\forall x)$

$$E^+\cos\vartheta\, e^{-j\beta x\,\text{sen}\vartheta} = E^-\cos\vartheta'\, e^{-j\beta x\,\text{sen}\vartheta'} \quad (\forall x)$$

↳ $\boxed{\vartheta = \vartheta'}$ e $\boxed{E^+ = E^-}$

↳ legge di Snell

$$\Longrightarrow \begin{cases} E_x(x,z) = -2j\,E^+\cos\vartheta\,\text{sen}(\beta z\cos\vartheta)\,e^{-j\beta x\,\text{sen}\vartheta} \\ E_z(x,z) = -2\,E^+\text{sen}\vartheta\,\cos(\beta z\cos\vartheta)\,e^{-j\beta x\,\text{sen}\vartheta} \\ H_y(x,z) = \dfrac{2E^+}{\eta}\cos(\beta z\cos\vartheta)\,e^{-j\beta x\,\text{sen}\vartheta} \end{cases}$$

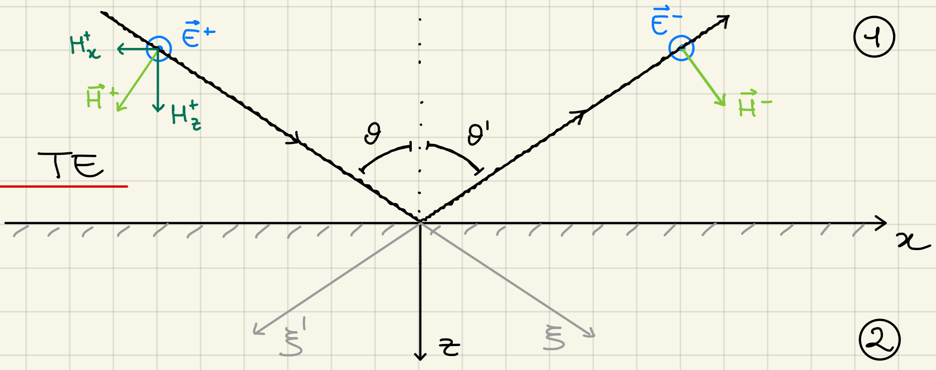
L'onda:

- progressiva in direzione $\vec{u}_x$
- puramente stazionaria in direzione $\vec{u}_z$

$E_x(x,z) = \ldots\,\text{sen}(\beta z\cos\vartheta)\ldots$ si annulla in $z=0$ e su tutti i piani paralleli al piano xy tali che $\beta z\cos\vartheta = n\pi$

↳ $\boxed{z = d_n = \dfrac{n\lambda}{2\cos\vartheta}}$ $\quad E_x(x,-d) = 0$ ↳ come se ci fosse un piano condut.

Incidenza TE



$$E_y = E^+ e^{-j\beta(x \operatorname{sen}\theta + z\cos\theta)} + E^- e^{j\beta(-x \operatorname{sen}\theta' + z\cos\theta')}$$

$$H_x = -\frac{E^+}{\eta}\cos\theta\, e^{-j\beta(x \operatorname{sen}\theta + z\cos\theta)} + \frac{E^-}{\eta}\cos\theta\, e^{j\beta(-x \operatorname{sen}\theta' + z\cos\theta)}$$

$$H_z = \frac{E^+}{\eta}\operatorname{sen}\theta\, e^{-j\beta(x \operatorname{sen}\theta + z\cos\theta)} + \frac{E^-}{\eta}\operatorname{sen}\theta\, e^{j\beta(-x \operatorname{sen}\theta' + z\cos\theta)}$$

Piano conduttore $(z=0)$ $\quad E_y(x,0)=0 \quad (\forall x)$

$$E^+ e^{-j\beta x \operatorname{sen}\theta} = -E^- e^{-j\beta x \operatorname{sen}\theta'} \quad (\forall x)$$

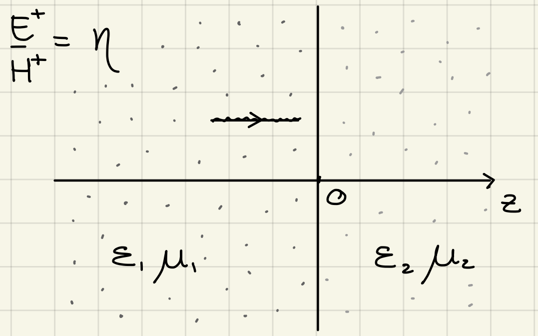
$\hookrightarrow$ $\boxed{\theta = \theta'}$ e $\boxed{E^+ = -E^-}$

$$\Longrightarrow \begin{cases} E_y = -2j\, E^+ \operatorname{sen}(\beta z\cos\theta)\, e^{-j\beta x \operatorname{sen}\theta} \\ H_x = -2\dfrac{E^+}{\eta}\cos\theta\, \operatorname{sen}(\beta z\cos\theta)\, e^{-j\beta x \operatorname{sen}\theta} \\ H_z = -2j\dfrac{E^+}{\eta}\operatorname{sen}\theta\, \operatorname{sen}(\beta z\cos\theta)\, e^{-j\beta x \operatorname{sen}\theta} \end{cases}$$

L'onda ha le stesse proprietà dell'incidenza TM.

$E_y(x,-d) = 0$ per $\boxed{d_n = \dfrac{n\lambda}{2\cos\theta}}$

Analogia tra linee di trasmissione e onde TEM

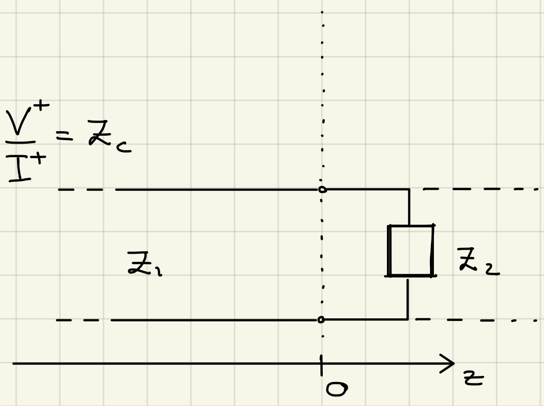


$$E(0) = E_2^+(0) = E_1^+(0) + E_1^-(0) = E^+(0)\,T(0)$$

$$E_1^-(0) = E_1^+(0)\,\Gamma(0)$$

$$\Gamma(0) = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{con} \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+(z) = E^+(0)\,e^{-j\beta z} \qquad \vec{S} = \frac{1}{2}\operatorname{Re}\{\bar{E} \times \bar{H}\}$$

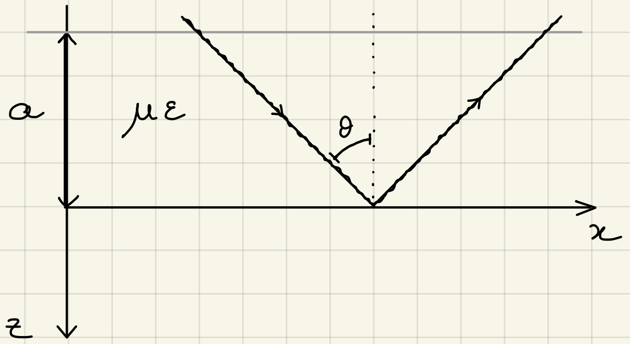


$$V(0) = V^+(0) + V^-(0) = V^+(0)\,T$$

$$V^-(0) = V^+(0)\,\Gamma \qquad \Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

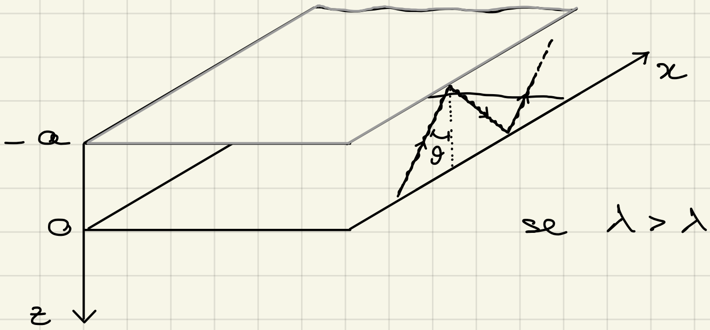
$$V^+(z) = V^+(0)\,e^{-j\beta z} \qquad P = \frac{1}{2}\operatorname{Re}\{V I^*\}$$

Onde guidate tra conduttori piani paralleli come sovrapposizione di onde piane



E_t (tangente al conduttore) (E_x o E_y) nullo a distanza

$\left[a = \frac{m\lambda}{2\cos\theta}\right]$ come se avessi un conduttore



$$\cos\theta = \frac{m\lambda}{2a} = \frac{\lambda}{\lambda_c} = \frac{\omega_c}{\omega}$$

l'angolo di incidenza dell'onda nella guida è fissato dalla frequenza

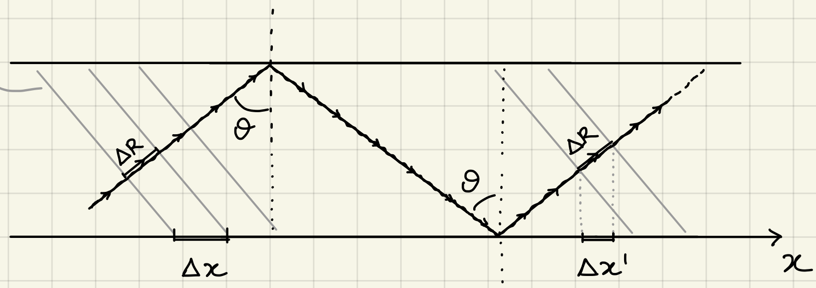
se $\lambda > \lambda_c$ $(f < f_c)$ $\boxed{\cos\theta > 1}$ impossib.

se $\lambda = \lambda_c = \frac{2a}{m}$ $\cos\theta = 1$ $\theta = 0°$

l'onda rimbalza ma non va avanti

$v = 1/\sqrt{\mu\varepsilon}$

$\Delta R = v \cdot \Delta t$ spostamento del fronte d'onda lungo la direzione di propagazione



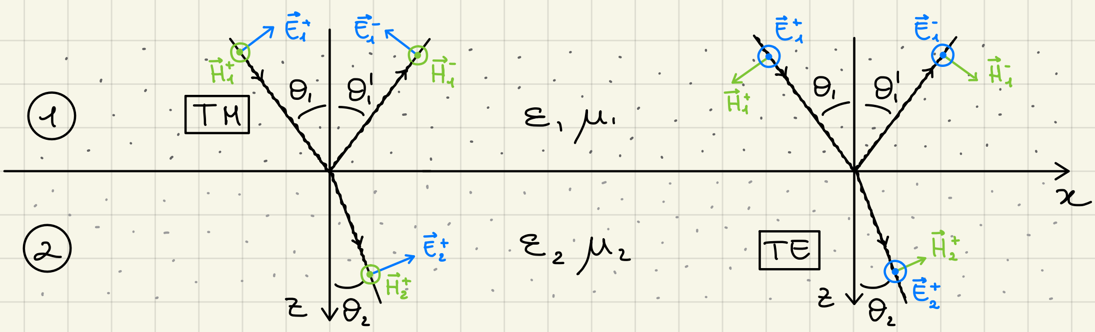
$\Delta x = \frac{\Delta R}{\operatorname{sen}\theta}$ spostamento delle superfici equifase lungo l'asse x (guida)

$$v_f = \frac{\Delta x}{\Delta t} = \frac{\Delta R}{\Delta t} \cdot \frac{1}{\operatorname{sen}\theta} = \frac{v}{\operatorname{sen}\theta} = \frac{v}{\sqrt{1-\cos^2\theta}} = \frac{v}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$ velocità di fase

$\Delta x' = \Delta R \operatorname{sen}\theta$ spostamento del fronte d'onda lungo l'asse x (guida)

$$v_g = \frac{\Delta x'}{\Delta t} = v \cdot \sqrt{1-\cos^2\theta} = v \cdot \sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$ velocità di gruppo

Incidenza obliqua su discontinuità tra dielettrici



$K = \omega\sqrt{\mu\varepsilon}$

$n = \sqrt{\mu_r \varepsilon_r}$

Condizioni al contorno: $E_{t_1} = E_{t_2}$ $H_{t_1} = H_{t_2}$ in $z = 0$

→ $K_1 \,\mathrm{sen}\,\theta_1 = K_1 \,\mathrm{sen}\,\theta_1' = K_2 \,\mathrm{sen}\,\theta_2$

$\boxed{\theta_1 = \theta_1'}$ $\boxed{n_1 \,\mathrm{sen}\,\theta_1 = n_2 \,\mathrm{sen}\,\theta_2}$ Leggi di Snell

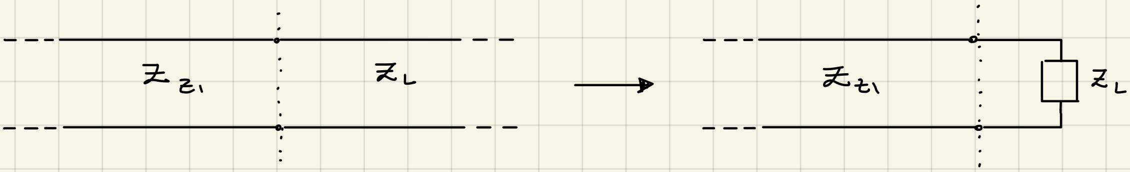
$\boxed{\text{TM}}$ Condizioni al contorno $(z=0)$:

$$\begin{cases} E_{x_1}^+ + E_{x_1}^- = E_{x_2}^+ \\ H_{y_1}^+ + H_{y_1}^- = H_{y_2}^+ \end{cases} \qquad \begin{cases} E_{x_1}^+ + E_{x_1}^- = E_{x_2}^+ \\ \dfrac{E_{x_1}^+}{Z_{z_1}} - \dfrac{E_{x_1}^-}{Z_{z_1}} = \dfrac{E_{x_2}^+}{Z_L} \end{cases}$$

Definiamo l' IMPEDENZA D'ONDA:

$$\begin{cases} Z_{z_1} = \dfrac{E_{x_1}^+}{H_{y_1}^+} = \dfrac{E_1^+ \cos\theta_1}{H_1^+} = \eta_1 \cos\theta_1 = -\dfrac{E_{x_1}^-}{H_{y_1}^-} \\ Z_L = \dfrac{E_{x_2}^+}{H_{y_2}^+} = \dfrac{E_2^+ \cos\theta_2}{H_2^+} = \eta_2 \cos\theta_2 = \eta_2 \sqrt{1 - \left(\dfrac{n_1}{n_2}\right)^2 \mathrm{sen}^2\theta_1} \end{cases}$$

$$\begin{cases} E_{x_1}^- = \Gamma \cdot E_{x_1}^+ \quad \text{con} \quad \Gamma = \dfrac{Z_L - Z_{z_1}}{Z_L + Z_{z_1}} \\ E_{x_2}^+ = T\, E_{x_1}^+ \quad \text{con} \quad T = \dfrac{2 Z_L}{Z_L + Z_{z_1}} = 1 + \Gamma \end{cases}$$



Campi totali ①

$$\begin{cases} E_x = E_1^+ \cos\theta_1 \, e^{-j\beta_{x_1} x} \left[e^{-j\beta_{z_1} z} + \Gamma e^{j\beta_{z_1} z} \right] \\ E_z = -E_1^+ \,\mathrm{sen}\,\theta_1 \, e^{-j\beta_{x_1} x} \left[e^{-j\beta_{z_1} z} - \Gamma e^{j\beta_{z_1} z} \right] \\ H_y = \dfrac{E_1^+}{\eta_1} e^{-j\beta_{x_1} x} \Big[\underbrace{e^{-j\beta_{z_1} z} - \Gamma e^{j\beta_{z_1} z}}_{\text{ONDA STAZIONARIA}} \Big] \end{cases}$$

$\beta_{x_1} = K_1 \,\mathrm{sen}\,\theta_1$

$\beta_{z_1} = K_1 \cos\theta_1$

$\boxed{\text{TE}}$ Condizioni al contorno $z=0$:

$$\begin{cases} E^+_{y_1} + E^-_{y_1} = E^+_{y_2} \\ H^+_{x_1} + H^-_{x_1} = H^+_{x_2} \end{cases}$$

IMPEDENZE D'ONDA:

$$\begin{cases} Z_{z_1} = \dfrac{E^+_{y_1}}{H^+_{x_1}} = \dfrac{E^+_1}{H^+_1 \cos\theta_1} = \dfrac{\eta_1}{\cos\theta_1} \\ Z_L = \dfrac{\eta_2}{\cos\theta_2} = \eta_2 \left[1 - \left(\dfrac{n_1}{n_2}\right)^2 \operatorname{sen}^2\theta_1\right]^{-1/2} \end{cases}$$

$$\Gamma = \frac{E^-_y}{E^+_{y_1}} = \frac{Z_L - Z_{z_1}}{Z_L + Z_{z_1}} \qquad T = \frac{E^+_{y_2}}{E^+_{y_1}} = \frac{2 Z_L}{Z_L + Z_{z_1}}$$

Campi totali ①

$$\begin{cases} E_y = E^+_1 e^{-j\beta_{x_1} x} \left[e^{-j\beta_{z_1} z} + \Gamma e^{j\beta_{z_1} z} \right] \\ H_x = -\dfrac{E^+_1}{\eta_1} \cos\theta_1 \, e^{-j\beta_{x_1} x} \left[e^{-j\beta_{z_1} z} - \Gamma e^{j\beta_{z_1} z} \right] \\ H_z = \dfrac{E^+_1}{\eta_1} \operatorname{sen}\theta_1 \, e^{-j\beta_{x_1} x} \left[e^{-j\beta_{z_1} z} + \Gamma e^{j\beta_{z_1} z} \right] \end{cases}$$

Riflessione totale : $|\Gamma| = 1$ $\quad Z_L = 0, \infty, jX$

cond. ideale

cond. magnetico ideale

Z_L immaginaria se

$$1 - \left(\frac{n_1}{n_2}\right)^2 \operatorname{sen}^2\theta_1 \leq 0$$

$$\boxed{\operatorname{sen}\theta_1 \geq \frac{n_2}{n_1}}$$ (si verifica solo se $\frac{n_2}{n_1} \leq 1$)

$$\theta_c = \operatorname{arcsen}\left(\frac{n_2}{n_1}\right)$$

θ_c: angolo critico

$|\Gamma| = 1$ → non c'é passaggio di densitá di potenza dal mezzo 1 al mezzo 2, ma i campi nel mezzo 2 non sono nulli ($T = 1 + \Gamma \neq 0$)

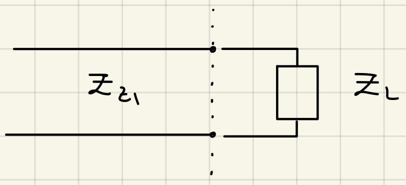
Nel mezzo ② c'é la cosiddetta "onda evanescente"

$$\beta_{z_2} = k_2 \cos\theta_2 = k_2 \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \operatorname{sen}^2\theta_1} = \underbrace{-jk_2 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \operatorname{sen}^2\theta_1 - 1}}_{\alpha_{z_2}}$$

$e^{-j\beta_{z_2} z} \Longrightarrow e^{-\alpha_{z_2} z}$ l'onda si consuma mano a mano che si propaga

Trasmissione (Rifrazione) totale: $\Gamma = 0$ $Z_L = Z_{z_1}$

$\varepsilon_1 , \varepsilon_2$ $\mu_1 = \mu_2 = \mu_0$ Incidenza TM



$\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$$\eta_1 \cos\vartheta_1 = \eta_2 \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \operatorname{sen}^2\vartheta_1}$$

$$\cos\vartheta_1 = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sqrt{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right) \operatorname{sen}^2\vartheta_1}$$

$$\boxed{\vartheta_p = \operatorname{arcsen} \sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} = \operatorname{arctg}\left(\frac{n_2}{n_1}\right)}$$

ϑ_p: angolo di BREWSTER

Teoria della Radiazione

Obbiettivo: un algoritmo SEMPLICE per il calcolo di $\vec{E}$ e $\vec{H}$.

→ SORGENTI SEMPLICI (da determinare)

1) Eq. di Maxwell (incluse le sorgenti)
2) Otteniamo l'eq. $\nabla^2 \bar{E} + \beta^2 \bar{E} =$ sorgenti ("complesse")
3) Troviamo la soluzione per sorgenti puntiformi elementari.
4) Introduciamo $\vec{A}$ (potenziale vettore) e otteniamo $\nabla^2 \vec{A} + \beta^2 \vec{A} =$ sorgenti ("semplici")
5) otteniamo $\vec{E}$ ed $\vec{H}$ da $\vec{A}$
6) Sorgente elementare: dipolo hertziano
7) Sorgenti composte

Dominio dei fasori

$$\begin{cases} \bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H} \\ \bar{\nabla} \times \bar{H} = j\omega\bar{E} + \bar{J} \quad \text{con} \quad \bar{J} = \sigma\bar{E} + \bar{J}_I \\ \bar{\nabla} \cdot \bar{E} = \frac{\rho}{\varepsilon} \\ \bar{\nabla} \cdot \bar{H} = 0 \\ \nabla \cdot \bar{J} = -j\omega\rho \end{cases}$$

$$\beta = \frac{\omega}{v} = \omega\sqrt{\mu\varepsilon}$$

$$\bar{\nabla} \times \bar{\nabla} \times \bar{E} = -j\omega\mu \, \bar{\nabla} \times \bar{H} = -j\omega\mu(j\omega\varepsilon\bar{E} + \bar{J})$$

ma $\bar{\nabla} \times \bar{\nabla} \times \bar{E} = \bar{\nabla}(\bar{\nabla} \cdot \bar{E}) - \nabla^2 \bar{E}$

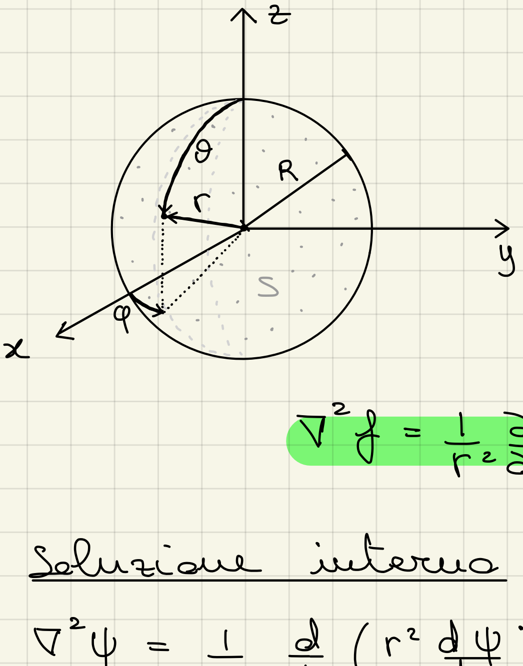
$$\longrightarrow \underbrace{\nabla^2 \bar{E} + \beta^2 \bar{E}}_{\left(\underline{\mathcal{L}}(\circ) = \nabla^2(\circ) + \beta^2(\circ) \quad \underline{\text{d'Alambertiano}}\right)} = \bar{\nabla}\left(\frac{\rho}{\varepsilon}\right) + j\omega\mu\bar{J} = \underbrace{-\frac{1}{j\omega\varepsilon}\bar{\nabla}(\bar{\nabla} \cdot \bar{J}) + j\omega\mu\bar{J}}_{\text{sorgenti}}$$

$$\left[\nabla^2 \psi + \beta^2 \psi = \text{sorgente puntiforme}\right]$$

(ψ: generica componente di $\bar{E}$)

Caso statico

$\omega = 0 \quad \beta = 0 \longrightarrow \nabla^2 \psi =$ sorgente puntiforme



$\nabla^2 \psi(r) = -S \qquad r \leq R$

$\nabla^2 \psi(r) = 0 \qquad r > R$ (no sorgenti)

$\psi(r, \varphi, \theta) \longrightarrow \psi(r)$ (simmetria sferica)

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \operatorname{sen}^2\theta}\frac{\partial^2 f}{\partial \varphi^2} + \frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial \theta}\left(\operatorname{sen}\theta \frac{\partial f}{\partial \theta}\right)$$

Soluzione interno ($r \leq R$)

$$\nabla^2 \psi = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) = -S \qquad d\left(r^2 \frac{d\psi}{dr}\right) = -Sr^2 dr$$

$$r^2 \frac{d\psi}{dr} = -S\frac{r^3}{3} + A \qquad \frac{d\psi}{dr} = -\frac{Sr}{3} + \frac{A}{r^2}$$

$$\boxed{\psi(r) = -\frac{Sr^2}{6} - \frac{A}{r} + B}$$

Soluzione esterna $(r > R)$

$$\frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) = 0 \qquad r^2 \frac{d\psi}{dr} = C \qquad d\psi = \frac{C}{r^2}\, dr$$

$$\boxed{\psi(r) = -\frac{C}{r} + D}$$

Condizioni al contorno

- se $r \to 0$ $\quad \psi(r)$ finito $\Longrightarrow A = 0$
- se $r \to +\infty$ $\quad \psi(r) \to 0 \Longrightarrow D = 0$

Impongo la continuità di $\psi(r)$ e $\frac{d\psi}{dr}(r)$ in $\underline{\underline{r = R}}$

$$\Longrightarrow \begin{cases} \psi(r) = s\left(\frac{1}{2}R^2 - \frac{1}{6}r^2\right) & r \leq R \\ \boxed{\psi(r) = \dfrac{sR^3}{3r}} & r > R \end{cases}$$

Volume della sfera vale $V = \frac{4}{3}\pi R^3$

$\psi(r) = \frac{V \cdot s}{4\pi r}$ se $V \to 0$ ma $\underbrace{V \cdot \underbrace{s}_{\text{densità di carica}}}_{\text{carica totale}} = \text{costante}$

Se ad esempio:

$\nabla^2 \psi = -\frac{\rho}{\varepsilon}$ cioè $s = \frac{\rho}{\varepsilon} \longrightarrow \frac{V \cdot \rho}{4\pi\varepsilon r} = \frac{Q}{4\pi\varepsilon r}$

potenziale elettrostatico

Caso dinamico

Soluzione esterna $(r > R)$

$$\nabla^2 \psi + \beta^2 \psi = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) + \beta^2 \psi = 0$$

poniamo $X = r \cdot \psi$

ossia $\psi = \frac{X}{r}$

$$\frac{d\psi}{dr} = \frac{1}{r}\frac{dX}{dr} - \frac{X}{r^2} \qquad r^2 \frac{d\psi}{dr} = r\frac{dX}{dr} - X$$

$$\frac{d}{dr}\left(r\frac{dX}{dr} - X\right) = \cancel{\frac{dX}{dr}} + r\frac{d^2X}{dr^2} - \cancel{\frac{dX}{dr}}$$

$$\frac{1}{r}\frac{d^2x}{dr^2} + \beta^2 \frac{x}{r} = 0 \qquad \frac{d^2x}{dr^2} + \beta^2 x = 0$$

$$\boxed{x(r) = M e^{-j\beta r} + N e^{+j\beta r}}$$

soluzione "centrifuga" e "centripeta"

Condizioni al contorno

- ∄ soluzione centripeta (é anticausale) $\Longrightarrow N = 0$

Caso dinamico tende al caso statico per $f \to 0$ cioé $\beta \to 0$

$$x(r) = M e^{-j\beta r} \longrightarrow \psi(r) = \frac{M e^{-j\beta r}}{r}$$

$$\lim_{\beta \to 0} \psi(r) = \frac{M}{r} = \frac{sV}{4\pi r} \Longrightarrow M = \frac{sV}{4\pi}$$

$$\Longrightarrow \boxed{\psi(r) = \frac{sV e^{-j\beta r}}{4\pi r}}$$

potenziale ritardato

Onda sferica:

* $|\psi(r)| \propto \frac{1}{r}$

* le sup. equifase sono delle sfere

Condizione di Sommerfeld:

qualsiasi effetto elettromagnetico ψ generato da una distribuzione di sorgenti di estensione limitata, deve soddisfare la seguente condizione:

$$\lim_{r \to +\infty} r \cdot \left(\frac{\partial \psi}{\partial r} + j\beta\psi \right) = 0$$

La soluzione centripeta NON soddisfa questa condizione.

Metodo dei potenziali

sorgenti vorticose di $\bar{A}$

$\bar{\nabla} \cdot \bar{H} = 0$ possiamo scrivere $\bar{H} = \bar{\nabla} \times \bar{A}$ poiché $\bar{\nabla} \cdot (\bar{\nabla} \times \bar{A}) = 0$

$$\bar{\nabla} \times \bar{E} = -j\omega\mu \bar{H} = -j\omega\mu\, \bar{\nabla} \times \bar{A} \qquad \bar{\nabla} \times \underbrace{(\bar{E} + j\omega\mu \bar{A})}_{\bar{F}} = 0$$

$\bar E + j\omega\mu\bar A = -\bar\nabla\phi$

$\bar\nabla\times\bar F = 0$ possiamo scrivere $\bar F = -\bar\nabla\phi$ poiché $\bar\nabla\times(\bar\nabla\phi) = 0$

$\bar\nabla\times\bar H = j\omega\varepsilon\bar E + \bar J$ $\quad\quad$ $\bar\nabla\times\bar\nabla\times\bar A = j\omega\varepsilon(-j\omega\mu\bar A - \bar\nabla\phi) + \bar J$

ma $\bar\nabla\times\bar\nabla\times\bar A = \bar\nabla(\bar\nabla\cdot\bar A) - \nabla^2\bar A$

$\longrightarrow \bar\nabla(\bar\nabla\cdot\bar A) - \nabla^2\bar A = \beta^2\bar A - j\omega\varepsilon\bar\nabla\phi + \bar J$ sorgenti pezzo di $\bar A$

Possiamo fissare a piacere $\bar\nabla\cdot\bar A$: $\bar\nabla\cdot\bar A = -j\omega\varepsilon\phi$

$\bar\nabla(\bar\nabla\cdot\bar A) = -j\omega\varepsilon\bar\nabla\phi$

$$\boxed{\nabla^2\bar A + \beta^2\bar A = -\bar J} \quad \longrightarrow \quad \boxed{\bar A(r) = \bar J\cdot\frac{V e^{-j\beta r}}{4\pi r}}$$

$$\textcircled{\#}\begin{cases}\bar H = \bar\nabla\times\bar A\\ \bar E = -j\omega\mu\bar A - \bar\nabla\phi = -j\omega\mu\bar A + \dfrac{1}{j\omega\varepsilon}\bar\nabla(\bar\nabla\cdot\bar A)\end{cases}$$

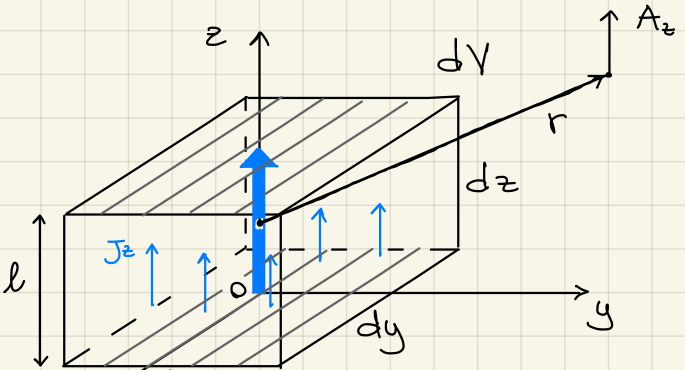
L'altra equazione (che usa la densità di carica invece che la densità di corrente) è:

$$\boxed{\nabla^2\phi + \beta^2\phi = -\frac{\rho}{\varepsilon}}$$

(più difficile da usare perché J è più conoscibile di ρ)

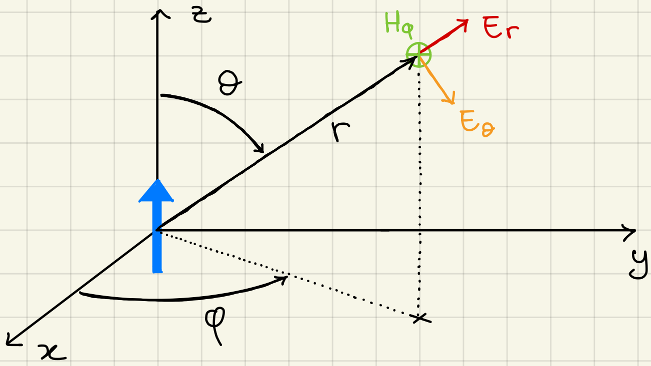
Sorgente elementare (dipolo elettrico o hertziano)

Hp: $\bar J = J\vec u_z$



$$\bar J\,dV = J_z\,dx\,dy\,dz\,\vec u_z$$
$$= I\,dz\,\vec u_z$$
$\parallel$
ℓ : lunghezza del dipolo

$\left\{A_z = \dfrac{I\ell\, e^{-j\beta r}}{4\pi r}\right\}$ introducendolo in $\textcircled{\#}$ ricavo le componenti di campo elettrico e magnetico

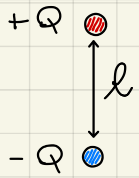


$$\begin{cases} E_r = \frac{I\ell}{2\pi} e^{-j\beta r}\left(\frac{\eta}{r^2} + \frac{1}{j\omega\varepsilon r^3}\right)\cos\vartheta \\ E_\vartheta = \frac{I\ell}{4\pi} e^{-j\beta r}\left(\frac{j\omega\mu}{r} + \frac{\eta}{r^2} + \frac{1}{j\omega\varepsilon r^3}\right)\operatorname{sen}\vartheta \\ H_\varphi = \frac{I\ell}{4\pi} e^{-j\beta r}\left(\frac{j\beta}{r} + \frac{1}{r^2}\right)\operatorname{sen}\vartheta \end{cases}$$

Campo "vicino" $r \to 0$

H_φ trascurabile

$$\left\{ E_r = \frac{I\ell\, e^{-j\beta r}}{2\pi j\omega\varepsilon r^3}\cos\vartheta \qquad E_\vartheta = \frac{I\ell\, e^{-j\beta r}}{4\pi j\omega\varepsilon r^3}\operatorname{sen}\vartheta \right\}$$



$$E_r = \frac{Q\ell\cos\vartheta}{2\pi\varepsilon r^3} \qquad E_\vartheta = \frac{Q\ell\operatorname{sen}\vartheta}{4\pi\varepsilon r^3}$$ caso statico

$$I = j\omega Q \qquad Q = \frac{I}{j\omega} \qquad r \to 0: \ \beta r = \frac{2\pi}{\lambda} r \longrightarrow 0 \qquad e^{-j\beta r} \to 1$$

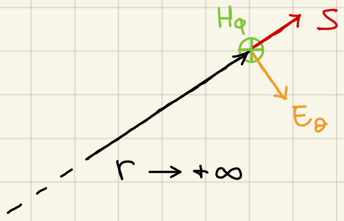
$$\left\{ E_r \simeq \frac{Q\ell\cos\vartheta}{\varepsilon 2\pi r^3} \qquad E_\vartheta \simeq \frac{Q\ell\operatorname{sen}\vartheta}{4\pi\varepsilon r^3} \right\}$$ campo $\bar{E}$ quasi-statico

Campo "lontano" $r \to +\infty \quad (r \gg \lambda)$

E_r trascurabile

$$\left\{ E_\vartheta = j\frac{\omega\mu I\ell\, e^{-j\beta r}}{4\pi r}\operatorname{sen}\vartheta \qquad H_\varphi = j\frac{\beta I\ell}{4\pi r} e^{-j\beta r}\operatorname{sen}\vartheta \right\}$$

$$E_\vartheta \perp H_\varphi \qquad \frac{E_\vartheta}{H_\varphi} = \frac{\omega\mu}{\beta} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$



Vettore di Poynting: $\bar{S} = \frac{\bar{E} \times \bar{H}^*}{2}$

$$\bar{E} \times \bar{H}^* = \det\begin{bmatrix} \vec{u}_r & \vec{u}_\vartheta & \vec{u}_\varphi \\ E_r & E_\vartheta & 0 \\ 0 & 0 & H_\varphi^* \end{bmatrix} = \underbrace{E_\vartheta H_\varphi^*}_{2\cdot S_r}\vec{u}_r - \underbrace{E_r H_\varphi^*}_{2\cdot S_\vartheta}\vec{u}_\vartheta$$

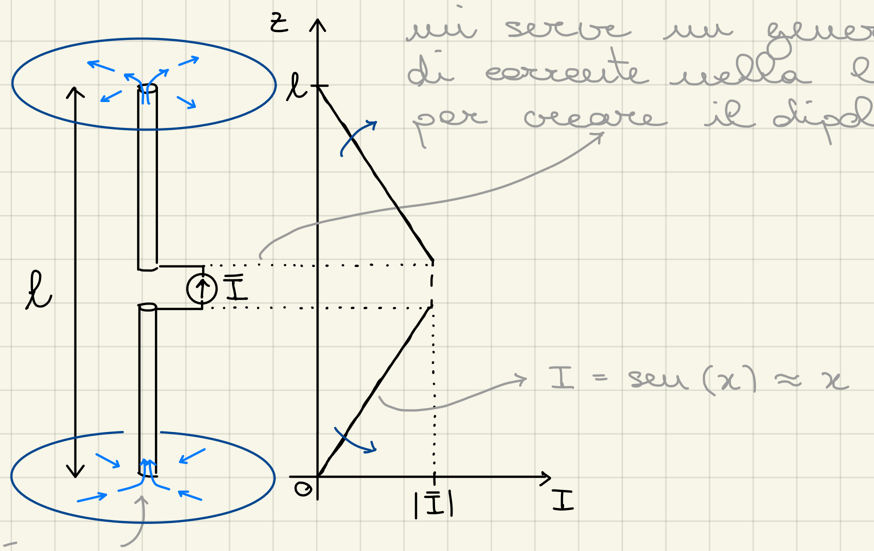
Ricordando che $\frac{j\omega\mu}{4\pi} = j\frac{\eta}{2\lambda}$ risulta:

$$\begin{cases} S_r = \frac{1}{2}\,\eta \frac{|I\ell|^2}{4\lambda^2}\left(\frac{1}{r^2} + \frac{1}{j\beta^3 r^5}\right) \text{sen}^2\vartheta \\ S_\vartheta = \frac{1}{2}\,\eta \frac{|I\ell|^2}{8\pi^2}\left(-\frac{j\beta}{r^3} + \frac{1}{j\beta^3 r^5}\right) \text{sen}\vartheta\cos\vartheta \end{cases}$$

componente solo immaginaria

$$\Longrightarrow \quad \text{Re}\{\bar{S}\} = \frac{1}{2} \cdot \frac{\eta}{4\lambda^2} \frac{|I\ell|^2}{r^2} \text{sen}^2\vartheta\, \vec{u}_r = \frac{E_\vartheta H_\varphi^*}{2}\, \vec{u}_r$$

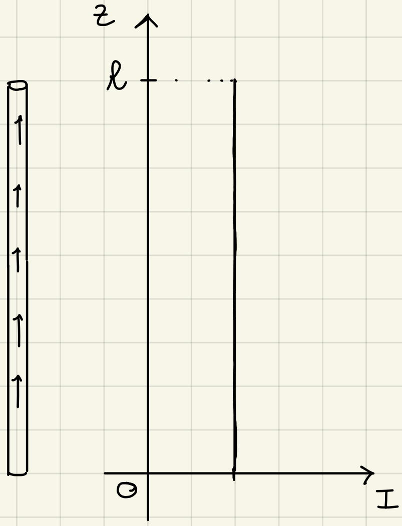
Le uniche componenti che contribuiscono al trasporto di potenza dell'onda sono E_ϑ e H_φ cioé le componenti del campo lontano (che per questo é anche detto "campo di radiazione")



mi serve un generatore di corrente nella linea per creare il dipolo

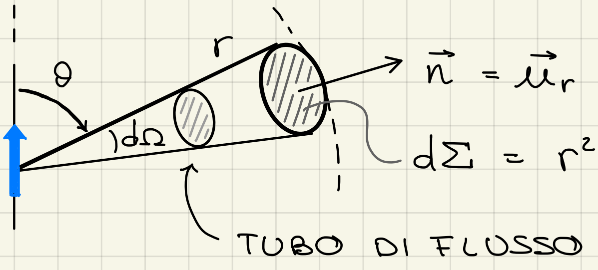
I = sen (x) ≈ x

é come una linea di trasmissione dove ℓ << λ



Per rendere il circuito di sinistra come quello di destra lo chiudo con un condensatore.

$$\boxed{\vec{S}(r,\vartheta) = \frac{\eta_0}{8} \frac{(I\ell)^2}{\lambda^2 r^2} \text{sen}^2\vartheta\, \vec{u}_r}$$



$\vec{n} = \vec{u}_r$

$d\Sigma = r^2 d\Omega = r^2 \text{sen}\vartheta\, d\vartheta\, d\varphi$

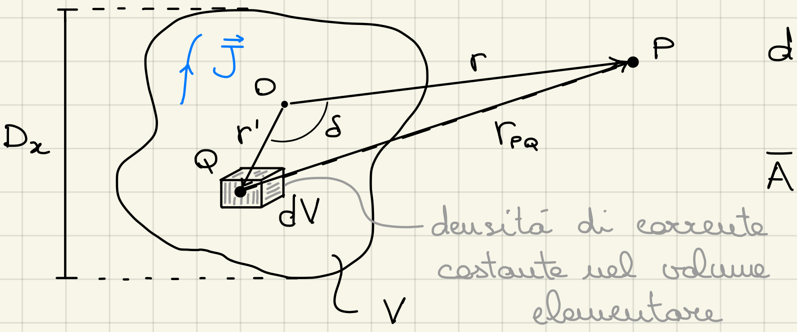
la potenza in un tubo di flusso é costante

$$dP = \vec{S} \cdot d\vec{\Sigma} = \eta_0 \frac{(I\ell)^2}{8\lambda^2 \cancel{r^2}} \text{sen}^2\vartheta\, \cancel{r^2}\, \text{sen}\vartheta\, d\vartheta\, d\varphi$$

$$P = \oint_{sfera} \vec{S}\cdot d\vec{\Sigma} = \int_0^{2\pi} d\varphi \int_0^{\pi} \eta_0 \frac{(Il)^2}{8\lambda^2} \mathrm{sen}^3\theta\, d\theta = 2\pi\eta_0 \frac{(Il)^2}{8\lambda^2} \underbrace{\int_0^{\pi} \mathrm{sen}^3\theta\, d\theta}_{4/3} =$$

$$\boxed{P = \frac{\pi}{3}\eta_0 I^2 \frac{l^2}{\lambda^2}} \quad [W]$$

Radiazione da un insieme di sorgenti



$$d\bar{A} = \frac{\bar{J}(Q)\,dV}{4\pi r_{PQ}} e^{-j\beta r_{PQ}}$$

$$\bar{A}(P) = \int_V d\bar{A} = \int_V \frac{\bar{J}(Q)\, e^{-j\beta r_{PQ}}}{4\pi r_{PQ}} dV$$

potenziale vettore totale = somma di potenziali vettori element.

Se $r \gg D_x \longrightarrow r_{PQ} \approx r - \underbrace{r'\cos\delta}_{\ll r}$ (poiché $r_{PQ} /\!/ r$ circa)

$$\bar{A}(P) = \int_V \frac{\bar{J}(Q)}{4\pi r} e^{-j\beta r} e^{+j\beta r'\cos\delta} dV = \frac{e^{-j\beta r}}{4\pi r} \underbrace{\int_V \bar{J}(Q)\, e^{+j\beta r'\cos\delta} dV}_{(I\cdot l)\ [A\cdot m]\ \text{momento di dipolo}} =$$

$$= \underbrace{\bar{N}(\theta,\varphi)}_{\text{vettore di radiazione}} \frac{e^{-j\beta r}}{4\pi r}$$

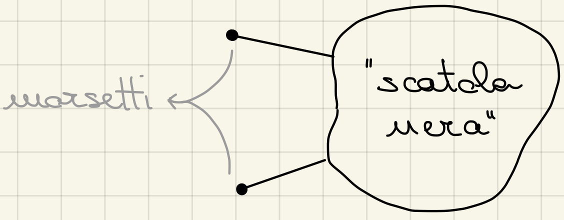
Si dimostra che il campo $\bar{E}$ di radiazione vale:

$$\bar{E} = -j\frac{\omega\mu}{4\pi r} e^{-j\beta r}\left[N_\theta(\theta,\varphi)\,\vec{u}_\theta + N_\varphi(\theta,\varphi)\,\vec{u}_\varphi\right]$$

$$\bar{E} = \eta_0 (\bar{H} \times \vec{u}_r)$$

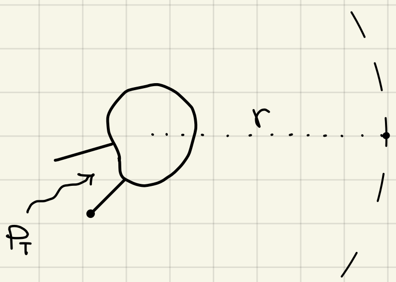
$$\bar{H} = \frac{j\beta}{4\pi r} e^{-j\beta r}\left[N_\varphi(\theta,\varphi)\,\vec{u}_\theta - N_\theta(\theta,\varphi)\,\vec{u}_\varphi\right]$$

Antenne come TRASDUTTORI → convertono una forma di energia in un'altra



- Passive
- Reciproche

Antenna isotropa (ideale, senza perdite - non esiste)



$$S(r) = \frac{P_T}{4\pi r^2} \left[\frac{W}{m^2}\right]$$

(S isotropa cioè non dipende da φ e θ)

$$\oint_{sfera} S(r)\cdot d\Sigma = P_T$$

Infatti: $\oint_{sfera} S(r)\cdot r^2 d\Omega = \oint_{sfera} \frac{P_T}{4\pi r^2}\cdot r^2 d\Omega = \frac{P_T}{4\pi}\oint_{sfera} d\Omega = \frac{P_T}{4\pi}\cdot 4\pi = P_T$

Le antenne reali sono direttive:

$$S(r,\theta,\varphi) = \frac{P_T}{4\pi r^2} f(\theta,\varphi)\cdot D$$

$f(\theta,\varphi)$: funzione di direttività

D: direttività

$0 \le f(\theta,\varphi) \le 1$ ∃ (almeno) una direzione $(\bar\theta,\bar\varphi)$ t.c. $f(\bar\theta,\bar\varphi) = 1$
(direzione di massima radiazione)

Per $\bar\theta, \bar\varphi$: $S_{MAX}(r,\bar\theta,\bar\varphi) = \frac{P_T}{4\pi r^2} D$ $S_{iso} = \frac{P_T}{4\pi r^2}$

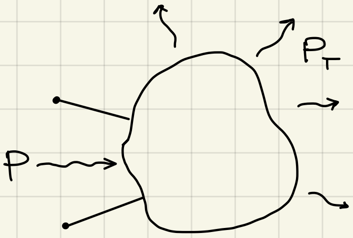
$\frac{S_{MAX}}{S_{iso}} = D\ (\ge 1)$ ma vale sempre $\oint_{sfera} S(r,\theta,\varphi)\, d\Sigma = P_T$

$$\oint_{sfera} \frac{\cancel{P_T}}{4\pi r^2} f(\theta,\varphi)\, D\cdot d\Sigma = \cancel{P_T}$$

$$\boxed{D = \frac{4\pi}{\oint_{sfera} f(\theta,\varphi)\, d\Omega}} \longleftarrow \frac{D}{4\pi}\cdot\oint_{sfera} f(\theta,\varphi)\, d\Omega = 1$$

sempre valida

∃ sempre perdite:



$P_T \le P$

$P_T = \nu P$

con $0 \le \nu \le 1$ rendimento

$$S(r,\theta,\varphi) = \frac{P\,\overbrace{\nu D}^{G}}{4\pi r^2} f(\theta,\varphi)$$

$\nu\cdot D = G \ge 0$ guadagno

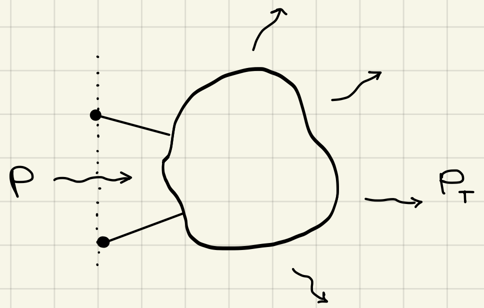
Di solito il guadagno è espresso in dB_I $10\log_{10}\frac{G}{\underset{=1}{G_{iso}}} = 10\log_{10} G$
rispetto all'isotropo

rispetto al dipolo risonante

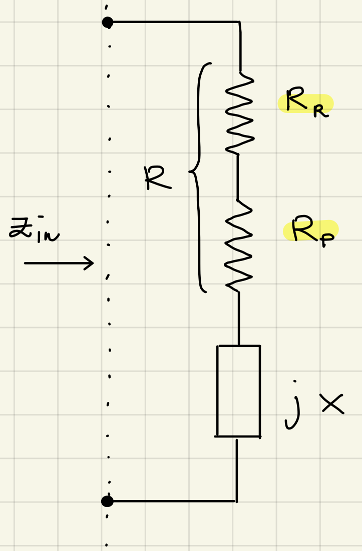
A volte in dB_d $\quad 10\log_{10}\frac{G}{1{,}64}$.

Parametri delle antenne: $D,\ f(\theta,\varphi),\ \nu,\ G,\ R,\ A_e,\ l_e$

antenna trasmittente



equivalente circuitale



R_R resistenza di radiazione

R_p resistenza di perdita

jX reattanza parassita

$$\nu = \frac{P_T}{P} \quad \text{ma} \quad P_T = \frac{|I|^2}{2}R_R$$

$$P_p = \frac{|I|^2}{2}R_p$$

$$P = P_T + P_p$$

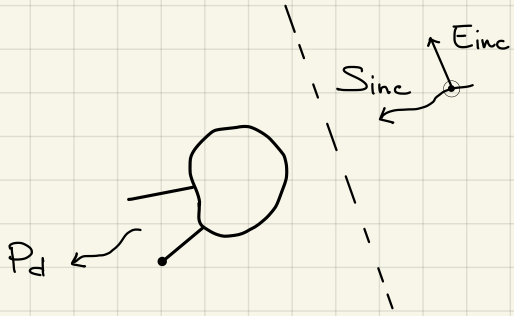
$$Z_{in} = R + jX$$

$$\Longrightarrow \quad \nu = \frac{R_R}{R_R + R_p}$$

il rendimento è solitamente maggiore per antenne più grandi

poiché R_R cresce più rapidamente di R_p al crescere delle dimensioni

Antenne riceventi



Hp:
- onda incidente è TEM (localmente) piana
- c'è adattamento di polarizzazione

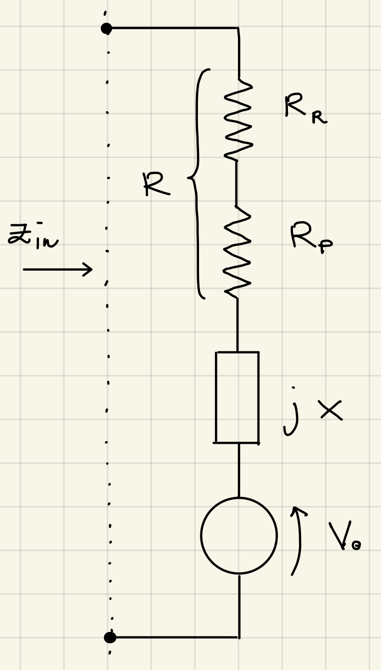
$$P_R = P_d = S_{inc} \cdot A_e \cdot f_R(\theta,\varphi)$$
$$\left[\frac{W}{m^2}\right] \quad [m^2]$$
$$\downarrow$$

area efficace

Per antenne reciproche si dimostra che:

$$f_R(\theta,\varphi) = f_T(\theta,\varphi)$$

Relazione universale: $\boxed{\frac{G}{A_e} = \frac{4\pi}{\lambda^2}}$



Tensione a vuoto: $V_0 = E_{inc} \cdot l_e \cdot \sqrt{f_R(\theta,\varphi)}$

$$\underset{\left[\frac{V}{m}\right]}{E_{inc}} \quad \underset{[m]}{l_e} \downarrow$$

lunghezza efficace
(numero complesso)

$$P_d = \frac{|V_0|^2}{8R} = \frac{|E_{inc}|^2 |l_e|^2}{8R} f_R(\theta,\varphi)$$

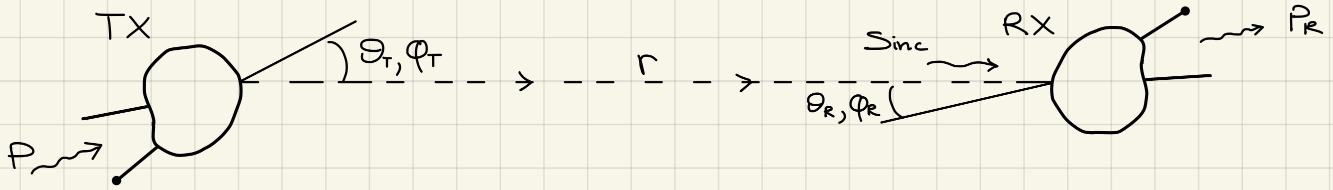
ma $P_d = S_{inc} A_e f_R(\theta,\varphi)$

$$\Longrightarrow \cancel{S_{inc}} A_e \cancel{f_R(\theta,\varphi)} = \frac{|E_{inc}|^2}{2\eta_0} \frac{|l_e|^2}{4R} \eta_0 \cancel{f_R(\theta,\varphi)}$$

$$\boxed{A_e = \frac{|l_e|^2 \eta_0}{4R}}$$

equivalente circuitale
dell'antenna ricevente

Link Budget (spazio libero)



$P_R = S_{inc} A_e f_R(\theta_R,\varphi_R)$ ma $S_{inc} = \frac{P G_{TX} f_T(\theta_T,\varphi_T)}{4\pi r^2}$ e $A_e = \frac{G\lambda^2}{4\pi}$

$$\Longrightarrow \left[P_R = \overset{TX}{\boxed{P \cdot G_{TX} f_T(\theta_T,\varphi_T)}} \underbrace{\left(\frac{\lambda}{4\pi r}\right)^2}_{\text{attenuazione di spazio libero}} \overset{RX}{\boxed{G_{RX} f_R(\theta_R,\varphi_R)}} \right]$$

Se le antenne sono "ben puntate": $f_T(\theta_T,\varphi_T) = 1$

$f_R(\theta_R,\varphi_R) = 1$

$$P_R = \underbrace{P G_{TX}}_{} \left(\frac{\lambda}{4\pi r}\right)^2 G_{RX}$$

E.I.R.P. [W] = Equivalent
Isotropically
Radiated
Power

(ad es.: WiFi
EIRP = 0,1 W)

Parametri del dipolo hertziano (ideale, $\nu = 1$)

$$S = \frac{\eta_0 (Il)^2}{8\lambda^2 r^2} \operatorname{sen}^2\theta \quad e \quad P_T = \frac{\pi}{3}\eta_0 \frac{(Il)^2}{\lambda^2}$$

$$\Longrightarrow \quad S = \underbrace{\frac{P_T}{4\pi r^2} \cdot \frac{3}{2} \cdot \operatorname{sen}^2\theta}_{\text{densità di potenza del dipolo}} \quad \text{confronto con} \quad S = \underbrace{\frac{P_T}{4\pi r^2} D\, f(\theta,\varphi)}_{\text{densità di potenza di un'antenna}}$$

Per il dipolo hertziano: $\begin{cases} f(\theta,\varphi) = \operatorname{sen}^2\theta \\ D = \dfrac{3}{2} = G \quad (\nu = 1) \end{cases}$

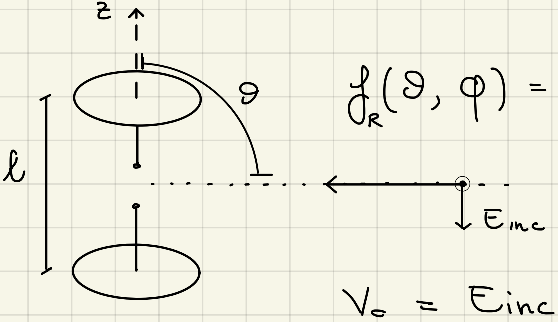
$\{R_p \propto l\}$

$\{R_R \propto l^2\}$

$$R_R \longrightarrow P_T = \frac{|I|^2}{2} R_R = \frac{\pi}{3}\eta_0 \frac{(Il)^2}{\lambda^2} \longrightarrow \left\{ R_R = \frac{2}{3}\pi\eta_0 \left(\frac{l}{\lambda}\right)^2 \right\}$$

NB: potevo anche ricavare D usando la formula

$$D = \frac{4\pi}{\iint_{sfera} f(\theta,\varphi)\, d\Omega} = \frac{4\pi}{\int_0^{2\pi} d\varphi \int_0^{\pi} \operatorname{sen}^2\theta \operatorname{sen}\theta\, d\theta}$$



$$f_R(\theta,\varphi) = 1 \longrightarrow \theta = \frac{\pi}{2}$$

è in adattamento di polarizzazione

$$V_0 = E_{inc} \cdot l \quad \text{ma} \quad V_0 = E_{inc} \cdot l_e \longrightarrow \{ l_e = l \}$$

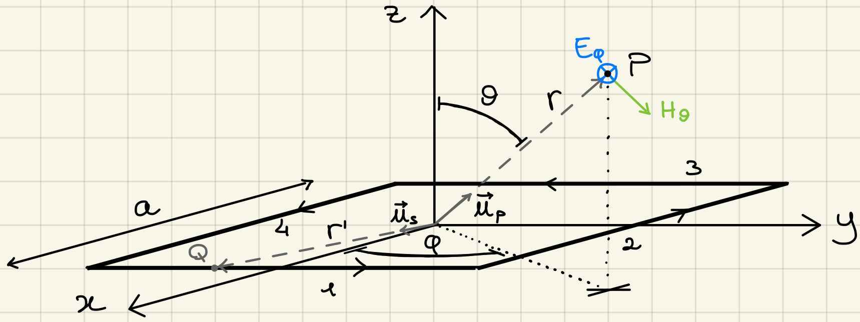
$$\left\{ A_e = \frac{|l_e|^2 \eta_0}{4 R_R} = \frac{|l|^2 \cancel{\eta_0}}{4 \cdot \frac{2}{3}\pi \cancel{\eta_0} \left(\frac{l}{\lambda}\right)^2} = \frac{3\lambda^2}{8\pi} \right\} \quad [m^2]$$

Potevo ricavare A_e dalla formula universale

$$\frac{A_e}{G} = \frac{\lambda^2}{4\pi}, \quad G = \frac{3}{2}$$

Sorgente composita (spira piccola)

↳ $a << \lambda$



Calcoleremo $\bar{N}(\theta,\varphi)$ e quindi $\bar{A}$ in P.

$\bar{N}(\vartheta,\varphi) = \int_V \bar{J}(Q)\, e^{+j\beta r'\cos\delta}\, dV$ con $\cos\delta = \vec{u}_p \cdot \vec{u}_s$

Se la spira é piccola: $r' << r$, $r' \leq a \rightarrow r' << \lambda$

$\Longrightarrow e^{+j\beta r'\cos\delta} \approx 1 + j\beta r'\cos\delta$ e $r' \approx \frac{a}{2}$

$$\bar{N}(\vartheta,\varphi) = \sum_{i=1}^{4} \underbrace{\bar{I}_i \cdot a}_{\bar{J}(Q)\cdot dV} \left(1 + j\beta \frac{a}{2} \cdot \vec{u}_p \cdot \vec{u}_{s_i}\right)$$

$$\vec{u}_p = \text{sen}\,\vartheta \cos\varphi\, \vec{u}_x + \text{sen}\,\vartheta\, \text{sen}\,\varphi\, u_y + \cos\vartheta\, u_z$$

lato	$\vec{I}_i$	$\vec{u}_{s_i}$	$\vec{u}_{s_i} \cdot \vec{u}_p$
1	$I\vec{u}_y$	u_x	$\text{sen}\,\vartheta \cos\varphi$
2	$-I\vec{u}_x$	u_y	$\text{sen}\,\vartheta\, \text{sen}\,\varphi$
3	$-I\vec{u}_y$	$-u_x$	$-\text{sen}\,\vartheta \cos\varphi$
4	$I\vec{u}_x$	$-u_y$	$-\text{sen}\,\vartheta \cos\varphi$

$$\Longrightarrow \bar{N}(\vartheta,\varphi) = j\beta a^2 I\, \text{sen}\,\vartheta \left(\underbrace{-\text{sen}\,\varphi\, \vec{u}_x + \cos\varphi\, \vec{u}_y}_{\vec{u}_\varphi}\right)$$
$$= j\beta a^2 I\, \text{sen}\,\vartheta\, \vec{u}_\varphi$$

$$\left\{ \bar{A}(r,\vartheta,\varphi) = \bar{N}(\vartheta,\varphi) \frac{e^{-j\beta r}}{4\pi r} = j\beta a^2 I\, \text{sen}\,\vartheta \frac{e^{-j\beta r}}{4\pi r}\, \vec{u}_\varphi \right\}$$

Sostituendo $\bar{A}$ nelle equazioni # ricavo $\bar{E}$ e $\bar{H}$:

$$\begin{cases} H_r = \dfrac{j\omega\mu I \cdot S}{2\pi\eta_0} \left(\dfrac{1}{r^2} - \dfrac{j}{\beta r^3}\right) \cos\vartheta\, e^{-j\beta r} \\[2ex] H_\vartheta = \dfrac{j\omega\mu I S}{4\pi\eta_0} \left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} - \dfrac{j}{\beta r^3}\right) \text{sen}\,\vartheta\, e^{-j\beta r} \\[2ex] E_\varphi = -\dfrac{j\omega\mu I S}{4\pi} \left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right) \text{sen}\,\vartheta\, e^{-j\beta r} \end{cases}$$

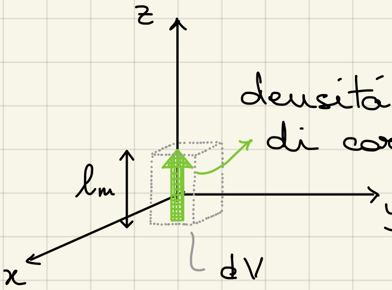
sono gli stessi per una spira di forma qualsiasi

con S: area della spira "piccola"

I campi elettrico e magnetico sono invertiti rispetto al dipolo hertziano.

Per questo si dice che dipolo e spira sono sorgenti duali.

La spira é equivalente a un dipolo attraversato da corrente magnetica



densitá volumetrica di corrente magnetica $\bar{J}_m \left[\frac{V}{m^2}\right]$

$$\bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H} (-\bar{J}_m)$$

$$\bar{\nabla} \times \bar{H} = j\omega\varepsilon\bar{E} + \bar{J}$$

$$J_m \cdot dV = V_0 \cdot l_m \leftarrow \text{momento di dipolo magnetico}$$

$$\frac{V}{m^2} \cdot m^3 = V \cdot m$$

Campo lontano

$$\left\{ H_\theta = j\frac{\omega\mu I S}{4\pi\eta_0}\left(\frac{j\beta}{r}\right) \operatorname{sen}\theta \, e^{-j\beta r} \qquad E_\varphi = -j\frac{\omega\mu I S}{4\pi}\left(\frac{j\beta}{r}\right) \operatorname{sen}\theta \, e^{-j\beta r} \right\}$$

$$\operatorname{Re}\left\{\frac{\bar{S}}{2}\right\} = \frac{1}{2} E_\varphi \cdot H_\theta^*$$

il vettore di Poynting dipende solo dalle componenti di campo di radiazione

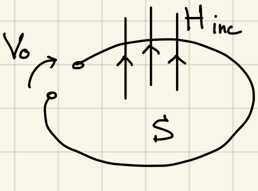
$$\frac{E_\varphi}{H_\theta} = -\eta_0$$

$$\frac{1}{2}\operatorname{Re}\{\bar{S}\} = \frac{1}{2}\frac{|E_\varphi|^2}{\eta_0} = \frac{1}{2}\eta_0 |H_\theta|^2$$ (come nel dipolo)

funzione di direttività — direttivitá — area efficacie

$$\{f(\theta) = \operatorname{sen}^2\theta\} \longrightarrow \left\{D = \frac{3}{2}\right\} \longrightarrow \left\{A_e = D \cdot \frac{\lambda^2}{2\pi} = \frac{3\lambda^2}{8\pi}\right\}$$

$$E_{inc} \cdot l_e = V_0$$



$\longleftarrow$ Adattamento di polarizzazione $H_{inc} \perp$ spira

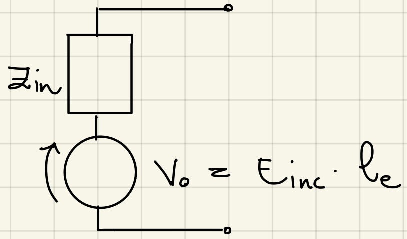
$$V_0 = j\omega\mu H_\perp S$$

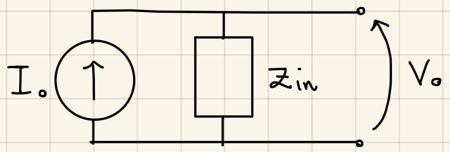
$$|e| = \frac{d\Phi(\vec{B})}{dt}$$

$$\text{ma } H_\perp = H_{inc} = \frac{E_{inc}}{\eta_0} \qquad V_0 = j\omega\mu\frac{E_{inc}}{\eta_0}S$$

$$\longrightarrow \left\{ l_e = j\frac{\omega\mu S}{\eta_0} = j\beta S \right\}$$ lunghezza efficacie (elettrica)

Lunghezza efficacie magnetica ℓ_m





$I_0 = H_{inc}\,\ell_m$

$A = \frac{A}{m} \cdot m$

$I_0 Z_{in} = V_0$ $\quad H_{inc} \cdot \ell_m \cdot Z_{in} = E_{inc} \cdot \ell_e$ $\quad$ ma $\quad H_{inc} = \frac{E_{inc}}{\eta_0}$

$$\boxed{\frac{\ell_e}{\ell_m} = \frac{Z_{in}}{\eta_0}}$$

Ora dimostreremo che: $j\omega\mu I S = V \ell_m$

$$j\omega\mu I_0 S = j\omega\mu \frac{V_0}{Z_{in}} S = j\omega\mu \frac{\ell_m}{\ell_e \eta_0} V_0 S = \cancel{j\beta S} \frac{\ell_m}{\cancel{\ell_e}} V_0 = \ell_m V_0$$

$\rightarrow \ell_e = j\beta S$

$$R_R = \frac{|\ell_e|^2 \eta_0}{4 A_e} = \eta_0 \frac{8\pi^3}{3}\left(\frac{S}{\lambda^2}\right)^2 \quad [\Omega]$$ resistenza di radiazione

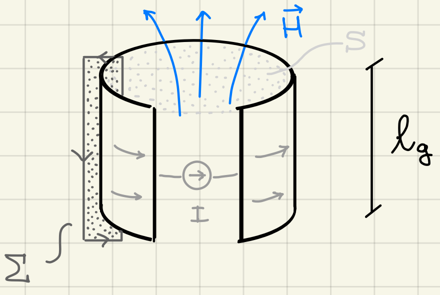
$Z_{in} = R_R + jX \rightarrow X = ?$ Senza delle specifiche sulla geometria della spira non siamo in grado di determinarne la reattanza.

Senza la reattanza non siamo in grado di determinare la lunghezza efficacie magnetica poiché

$$\ell_m = \ell_e \cdot \frac{\eta_0}{Z_{in}}$$

Nastro di corrente

Verifichiamo che: $\ell_g = \ell_m$



$$\int_\Sigma \bar\nabla \times \bar H \cdot d\bar\Sigma = \int_\Sigma \bar J \cdot d\bar\Sigma = I$$

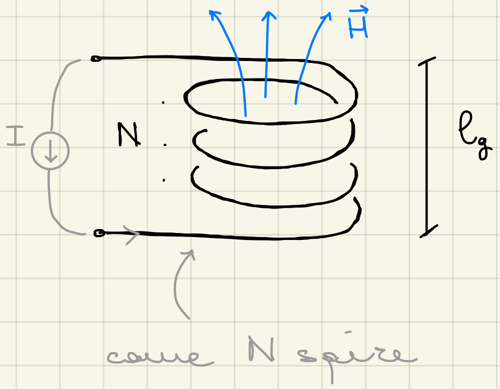
($j\omega\mu \bar E$ manca: $E_t = 0$)

$H \cdot \ell_g = I$ ma $V \ell_m = j\omega\mu I S = j\omega\mu H \ell_g S$

$j\omega \phi_B \ell_g = V_{fem} \cdot \ell_g$; se il conduttore è ideale

$V_{fem} = V \longrightarrow \boxed{\ell_m = \ell_g}$ $\quad L = \mu_0 \frac{S}{\ell_g}$

Solenoide

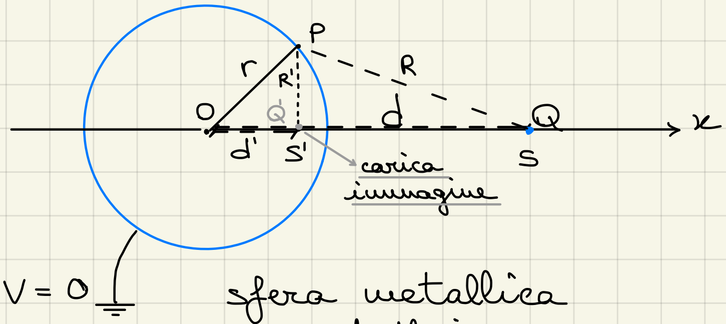


$$\frac{l_g}{N} = l_m$$

in TX:	spira	solenoide
campo	E_0	$N E_0$
densità di potenza	$S_0 = \frac{\lvert E_0 \rvert^2}{2\eta_0}$	$S_s = \frac{N^2 \lvert E_0 \rvert}{2\eta_0} = N^2 S_0$
potenza trasmessa	P_0	$N^2 P_0$
resistenza di radiazione	R_{R_0}	$N^2 R_{R_0}$
tensione a vuoto	V_0	$N V_0$
potenza disponibile	P_{d_0}	P_{d_0}

$$P_{d_s} = \frac{\lvert N V_0 \rvert^2}{8 N^2 R_{g_0}} = \frac{\cancel{N^2} \lvert V_0 \rvert^2}{8 \cancel{N^2} R_{R_0}} = \frac{\lvert V_0 \rvert^2}{8 R_{R_0}} = P_{d_0}$$

Esercizi



sfera metallica conduttrice

$\overline{OS} = d$

$\overline{OS'} = d'$

$\overline{S'P} = r$

$$V(P) = 0 = \frac{Q}{4\pi\varepsilon_0 R} + \frac{Q'}{4\pi\varepsilon_0 R'} \quad \forall P \in \text{sfera}$$

$$\frac{Q}{R} = -\frac{Q'}{R'}$$

$$\underbrace{\frac{Q}{Q'}}_{\text{costante}} = -\frac{R}{R'}$$

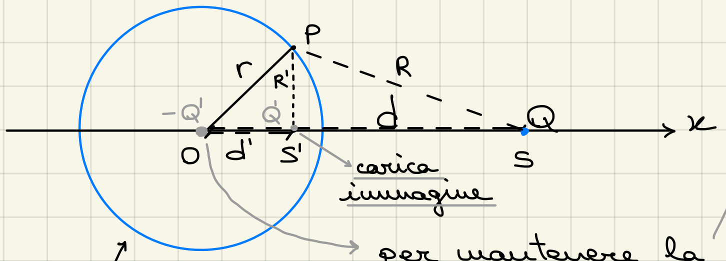
costante se $O\hat{S}P$ e $O\hat{S'}P$ sono simili

$$\frac{\overline{OP}}{\overline{OS}} = \frac{\overline{S'P}}{\overline{SP}} = \frac{\overline{OS'}}{\overline{OP}} \Rightarrow \underbrace{\frac{r}{d}}_{\text{costante}} = \frac{R'}{R} = \frac{d'}{r}$$

$$\rightarrow \boxed{d' = \frac{r^2}{d}} \quad \boxed{Q' = -Q\frac{r}{d}}$$

posizione e valore della carica per il metodo delle immagini

E se la sfera non fosse a potenziale nullo ?



V = costante = ?

per mantenere la neutralità di carica

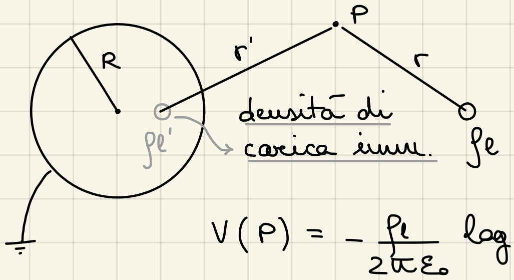
V(P) sulla sfera ?

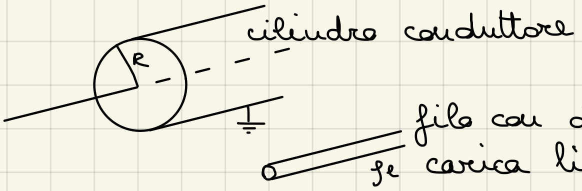
$\boxed{Q_{tot} = 0}$ sulla sfera

$$Q' = -Q\frac{r}{d} \qquad d' = \frac{R^2}{d}$$

$V(P) = 0$ $\quad$ V(P) 3 contributi: $\begin{cases} Q \\ Q' \end{cases} = 0$, $-Q'$

$$V(P) = \frac{-Q'}{4\pi\varepsilon_0 r} = \frac{Q\,^r/_d}{4\pi\varepsilon_0 r} = +\frac{Q}{4\pi\varepsilon_0 d}$$





filo con densità di carica lineare

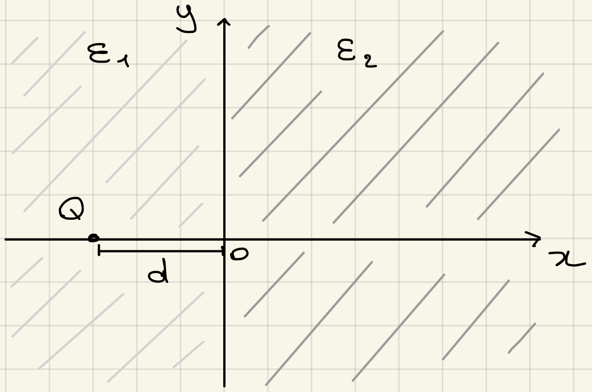
$$V(P) = -\frac{\rho_l}{2\pi\varepsilon_0}\log r - \frac{\rho_l'}{2\pi\varepsilon_0}\log r' + c$$

$\exists$ soluzione se $\rho_l = -\rho_l'$

$$V(P) = -\frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{r'}{r}\right) + c = 0 \qquad \forall P \in \text{cilindro}$$

$$\frac{r}{r'} = \text{costante } (\forall P \in \text{cilindro}) \Longrightarrow \quad d' = \frac{R^2}{d}$$

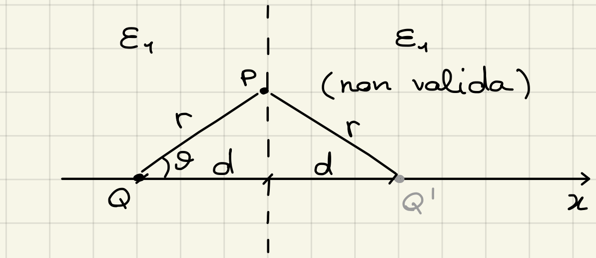
$$\frac{r}{r'} = \frac{d}{R} \Rightarrow -\frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{d}{R}\right) + c = 0$$

- 

$\rightarrow E_{t1} = E_{t2}$

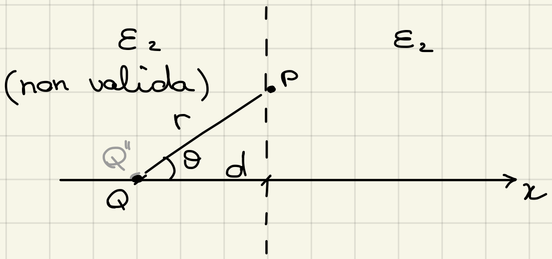
$(D_{n1} = D_{n2})$

$\rightarrow \varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$



$$E_{t1} = \frac{Q\sin\theta}{4\pi\varepsilon_1 r} + \frac{Q'\sin\theta}{4\pi\varepsilon_1 r}$$

$$\varepsilon_1 E_{n1} = \cancel{\varepsilon_1}\left(\frac{Q\cos\theta}{4\pi\cancel{\varepsilon_1} r} - \frac{Q'\cos\theta}{4\pi\cancel{\varepsilon_1} r}\right)$$



$$E_{t2} = \frac{Q\sin\theta}{4\pi\varepsilon_2 r} + \frac{Q''\sin\theta}{4\pi\varepsilon_2 r}$$

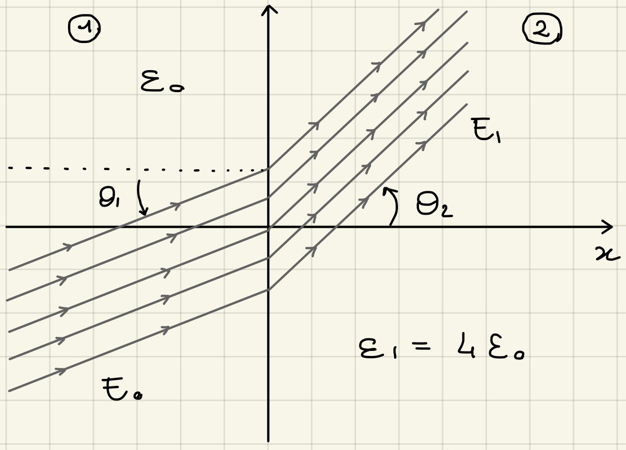
$$\varepsilon_2 E_{n2} = \cancel{\varepsilon_2}\left(\frac{Q\cos\theta}{4\pi\cancel{\varepsilon_2} r} + \frac{Q'\cos\theta}{4\pi\cancel{\varepsilon_2} r}\right)$$

$$\frac{Q\cancel{\sin\theta}}{4\pi\varepsilon_1\cancel{r}} + \frac{Q'\cancel{\sin\theta}}{4\pi\varepsilon_1\cancel{r}} = \frac{Q\cancel{\sin\theta}}{4\pi\varepsilon_2\cancel{r}} + \frac{Q''\cancel{\sin\theta}}{4\pi\varepsilon_2\cancel{r}}$$

$$\frac{Q\cancel{\cos\theta}}{\cancel{4\pi r}} - \frac{Q'\cancel{\cos\theta}}{\cancel{4\pi r}} = \frac{Q\cancel{\cos\theta}}{\cancel{4\pi r}} + \frac{Q''\cancel{\cos\theta}}{\cancel{4\pi r}}$$

$$\begin{cases} \dfrac{Q+Q'}{\varepsilon_1} = \dfrac{Q+Q''}{\varepsilon_2} \\ Q - Q' = Q + Q'' \end{cases} \Longrightarrow \boxed{\begin{array}{l} Q'' = Q\,\dfrac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1} \\ Q' = -Q'' \end{array}}$$

$$V(x,y) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0\sqrt{(x+d)^2+y^2}} - \dfrac{Q\,\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}}{4\pi\varepsilon_0\sqrt{(x-d)^2+y^2}}, & x < 0 \\ \dfrac{Q}{4\pi\varepsilon_0\sqrt{(x+d)^2+y^2}} + \dfrac{Q\,\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}}{4\pi\varepsilon_0\sqrt{(x+d)^2+y^2}}, & x > 0 \end{cases}$$



Quanto deve valere θ_1 di modo che $\theta_2 = \frac{\pi}{4}$?

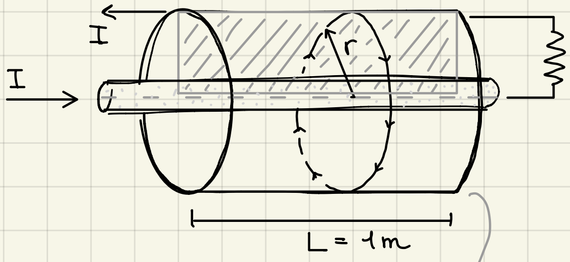
Condizioni al contorno:

$$\begin{cases} E_{t_1} = E_{t_2} \\ D_{n_1} = D_{n_2} \end{cases} \quad (\rho_s = 0) \rightarrow \qquad \rightarrow \begin{cases} E_{t_1} = E_{t_2} \\ \varepsilon_0 E_{n_1} = \varepsilon_1 E_{n_2} \end{cases}$$

$$\rightarrow \begin{cases} \underline{E_0 \operatorname{sen} \theta_1} = \underline{E_1 \operatorname{sen} \theta_2} \\ \varepsilon_0 E_0 \cos \theta_1 = \varepsilon_1 E_1 \cos \theta_2 \end{cases} \Longrightarrow \frac{1}{\varepsilon_0} \operatorname{tg} \theta_1 = \frac{1}{\varepsilon_1} \underset{\uparrow \, \pi/4}{\operatorname{tg} \theta_2}$$

$$\rightarrow \operatorname{tg} \theta_1 = \frac{1}{4} \Longrightarrow \boxed{\theta_1 \simeq 14^\circ}$$

Induttanza (esterna) di un coassiale



ricavato dalla circuitazione

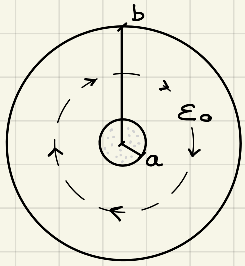
$$H_\phi = \frac{I}{2\pi r} \qquad B_\phi = \mu_0 H_\phi$$

(× 1m)

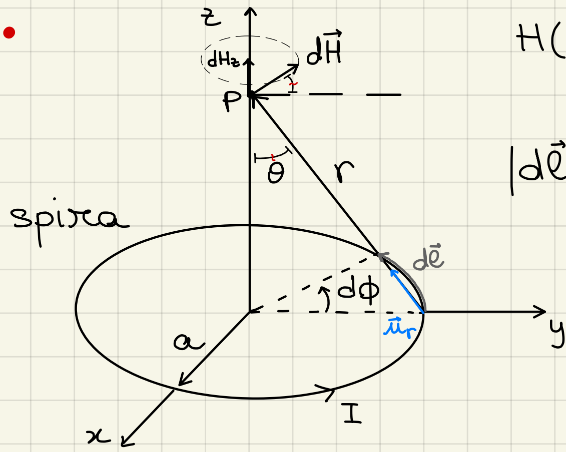
$$L = \frac{\Phi(B)}{I} = \frac{\mu_0}{I} \int_a^b \frac{I}{2\pi r} dr =$$

$$= \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \qquad \left[\frac{H}{m}\right]$$

induttanza per unità di lunghezza



il conduttore coassiale è infinitamente esteso; ne calcolo l'induttanza su una sezione (L = 1m) e poi la generalizzo per unità di lunghezza



H(z) lungo l'asse z ?

$|d\vec{\ell}| = a \cdot d\phi$ è perpendicolare a $\vec{u}_r$

$$d\vec{H} = \frac{I \, d\vec{\ell} \times \vec{u}_r}{4\pi r^2}$$ è perpendicolare a $\vec{u}_r$ e $d\vec{\ell}$

$$dH = \frac{I a \, d\phi}{4\pi r^2} = \frac{I a \, d\Phi}{4\pi (a^2+z^2)}$$

con $r^2 = a^2 + z^2$ e $0 \le \phi \le 2\pi$

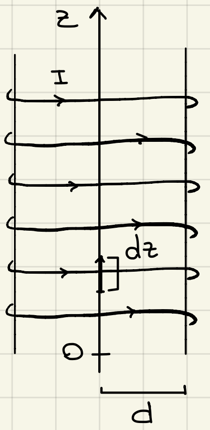
$$dH_z = \frac{I \cdot a \cdot d\phi}{4\pi(a^2+z^2)} \operatorname{sen}\vartheta \quad \text{ma} \quad \operatorname{sen}\vartheta = \frac{a}{r} = \frac{a}{\sqrt{a^2+z^2}}$$

$$H_z = \int_0^{2\pi} dH_z = \frac{I a^2}{4\pi(a^2+z^2)^{3/2}} \cdot 2\pi = \frac{I a^2}{2(a^2+z^2)^{3/2}} \quad \left[\frac{A}{m}\right]$$

le componenti radiali di H sono complessivamente nulle

$$\boxed{z = 0} \rightarrow H_z = \frac{I}{2a}$$ sul piano della sfera

- Campo H all'interno di un solenoide infinito



Campo $\vec{H}$ sull'asse z ?

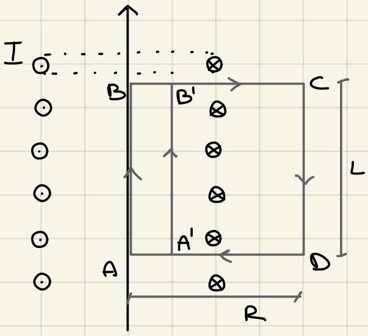
$\}$ N [spire/metro]

dai risultati precedenti

N·dz spire $\quad dH_z = \frac{I a^2 N dz}{2(a^2+z^2)^{3/2}}$

$$H_z = \int_{-\infty}^{+\infty} \frac{I a^2 N}{2(a^2+z^2)^{3/2}} dz = \frac{I a^2 N}{2} \underbrace{\int_{-\infty}^{\infty} (a^2+z^2)^{-3/2} dz}_{2/a^2}$$

$$= N \cdot I$$



R > d

$$\oint_{ABCD} \vec{H} \cdot d\vec{l} = \underbrace{\int_{\Sigma} \vec{J} \cdot d\vec{\Sigma}}_{N \cdot I}$$

$$\oint_{ABCD} \vec{H} \cdot d\vec{l} = \int_A^B \vec{H} \cdot d\vec{l} + \cancel{\int_B^C \vec{H} \cdot d\vec{l}} + \int_C^D \vec{H} \cdot d\vec{l} + \cancel{\int_D^A \vec{H} \cdot d\vec{l}}$$

$$= \underbrace{\cancel{H_z \cdot L}}_{N \cdot I} + \int_C^D H(R) dz = \cancel{N \cdot I}$$

$\longrightarrow H(R) = 0$

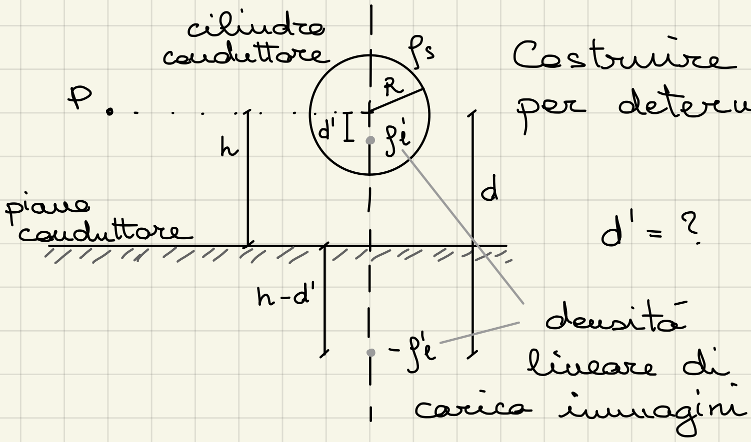
il campo magnetico all'esterno del solenoide è nullo

$$\oint_{A'B'CD} \vec{H} \cdot d\vec{l} = \int_{A'}^{B'} \vec{H} \cdot d\vec{l} + \cancel{\int_{B'}^{C} \cdots} + \int_C^D \cdots + \cancel{\int_D^{A'} \cdots}$$

$$= \int_{A'}^{B'} \vec{H} \cdot d\vec{l} + 0 = N \cdot I$$

$\longrightarrow H_z = N \cdot I$

il campo magnetico all'interno del solenoide è uniforme



Costruire il sistema di immagini (2D) per determinare V(P)

$d' = ?$ $d' = \frac{R}{d}$ (da risultati precedenti)

$d = 2h - d'$

$d' = \frac{R^2}{2h-d'} \rightarrow d'^2 - 2hd' + R^2 = 0$

ρ'_l deve essere tale da mantenere invariata la carica totale nel conduttore:

$$\rho_s \cdot 2\pi R \cdot \underset{\substack{\uparrow \\ \text{lunghezza del cilindro}}}{\cancel{L}} = \rho'_l \cdot \cancel{L} \longrightarrow \rho'_L = 2\pi R \rho_s$$

- Regione di spazio vuoto (μ_0, ε_0)

$\vec{E}(x,y,z,t)$ $t = 0 \rightarrow \vec{E} = 0$

$t > 0 \rightarrow$ si genera: $B_x = -Ky$, $B_y = Kx$, $B_z = 0$

$\vec{E}(t>0) = ?$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

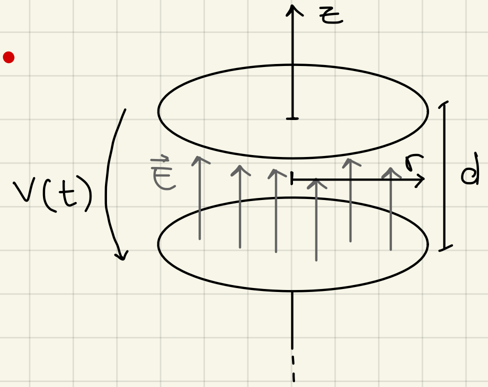
$$\vec{\nabla} \times \vec{B} = \det \begin{bmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -Ky & Kx & 0 \end{bmatrix} \longrightarrow \begin{cases} -\frac{\partial(Kx)}{\partial z} = \mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t} = 0 \\ -\frac{\partial(-Ky)}{\partial z} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} = 0 \\ \frac{\partial(Kx)}{\partial x} - \frac{\partial(-Ky)}{\partial y} = \mu_0 \varepsilon_0 \frac{\partial E_z}{\partial t} = 2K \end{cases}$$

$$\Longrightarrow \begin{cases} E_x = C_1 \\ E_y = C_2 \\ E_z = \frac{2K}{\mu_0 \varepsilon_0} \cdot t + C_3 \end{cases} \longrightarrow \vec{E} = \vec{E}_0 + \frac{2K}{\mu_0 \varepsilon_0} \cdot t \, \vec{u}_z$$

$\frac{1}{\mu_0 \varepsilon_0} \rightarrow \frac{1}{C^2}$

condizione iniziale
$\vec{E}(t=0) = 0 = \vec{E}_0$

$$\Longrightarrow \vec{E}(x,y,z,t) = 2Kc^2 t \, \vec{u}_z$$



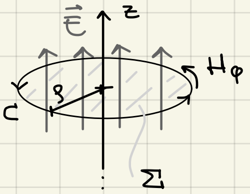
Trascurare gli effetti di bordo.

$$V(t) = V_0 \cos\omega t$$

$$\vec{E}(t) = \frac{V(t)}{d}\vec{u}_z = \frac{V_0}{d}\cos\omega t \, \vec{u}_z$$

$\vec{H}$ tra le armature?

$$\vec{\nabla}\times\vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \cancel{\vec{J}_c} \longrightarrow \text{ non c'é corrente di conduzione}$$



$$\oint \vec{H}\cdot d\vec{\ell} = \varepsilon_0 \frac{\partial}{\partial t}\int_\Sigma \vec{E}\cdot d\vec{\Sigma} \longrightarrow H_\varphi 2\cancel{\pi}\cancel{\rho} = \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{V_0}{d}\cos\omega t\right)\cancel{\pi}\rho^{\cancel{2}}$$

$$H_\varphi \cdot 2 = \varepsilon_0\left(-\omega \frac{V_0}{d}\,\text{sen}\,\omega t\right)\cdot\rho$$

Altro metodo:

$$\Longrightarrow \vec{H_\varphi}(\rho) = -\rho\,\frac{\varepsilon_0 V_0 \omega}{2d}\,\text{sen}\,\omega t\,\vec{u}_\varphi$$

diretto solo come $\vec{u}_z$

$$\vec{\nabla}\times\vec{H} = \varepsilon_0\frac{\partial\vec{E}}{\partial t} \text{ in coordinate cilindriche}$$

$$\vec{\nabla}\times\vec{H} = \frac{1}{\rho}\left[\frac{\partial(\rho H_\varphi)}{\partial\rho} - \frac{\partial H_\rho}{d\varphi}\right]\vec{u}_z = \varepsilon_0\frac{\partial E}{\partial t}\vec{u}_z, \quad \text{per simmetria } \frac{\partial H_\rho}{\partial\varphi} = 0$$

$$\frac{1}{\rho}\frac{\partial(\rho H_\varphi)}{\partial\rho} = \varepsilon_0\frac{\partial E}{\partial t} \longrightarrow \partial(\rho H_\varphi) = -\rho\,\varepsilon_0\frac{V_0}{d}\omega\,\text{sen}(\omega t)\,\partial\rho$$

$$\longrightarrow \rho H_\varphi = -\frac{\rho^2}{2}\varepsilon_0\frac{V_0}{d}\omega\,\text{sen}(\omega t) \Longrightarrow H_\varphi(\rho) = -\rho\,\frac{\varepsilon_0 V_0\omega}{2d}\,\text{sen}\,\omega t$$

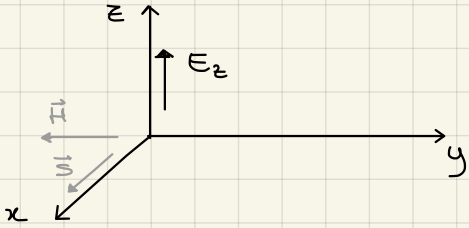
$$\vec{\nabla}\times\vec{F} = \left(\frac{1}{\rho}\frac{\partial F_z}{\partial\varphi} - \frac{\partial F_\varphi}{\partial z}\right)\vec{u}_\rho + \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho}\right)\vec{u}_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho F_\varphi)}{\partial\rho} - \frac{\partial F_\rho}{\partial\varphi}\right)\vec{u}_z$$

$$\vec{E}(x,t) = 15\cdot 10^{-2}\cos\left(1{,}7\pi\cdot 10^9 t - \beta x\right)\vec{u}_z \quad \left[\frac{V}{m}\right]$$

Calcolare β, λ, direzione di propagazione (nel vuoto).

$$\vec{E}(x,t) = A\cos(\omega t - \beta x)\vec{u}_z$$

la direzione di propagazione é l'asse x, il verso è dato dal segno ("−") per cui é un'onda progressiva che propaga verso le x positive



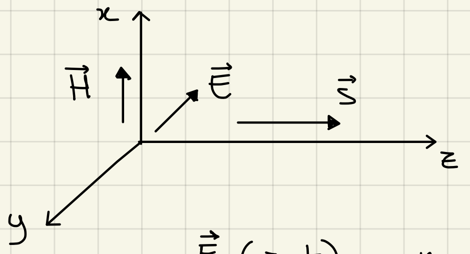
$$\vec{S} = \vec{E}\times\vec{H}$$

μ_0, ε_0

$$\omega = 2\pi f = 1{,}7\pi\cdot 10^9 \frac{\text{rad}}{\text{s}} \longrightarrow \begin{cases} f = 850 \text{ MHz} \\ \lambda = \dfrac{c}{f} = \dfrac{3\cdot 10^8}{8{,}5\cdot 10^8} = 0{,}353 \text{ m} \\ \beta = \dfrac{2\pi}{\lambda} = 17{,}8 \dfrac{\text{rad}}{\text{m}} \end{cases}$$

$\vec{H}(z,t) = 0{,}1 \operatorname{sen}(\omega t - 9{,}3z)\, \vec{u}_x \left[\frac{mA}{m}\right]$

Calcolare f, $\vec{E}(z,t)$ al $t = 10^{-4}$ s in $z = 2$ m, $|\vec{S}|$ (nel vuoto)



$\beta = \frac{2\pi}{\lambda} = 9{,}3 \ \frac{rad}{m} \longrightarrow \lambda = 0{,}676\,m$

$f = \frac{c}{\lambda} = 444 \ MHz$

$\omega = 2\pi f = 2{,}79 \cdot 10^9 \frac{rad}{s}$

$\vec{E}(z,t) = \eta_0 H(z,t) \cdot (-\vec{u}_y) =$

$= -37{,}7 \operatorname{sen}(\omega t - 9{,}3z)\, \vec{u}_y \left[\frac{mV}{m}\right]$

$\vec{E}(2m, 10^{-4}s) = -28{,}5 \ \vec{u}_y \left[\frac{mV}{m}\right]$

$|\vec{S}| = |\vec{E} \times \vec{H}| = \frac{1}{2} \frac{|\vec{E}|^2}{\eta_0} = \frac{1}{2} \eta_0 |\vec{H}|^2 = \frac{0{,}00142}{2 \cdot 377} = 1{,}88 \cdot 10^{-6} \frac{W}{m^2}$

- $\vec{E} = 0{,}5 \cdot 10^8 \operatorname{sen}\left[2\pi(10^8 t - 0{,}5x - 0{,}125)\right] \vec{u}_z$

Calcolare f, λ, v, direzione di propagazione, ε_r nel mezzo.

$(\mu_0, \ \varepsilon = \varepsilon_0 \varepsilon_r)$ $\quad \vec{E} = A \operatorname{sen}(\omega t - \beta x + \varphi_0)$ direzione x pos.

$\vec{E} = 0{,}5 \cdot 10^8 \operatorname{sen}\left(2\pi \cdot 10^8 t - \pi x - \frac{\pi}{4}\right) \vec{u}_z$

$\omega = 2\pi\, 10^8 \ \frac{rad}{s} \longrightarrow f = 100 \ MHz$

$\beta = \pi \ \frac{rad}{m} \longrightarrow \lambda = \frac{2\pi}{\beta} = 2\,m$

$\Big\} \ v = \lambda f = 2 \cdot 10^8 \frac{m}{s}$

$v = \sqrt{\frac{1}{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}} = 2 \cdot 10^8 \frac{m}{s}$

$\sqrt{\varepsilon_r} = \frac{3}{2} \longrightarrow \varepsilon_r = 2{,}25$

- Onda TEM nel vuoto (dominio dei fasori)

$\bar{E} = (5 - j3)\, e^{-j2z}\, \vec{u}_x \left[\frac{V}{m}\right]$ (nel vuoto)

Calcolare il valore del campo elettrico in $z = 0{,}4$ m al tempo $t = 3 \cdot 10^{-9}$ s.

$\bar{E} = \bar{E}_0(0)\, e^{-j\beta z}$ $\quad E_0(0) = 5 - j3$, $\beta = 2 \ \frac{rad}{m} = \frac{2\pi}{\lambda} = \frac{\overbrace{2\pi f}^{\omega}}{c}$

$\longrightarrow \omega = 6 \cdot 10^8 \ \frac{rad}{s}$

$$\vec{E}(z,t) = \text{Re}\left\{ \bar{E}\, e^{j\omega t} \right\} \vec{u}_x = \text{Re}\left\{ (5-j3)\, e^{-j2z} e^{j6\cdot10^8 t} \right\} \vec{u}_x$$

$$\vec{E}(0{,}4m, 3\cdot10^{-9}s) = 5{,}22\ \vec{u}_x \left[\frac{V}{m}\right]$$

- Calcolare la profondità di penetrazione (spessore pelle) di un'onda con $f = 10\ KHz$ nei seguenti materiali:

a) acqua marina: $\varepsilon_r = 81$ $\quad\sigma = 4\ S/m$

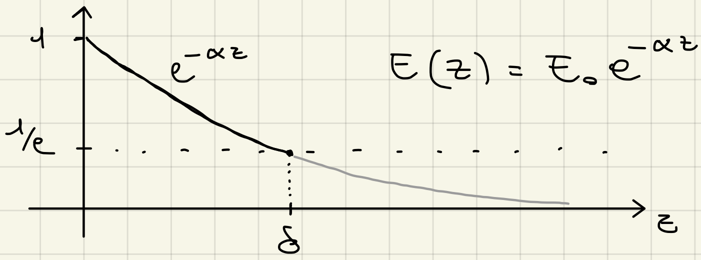
b) terreno umido: $\varepsilon_r = 10$ $\quad\sigma = 10^{-2}\ S/m$

c) " asciutto: $\varepsilon_r = 3$ $\quad\sigma = 10^{-4}\ S/m$

$\hookrightarrow$ minore è la presenza di acqua, minore è la cost. dielettrica

I materiali sono buoni conduttori se:

$\sigma \gg \omega\varepsilon \longrightarrow \sigma \gg 2\pi f \varepsilon_0 \varepsilon_r$ vero per a), b) e c)



$E(z) = E_0 e^{-\alpha z}$

spessore pelle $\delta = \frac{1}{\alpha}$

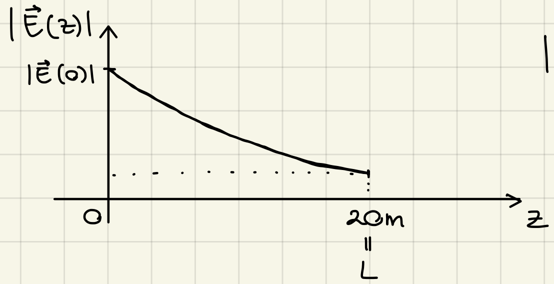
$$\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} \begin{cases} \delta_a = 2{,}5\,m \\ \delta_b = 50{,}3\,m \\ \delta_c = 503\,m \end{cases}$$

- Onda TEM piana $f = 1\ GHz$, si propaga in un mezzo con $\varepsilon_r = 3 - j0{,}01$.

Valutare di quanti dB si è ridotto il campo $\vec{E}$ dopo essersi propagato per 20 m.
Valutare λ!

$$\boxed{\begin{aligned} \beta &= \pm\sqrt{\frac{\sqrt{x^2+y^2} - x}{2}} \\ \alpha &= y/2\beta \end{aligned}}$$

$$\gamma = \sqrt{-\omega^2 \mu_0 \varepsilon_0 \varepsilon_r} = \sqrt{\underset{x}{-1318} \underset{+}{+} \underset{jy}{j4{,}4}} = (\underset{\alpha}{0{,}06} \underset{+}{+} \underset{j\beta}{j36{,}3})\ m^{-1}$$

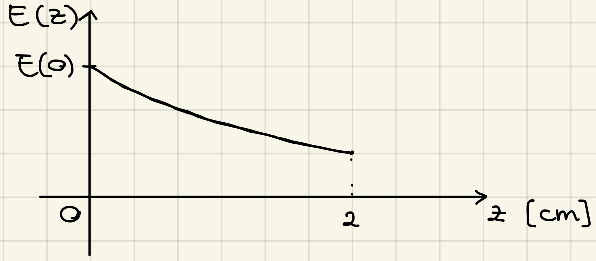


$$|\vec{E}(L)| = |\vec{E}(0)|\, e^{-\alpha L} = |\vec{E}(0)| \cdot 0{,}3$$

$$20\log_{10} \frac{|\vec{E}(L)|}{|\vec{E}(0)|} = 20\log_{10} 0{,}3 = -10{,}4\,dB$$

$$\beta = \frac{2\pi}{\lambda} = 36{,}3 \longrightarrow \lambda = 0{,}17\,m$$

Onda TEM $f = 1 MHz$ in mezzo con con $\varepsilon_r = 4$ e $\sigma = 10^3 \frac{S}{m}$



$E(0) = 1 \frac{V}{m}$ $\quad E(z = 2cm) = ?$

$\lambda = ?$

Buon conduttore ?

$\overbrace{\omega \varepsilon_0 \varepsilon_r}^{2,2 \cdot 10^{-4}} \ll \sigma \longrightarrow$ sí

$$\gamma = \frac{1+j}{2} \sqrt{\omega \mu_0 \sigma} = (62,8 + j62,8)\, m^{-1}$$

$$E(z) = E(0) e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z} = 0,284\, e^{-j1,256} \left(\frac{V}{m}\right) =$$

$(z = 2cm)$

$$= (0,088 - j0,27) \frac{V}{m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{62,8} = 0,1\, m = \frac{v}{f} \longrightarrow v = 10^5 \frac{m}{s}$$

NB: non sarebbe stato corretto calcolare la velocitá e la lunghezza d'onda con l'equazione

$v = \frac{c}{\sqrt{\varepsilon_r}} = 1,5 \cdot 10^8 \frac{m}{s}$ ✗ perché non considero la conducibilitá σ

- Onda TEM $f = 1GHz$ in acqua marina ($\varepsilon_r = 80$ $\sigma = 4 \frac{S}{m}$)

Valutare a quale distanza l'ampiezza di $\vec{E}$ si sará ridotta di $\frac{1}{10}$ del suo valore. Valutare λ.

$\omega \varepsilon_0 \varepsilon_r = 4,45 > \sigma \longrightarrow$ NON é un buon conduttore

$$\gamma = \sqrt{-\omega^2 \mu_0 \varepsilon_0 \varepsilon_r + j\omega\mu_0\sigma} = \sqrt{-35147 + j31582} = (77,8 + j202)\, m^{-1}$$

$$|E(z)| = |E(0)| e^{-\alpha L} = \frac{1}{10} |E(0)| \longrightarrow -\alpha L = -2,306$$

$$L = 0,0296\, m$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{202} = 0,0311\, m$$

Se $\sigma = 0$ e $\varepsilon_r = 80$ allora $\lambda = \frac{c}{\sqrt{\varepsilon_r} \cdot f} = 0,0335 m$

↓ la conducibilitá diversa da 0 cambia (poco) la lunghezza d'onda

- Onda TEM, mezzo: $\varepsilon_r = 36 \quad \mu_r = 4 \quad \sigma = 1\,\frac{S}{m}$

$$\vec{E} = 100\, e^{-\alpha x} \cos(10\pi \cdot 10^8 t - \beta x)\, \vec{u}_z \ \left[\frac{V}{m}\right]$$

Determinare α, β, $\vec{H}$.

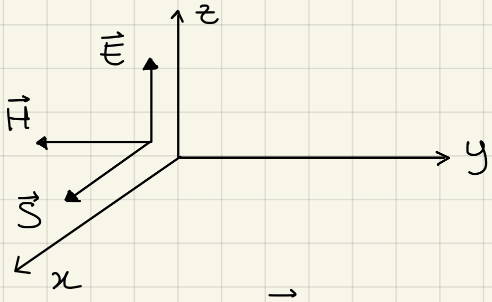
$$\gamma = \sqrt{-\omega^2 \mu_0 \mu_r \varepsilon_0 \varepsilon_r + j\omega \mu_0 \mu_r \sigma} = \sqrt{-15782 + j15775} =$$

$$= (57{,}15 + j138)\ m^{-1}$$

$$\eta = \sqrt{\frac{j\omega\mu_0\mu_r}{\sigma + j\omega\varepsilon_0\varepsilon_r}} =$$

$$= (97{,}6 + j40{,}4)\ \Omega$$

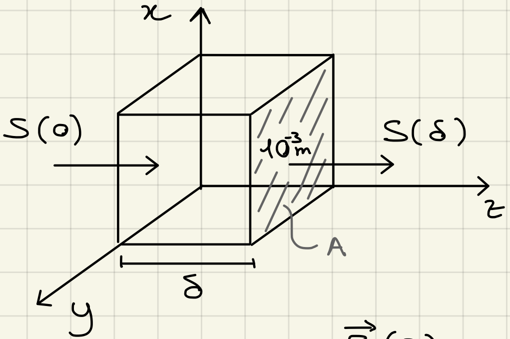
$$= 105{,}6\, e^{+j\frac{\pi}{8}}\ \Omega$$



$$\vec{H} = \frac{\vec{E}}{\eta}(-\vec{u}_y)$$

$$\vec{H} = -0{,}95\, e^{-57{,}15x} \cdot \cos(10\pi\, 10^8 t - 138x - \frac{\pi}{8})\, \vec{u}_y \ \left[\frac{V}{m}\right]$$

Onda TEM, $|\vec{E}| = 100\,\frac{V}{m}$ $f = 7\,GHz$ mezzo: $\varepsilon_r = 81$
$\sigma = 4\,\frac{S}{m}$



Calcolare la potenza dissipata in un blocco d'acqua avente una superficie di 10 cm² e uno spessore pari a δ (spessore pelle)

$$\vec{S}(z) = \frac{1}{2}\frac{|\vec{E}(z)|^2}{|\eta|}\cos\varphi_\eta\, \vec{u}_z = \frac{1}{2}\frac{|\vec{E}(0)|^2}{|\eta|} e^{-2\alpha z} \cos\varphi_\eta\, \vec{u}_z$$

$$\eta = \sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\varepsilon_0\varepsilon_r}} = \sqrt{1724 + j219} = (41{,}6 + j2{,}6)\ \Omega$$

$$|\eta| = 41{,}7\ \Omega \qquad \cos\varphi_\eta = \cos\left(\mathrm{arctg}\left(\frac{2{,}6}{41{,}6}\right)\right) = 0{,}998$$

Ricordando che $\delta = \frac{1}{\alpha}$: $P(0) = S(0) \cdot A = \frac{1}{2}\frac{|\vec{E}(0)|^2}{|\eta|}\cos\varphi_\eta \cdot 10^{-3}$

$$= 0{,}12\ W$$

$$\Rightarrow P_{dis} = P(0) - P(\delta) \qquad P(\delta) = \frac{1}{2}\frac{|\vec{E}(0)|^2}{|\eta|} e^{-2\cancel{\alpha}\frac{1}{\cancel{\alpha}}} \cos\varphi_\eta\, 10^{-3} =$$

$$= 0{,}104\ W \qquad = 0{,}016\ W$$

- Onda piana TEM che si propaga come $\vec{u}_z$

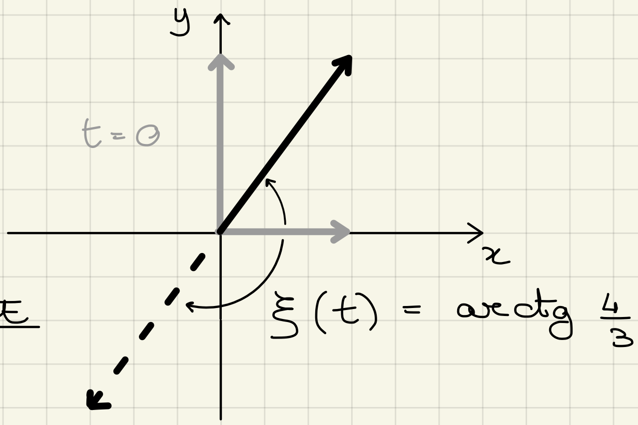
$$\vec{E} = 3\cos(2\pi f t - \beta z)\,\vec{u}_x + 4\cos(2\pi f t - \beta z)\,\vec{u}_y$$

Determinarne la polarizzazione.

Piano xy $(z=0)$:

$$\begin{cases} E_x(0,t) = 3\cos 2\pi f t \\ E_y(0,t) = 4\cos 2\pi f t \end{cases}$$

$\varphi_0 = 0$ → POL. LINEARE

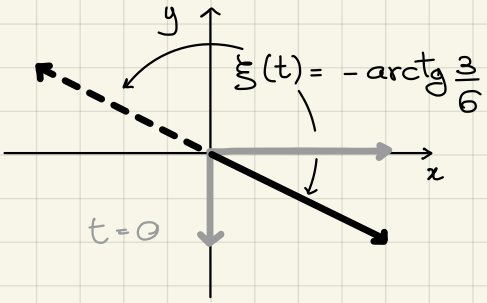


- Onda TEM $\vec{E} = 6\cos(2\pi f t - \beta z + \frac{\pi}{4})\,\vec{u}_x - 3\cos(2\pi f t - \beta z + \frac{\pi}{4})\,\vec{u}_y$

Polarizzazione?

In $z = 0$:
$$\begin{cases} E_x(0,t) = 6\cos(2\pi f t + \frac{\pi}{4}) \\ E_y(0,t) = -3\cos(2\pi f t + \frac{\pi}{4}) \end{cases}$$

$\varphi_0 = \pi$ → POL. LINEARE

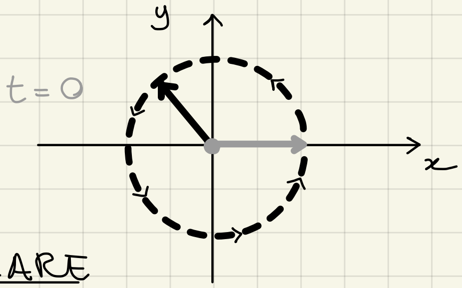


- Onda TEM $\vec{E} = 4\cos(2\pi f t - \beta z)\,\vec{u}_x + 4\cos(2\pi f t - \beta z - \frac{\pi}{2})\,\vec{u}_y$

Polarizzazione?

In $z = 0$:
$$\begin{cases} E_x(0,t) = 4\cos 2\pi f t \\ E_y(0,t) = 4\cos(2\pi f t - \frac{\pi}{2}) \end{cases}$$

$\varphi_0 = -\frac{\pi}{2}$ e $E_x = E_y$ → POL. CIRCOLARE DESTRA



$\xi(t) = +\omega t$

- Onda TEM $f = 300\text{ MHz}$ propaga come $\vec{u}_z$

mezzo: $\varepsilon_r = 6$ e $\sigma = 0{,}1\ S/m$

Sapendo che $|\vec{E}(1,0,1)| = 1 \frac{V}{m}$ calcolare:

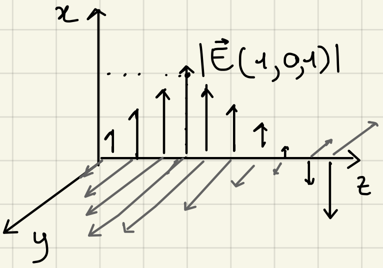
$|\vec{H}(1,0,1)|$ e $|\vec{E}(0,0,0)|$?

Buon conduttore ? $\omega \varepsilon_0 \varepsilon_r = 1{,}9 \cdot 10^9 \cdot 8{,}85 \cdot 10^{-12} \cdot 6 = 0{,}1 = \sigma$

$\longrightarrow$ non é un buon conduttore

$$\vec{H} = \frac{\vec{E}}{\eta}$$

$$|\eta| = \left|\sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\varepsilon_0\varepsilon_r}}\right| = \qquad \omega = 2\pi f = 1{,}9 \cdot 10^9 \frac{rad}{s}$$



$$= \left|\sqrt{\frac{j\, 2{,}4 \cdot 10^3}{0{,}1 + j0{,}1}}\right| =$$

$$= \left|\sqrt{\frac{240 - j240}{0{,}02}}\right| = \left|\sqrt{1200}\sqrt{1-j}\right| =$$

$$= 109{,}5 \cdot \sqrt{|1-j|} = 130\ \Omega$$

$$|\vec{H}(1,0,1)| = \frac{|\vec{E}(1,0,1)|}{|\eta|} = 7{,}69 \frac{mH}{m}$$

$$\vec{E} = \vec{E}(t,z) = |\vec{E}(1,0,1)|\, e^{-\alpha(z-1)}\, e^{-j\beta(z-1)}\, e^{j\omega t}$$

$$\alpha + j\beta = \gamma = \sqrt{j\omega\mu_0(\sigma + j\omega\varepsilon_0\varepsilon_r)} =$$

$$= \sqrt{j\, 2400 \cdot 0{,}1\,(1+j)} = 15{,}5\sqrt{-1+j} =$$

$$= 18{,}42\sqrt{e^{j\frac{3}{4}\pi}} = 18{,}42\left(\cos\frac{3}{8}\pi + j\,\mathrm{sen}\,\frac{3}{8}\pi\right)$$

$$= 7{,}05 + j\,17{,}02$$

$$|\vec{E}(0,0,0)| = |\vec{E}(1,0,1)|\,|e^{+\alpha}| = 1{,}15 \cdot 10^3 \frac{V}{m}$$

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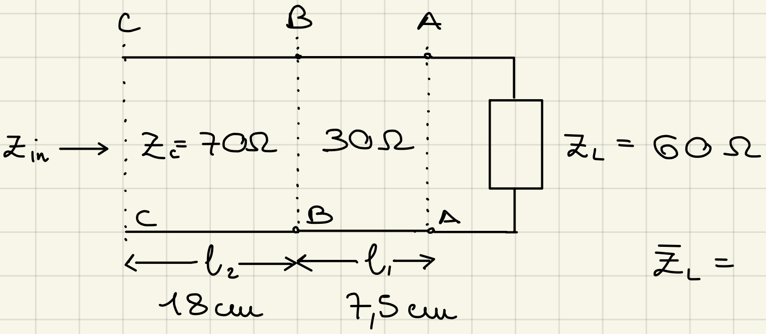


$f = 1\,GHz \rightarrow \lambda = 0{,}3\ m$

$Z_{in} = ?$

$$\overline{Z}_L = \frac{Z_L}{30} = \frac{60}{30} = 2$$

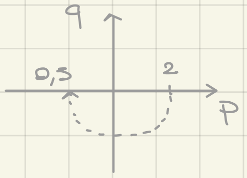
$$\frac{\ell_1}{\lambda} = \frac{7{,}5}{30} = 0{,}25 \qquad \frac{\ell_2}{\lambda} = \frac{18}{30} = 0{,}6 \longrightarrow 0{,}1$$

(ruotare di 0,5λ significa compiere una rotazione completa sulla carta di Smith)

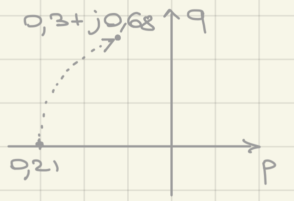
$$\Gamma_{AA} = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{1}{3}$$ parto da 0,25λ

$0{,}25\lambda + 0{,}25\lambda$

$$\bar{z}'_{BB} = 0{,}5 + j0 \qquad z'_{BB} = \bar{z}'_{BB} \cdot 30\Omega = 15\Omega$$



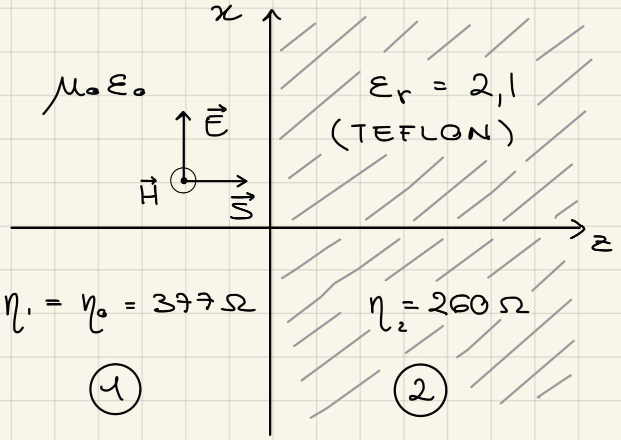
$$\Gamma_{BB} = \frac{z'_{BB} - z_c}{z'_{BB} + z_c} = \frac{15\Omega - 70\Omega}{15\Omega + 70\Omega} = -\frac{11}{17}$$ parto da 0λ



$0\lambda + 0{,}1\lambda$

$$\bar{z}''_{BB} = \frac{z'_{cc}}{70\Omega} = \frac{15}{70} = 0{,}21 \qquad \bar{z}_{cc} = 0{,}3 + j\,0{,}68$$

$$z_{cc} = \bar{z}_{cc}\ 70 = (22 + j\,47{,}5)\,\Omega = z_{in}$$

- 

Onda TEM $\quad f = 1\,GHz$

$$|\vec{E}_{inc}| = 1\,\frac{V}{m}$$

Calcolare S_{inc}, S_{rif}, S_{tra} e i campi $\vec{E}$ e $\vec{H}$ nel mezzo ②.

$$S_{inc} = \frac{1}{2}\,\frac{|\vec{E}_{inc}|^2}{\underbrace{\eta_1}_{\eta_0}} = 1{,}327\cdot 10^{-3}\,\frac{W}{m^2}$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} \simeq 260\,\Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0{,}184$$

$$S_{tra} = S_{inc}(1 - |\Gamma|^2) = 1{,}28\cdot 10^{-3}\,\frac{W}{m^2}$$

$$S_{rif} = S_{inc}\cdot|\Gamma|^2 = 4{,}4\cdot 10^{-5}\,\frac{W}{m^2}$$

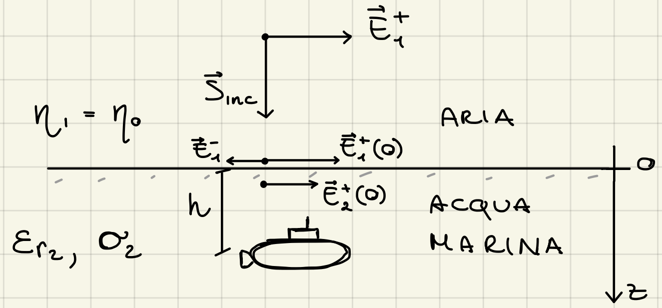
$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = 2\pi f\sqrt{\mu_0\varepsilon_0\varepsilon_r} = 30{,}4\,\frac{rad}{s}$$

Assumiamo fase zero per $\vec{E}_{inc}(0)$

$$\vec{E}_2^+(z) = E_2^+(0)\,e^{-j\beta_2 z}\,\vec{u}_x = \underbrace{E_{inc}(0)}_{E_1^+(0)}\cdot T\,e^{-j\beta_2 z}\,\vec{u}_x \qquad \text{con } T = 1 + \Gamma$$

$$= 0{,}816\,e^{-j30{,}4z}\,\vec{u}_x\ \frac{V}{m}$$

$$\vec{H}_2^+(z) = \frac{E_2^+(z)}{\eta_2}\,\vec{u}_y$$

- 

$f = 100\ kHz \qquad \varepsilon_{r_2} = 80$

$$S_{inc} = 1{,}33\cdot 10^{-5}\,\frac{W}{m^2} \qquad \sigma_2 = 4\,\frac{S}{m}$$

$$|\vec{E}_{min}| = 1\,\mu V/m$$

Calcolare h_{max} (massima profondità di immersione).

$\omega \varepsilon_0 \varepsilon_{r_2} << \sigma_2 \longrightarrow$ buon conduttore

$$\eta_2 \simeq (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = (1+j)\,0{,}314\,\Omega$$

$$S_{inc} = \frac{1}{2}\frac{|E_1^+|^2}{\eta_0} \longrightarrow |E_1^+| = \sqrt{2 \cdot S_{inc} \cdot \eta_0} = 0{,}1\ \frac{V}{m}$$

$\forall z \leq 0$ in quanto il mezzo 1 non è attenuativo

$$T = 1 + \Gamma = 1 + \frac{\eta_2 - \eta_1}{\eta_2 - \eta_1} = (1+j)\,1{,}66 \cdot 10^{-3}$$

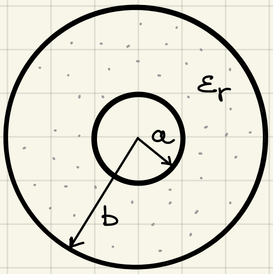
→ forte riflessione

$$|E_2^+(0)| = |E_1^+(0)| \cdot |T| = 235\ \frac{\mu V}{m}$$

$$|E_2^+(z)| = |E_2^+(0)| \cdot e^{-\alpha z} \quad \text{con} \quad \alpha = \frac{1}{\delta} = \sqrt{\frac{\omega \mu \sigma}{2}} = 1{,}2566\ \frac{Np}{m}$$

$$\underset{1\mu V/m}{|\vec{E}_{min}|} = \underset{235\mu V/m}{|E_2^+(0)|}\, e^{-\overset{1{,}2566}{\alpha} \cdot h_{max}} \Longrightarrow h_{max} = 4{,}3\,m$$

- Linea TEM - cavo coassiale



$a = 2mm \qquad \varepsilon_r = 2{,}5$

$b = 4{,}5mm$

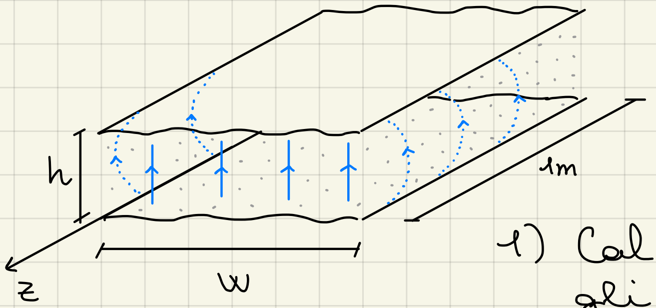
Calcolare C, L, Z_c.

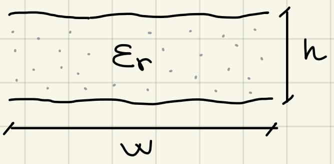
$CL = \mu \varepsilon$

$$C = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln\left(\frac{b}{a}\right)} = 171\ \frac{pF}{m} \qquad L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) = 1{,}62 \cdot 10^{-7}\ \frac{H}{m}$$

$$Z_c = \sqrt{\frac{L}{C}} = 30{,}8\,\Omega$$

Linea a striscia





$w = 10mm$

$h = 4mm$

$\varepsilon_r \approx 1$

1) Calcolare C, L, Z_c (trascurando gli effetti di bordo)

$C = \varepsilon_0 \varepsilon_r \frac{w \cdot 1m}{h} = 22 pF$ $\qquad$ $L = \frac{\mu_0 \varepsilon}{C} = 5{,}06 \cdot 10^{-7} H$

$Z_c = \sqrt{\frac{L}{C}} = \frac{1}{C}\sqrt{\mu_0 \varepsilon} = 151\ \Omega$

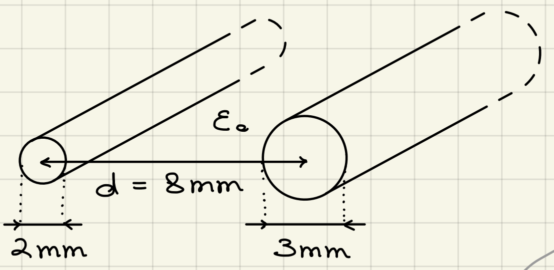
NB: C' (capacità reale) $> C \longrightarrow Z_c'$ (imped. reale) $< Z_c$ $\quad$ che considera gli effetti di bordo

2) Calcolare tensione e $|\vec{E}|$ di picco sapendo che nella linea transita un'onda (progressiva e diretta) $P_m^+ = 100\ W$

$$P_m^+ = \frac{1}{2}\frac{|V^+(z)|^2}{Z_c} \longrightarrow |V^+(z)| = \sqrt{2 Z_c P_m^+} = 174\ V$$

$$|V| = |E| \cdot h \longrightarrow |\vec{E}| = \frac{|V^+(z)|}{h} = 43{,}5\ \frac{kV}{m}$$

- linea bifilare

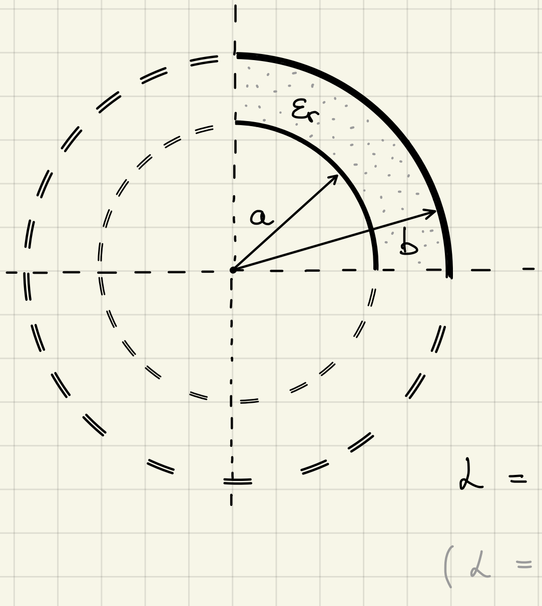


Calcolare Z_c con l'approssimazione dei conduttori sottili.

$$V = \frac{\rho_e}{2\pi\varepsilon_0} \ln\left(\frac{d^2}{R^+ \cdot R^-}\right)$$

$$\mathcal{C} = \frac{\rho_e}{V} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{d^2}{R^+ R^-}\right)} = 14{,}8\ \frac{pF}{m} \qquad Z_c = \frac{\sqrt{\mu_0\varepsilon_0}}{\mathcal{C}} = 225\ \Omega$$

-



a = 3mm

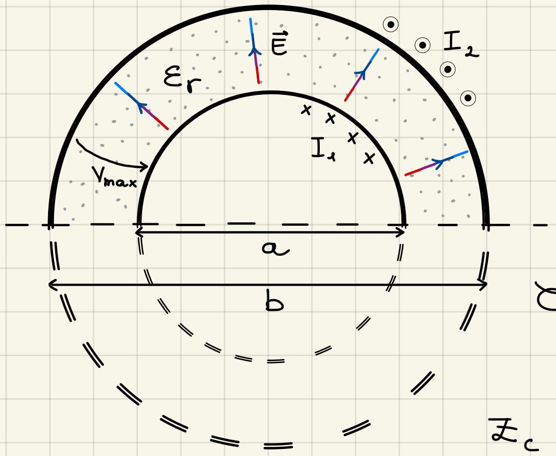
b = 5mm $\qquad$ Calcolare $\mathcal{C}$, $\mathcal{L}$, Z_c

$\varepsilon_r = 4$ $\qquad$ la capacità è $\frac{1}{4}$ di quella di un cavo coassiale

$$\mathcal{C}_0 = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \qquad \mathcal{C} = \frac{\mathcal{C}^\circ}{4} = \frac{\pi\varepsilon_0\varepsilon_r}{2\ln\left(\frac{b}{a}\right)} = 109\ \frac{pF}{m}$$

$$\mathcal{L} = \frac{\mu_0\varepsilon_0\varepsilon_r}{\mathcal{C}} = 4{,}08 \cdot 10^{-7}\ \frac{H}{m} \qquad Z_c = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = 61{,}2\ \Omega$$

$(\mathcal{L} = 4\mathcal{L}_0)$



$a = 1\,cm \qquad \varepsilon_r = 2$

1) Determinare b in modo che $Z_c = 50\,\Omega$

$$C = \frac{C_0}{2} = \frac{\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \qquad Z_c = \frac{\sqrt{\mu_0\varepsilon_0\varepsilon_r}}{C} = 50\,\Omega$$

$$Z_c = \frac{\eta}{\pi}\ln\left(\frac{b}{a}\right) = 50\,\Omega \quad \text{con} \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} = 267\,\Omega$$

$\hookrightarrow b = 1{,}8\,cm$

2) Calcolare I_1 e I_2 con $P_m^+ = 50\,W$

$$P_m^+ = \frac{1}{2}|I|^2 Z_c = 50\,W \longrightarrow |I| = 1{,}41\,A = |I_1| = |I_2|$$

correnti uguali e opposte

3) Calcolare P_m^+ sapendo che $|\vec{E}_{max}| = 1\,\frac{kV}{cm}$

il campo elettrico non è unif. nel coassiale!

Da $|\vec{E}_{max}|$ ricaviamo V_{max}:

$$V_{max} = E_{max}\ln\left(\frac{b}{a}\right)\frac{a}{2} = 10^5\,\frac{V}{m}\,\ln(1{,}8)\cdot\frac{10^{-2}}{2} = 294\,V$$

è maggiore dove il raggio è minore

$$P^+_{m,max} = \frac{1}{2}\frac{|V_{max}|^2}{Z_c} = 864\,W$$

in condizioni di adattamento cioè $P_m^- = 0$

4) Come al p.to 3, ma la linea è chiusa su un carico $Z_L = 300\,\Omega$ (non in adattamento)

$V_{max} = 294\,V \qquad V(z) = V^+(z) + V^-(z) \qquad |V(z)| = |V^+(z)||1+\Gamma(z)|$

$$|V_{max}| = |\overset{V^+(0)e^{-j\beta z}}{V^+(z)}|\,(1+|\Gamma_L|)$$

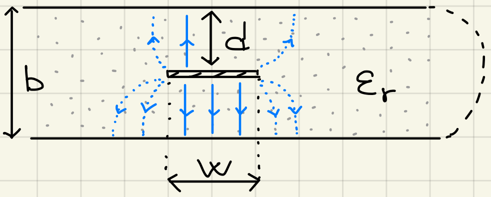
$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{300-50}{300+50} = 0{,}71 \qquad |V_{max}| = |V^+(0)|\,1{,}71$$

$$\longrightarrow |V^+(0)| = \frac{V_{max}}{1{,}71} = \frac{294}{1{,}71} = 172\,V$$

$$P^+_{max} = \frac{1}{2}\frac{|V^+(0)|^2}{Z_c} = 295\,W$$

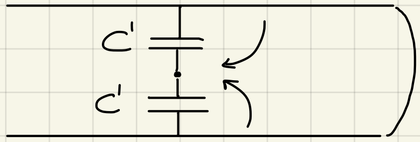
- Stripline

Calcolare w in modo che $Z_c = 50\,\Omega$.



$\varepsilon_r = 2,2$ (adottare l'approx. del condensatore ideale)

$b = 0,32$ cm



$C = 2C'$ con $C' = \varepsilon_0 \varepsilon_r \frac{w}{d}$

$b = 2d$

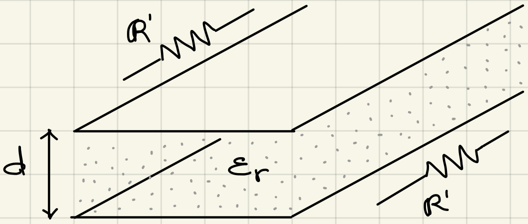
$$\Longrightarrow \quad C = \varepsilon_0 \varepsilon_r \frac{w}{b} 4 \qquad Z_c = \frac{\sqrt{\mu\varepsilon}}{C} \qquad C = \frac{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}{Z_c} \simeq 99 \frac{pF}{m}$$

$$w = \frac{bC}{4\varepsilon_0 \varepsilon_r} = 0,41 \text{ cm}$$ (senza effetti di bordo)

Senza trascurare gli effetti di bordo

$w = 0,26$ cm

-



$\sigma_c = 5 \cdot 10^7 \, S/m \qquad f = 500\,MHz \qquad v = \frac{c}{3}$

$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$ e $\frac{\varepsilon_r''}{\varepsilon_r'} = 10^{-4} \qquad Z_c = 100\,\Omega \qquad d = 2mm$

Calcolare w e $\alpha_{TOT} = \alpha_c + \alpha_D$ (in Np/m, dB/m)

$$v = \frac{c}{\sqrt{\varepsilon_r'}} = \frac{c}{3} \rightarrow \varepsilon_r' = 9 \qquad Z_c = \frac{\sqrt{\mu\varepsilon}}{C} \qquad C = \frac{\sqrt{\mu_0 \varepsilon_r \varepsilon_0}}{Z_c} = 100 \frac{pF}{m}$$

$$C = 100 \frac{pF}{m} = \varepsilon_0 \varepsilon_r \frac{w}{d} \longrightarrow w = 2,5mm$$

$$\alpha_c = \frac{R}{2Z_c} \text{ con } R = \overbrace{\frac{R_s}{w}}^{R'} \cdot 2, \quad R_s = \frac{1}{\sigma_c \delta} \text{ dove } \delta = \sqrt{\frac{1}{\pi f \mu_0 \sigma_c}} = 3,18\,\mu m$$

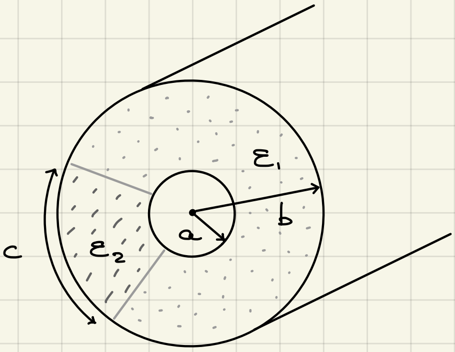
$$\longrightarrow R = 5,02 \frac{\Omega}{m} \text{ e } \alpha_c = 0,025 \frac{Np}{m}$$

$$\alpha_D = \frac{G Z_c}{2} \qquad G = \underbrace{\omega \varepsilon''}_{\sigma_{eq}} \frac{W}{d} = 3,14 \cdot 10^{-5} \frac{S}{m}$$

$$\longrightarrow \quad \alpha_D = 1,57 \cdot 10^{-3} \frac{Np}{m} \qquad \text{oppure} \qquad \alpha_D = \frac{\pi}{\lambda} \frac{\varepsilon_r''}{\varepsilon_r'} = \frac{\pi f}{v} \frac{\varepsilon_r''}{\varepsilon_r'} = 1,57 \cdot 10^{-3} \frac{Np}{m}$$

$$\Longrightarrow \quad \alpha_{TOT} = \alpha_c + \alpha_D = 0,0266 \frac{Np}{m}$$

$$\left(= 0,23 \frac{dB}{m} \right)$$

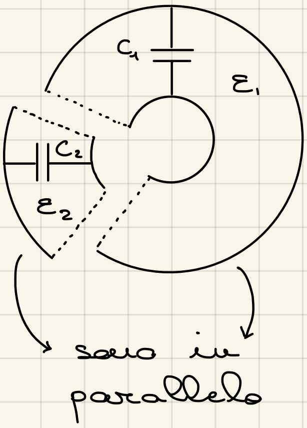


$\varepsilon_1 = 2\varepsilon_0 \qquad a = 0,5\ cm$

$\varepsilon_2 = 2\varepsilon_0 \qquad b = 1\ cm$

$c = 1\ cm$

Calcolare C, L, Z_c, ε_{eff}.



$$C = C_1 + C_2 =$$

$$= \frac{2\pi\varepsilon_1}{\ln\frac{b}{a}} \cdot \frac{(2\pi b - c)}{2\pi b} + \frac{2\pi\varepsilon_2}{\ln\frac{b}{a}} \cdot \frac{c}{2\pi b} = 2,62 \cdot 10^{-10} \frac{F}{m}$$

$L = L_0$ (l'induttanza non viene influenzata dal dielettrico con cui è riempita la linea)

$$C_0 = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}} = 8 \cdot 10^{-11} \frac{F}{m} \qquad L_0 C_0 = \mu_0 \varepsilon_0$$

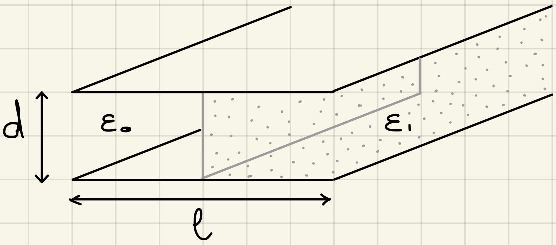
$$\longrightarrow \quad L_0 = 1,38 \cdot 10^{-7} \frac{H}{m}$$

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{L_0}{C}} = 23\,\Omega$$

$$v = \frac{1}{\sqrt{LC}} = 1,66 \cdot 10^8 \frac{m}{s}$$

$$CL = CL_0 = \mu \varepsilon_{eff}$$

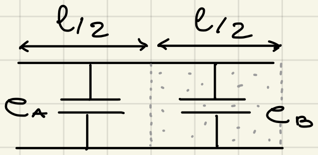
$$\longrightarrow \quad \varepsilon_{eff} = \frac{CL_0}{\mu_0} = 3,31\,\varepsilon_0$$



$d = 0,5\ cm \qquad l = 3\ cm$

$\varepsilon_1 = 3\varepsilon_0 \qquad \sigma = 3 \cdot 10^7 \frac{S}{m}$

Calcolare Z_c e α a 1GHz.



$$\mathcal{C} = \mathcal{C}_A + \mathcal{C}_B = \frac{\varepsilon_0 \ell}{2d} + \frac{\varepsilon_1 \ell}{2d} = 1{,}06\ \frac{F}{m}$$

$\mathcal{L} = \mathcal{L}_0$ $\quad \mathcal{C}_0 = \frac{\varepsilon_0 \ell}{d} = 5{,}32\cdot10^{-11}\ \frac{F}{m}$ $\quad \mathcal{C}_0\mathcal{L}_0 = \mu_0\varepsilon_0$

$\longrightarrow \mathcal{L}_0 = 2{,}09\cdot10^{-7}\ \frac{H}{m}$

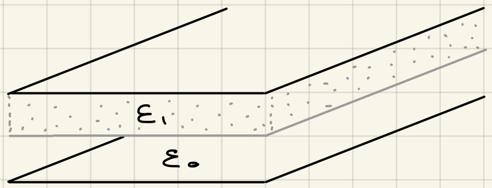
$$Z_c = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \sqrt{\frac{\mathcal{L}_0}{\mathcal{C}}} = \underline{\underline{44{,}4\ \Omega}}$$

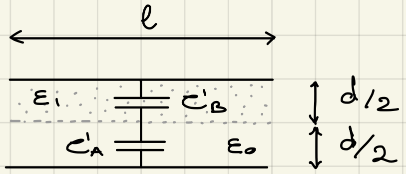
$v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = 2{,}12\cdot10^{8}\ \frac{m}{s}$ $\quad \varepsilon_{eff} = 2$

$R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} = 0{,}0115\ \frac{\Omega}{\square}$ $\quad R = 2R' = \frac{R_s}{\ell}\cdot 2 = 0{,}767\ \frac{\Omega}{m}$

$$\alpha = \frac{R}{2Z_c} = \underline{\underline{8{,}6\cdot10^{-3}}}\ \frac{Np}{m}$$

Stessa linea quasi-TEM, ma con il dielettrico disposto diversamente.





$\mathcal{C}' = \mathcal{C}'_A \,/\!/\, \mathcal{C}'_B$ con $\mathcal{C}'_A = \varepsilon_0 \frac{\ell}{d/2}$ e $\mathcal{C}'_B = \varepsilon_1 \frac{\ell}{d/2}$

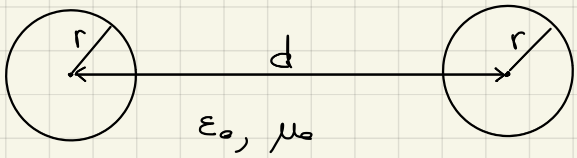
$= \frac{\mathcal{C}'_A \cdot \mathcal{C}'_B}{\mathcal{C}'_A + \mathcal{C}'_B} = 0{,}796\cdot10^{-10}\ \frac{F}{m}$

$\mathcal{L}' = \mathcal{L}_0$ come prima. $\quad$ R e R_s come prima.

$Z'_c = \sqrt{\frac{\mathcal{L}_0}{\mathcal{C}'}} = 51{,}3\Omega > Z_c$ $\quad \alpha' = \frac{R}{2Z'_c} = 7{,}5\cdot10^{-3}\ \frac{Np}{m} < \alpha$

$\downarrow$ minori perdite

$v' = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = 2{,}45\cdot10^{8}\ \frac{m}{s}$ $\quad \varepsilon'_{eff} = 1{,}5$



$d = 3$ cm $\quad f = 100$ MHz

$Z_c = 300\ \Omega$ $\quad \sigma = 5\cdot10^{7}\ \frac{S}{m}$

Determinare r (approx. conduttori sottili).
Calcolare α_c (Np/m).

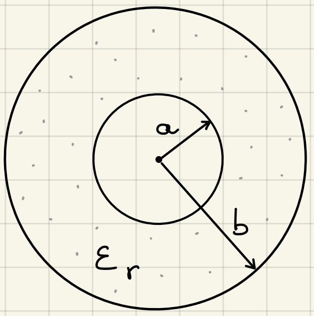
$$Z_c = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\frac{1}{\pi}\ln\frac{d}{r} = 300\,\Omega \rightarrow \frac{d}{r} = 12,18 \rightarrow r = 0,25\,\text{cm}$$

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} = 7,12\,\mu\text{m}$$

pochissimo dello spessore del conduttore viene effettivamente attraversato da corrente

$$R = 2R' = 2\frac{R_s}{2\pi r} = \frac{1}{\pi r \sigma \delta} = 0,36\,\frac{\Omega}{m}$$

$$\alpha_c = \frac{R}{2Z_c} = 6,05\cdot 10^{-4}\,\frac{Np}{m}$$

- 

Dimensionare la linea in modo che

$v = \frac{c}{2}$ $Z_c = 75\,\Omega$ $b = 0,5\,\text{cm}$

Calcolare α_c e α_D a 500 MHz con $\sigma = 5\cdot 10^7\,\frac{S}{m}$ e $\varepsilon_r'' = 10^{-3}\varepsilon_r'$

$$Z_c = \frac{1}{vC} \longrightarrow C = \frac{1}{\frac{c}{2}Z_c} = 89\,pF/m$$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}} = \frac{c}{2} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\cdot\frac{1}{2} \longrightarrow \sqrt{\varepsilon_r} = 2 \qquad \overset{\varepsilon_r'}{\varepsilon_r} = 4$$

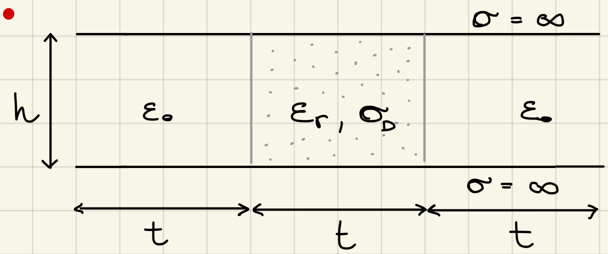
$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\frac{b}{a}} \longrightarrow \ln\frac{b}{a} = 2,5 \qquad \underline{\underline{a = 0,041\,\text{cm}}}$$

$$R_s = \frac{1}{\sigma\delta} \overset{\sqrt{\frac{\pi f \mu_0}{\sigma}}}{=} 6,28\cdot 10^{-3}\,\frac{\Omega}{\square} \qquad R = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = 2,64\,\frac{\Omega}{m}$$

× le linee TEM

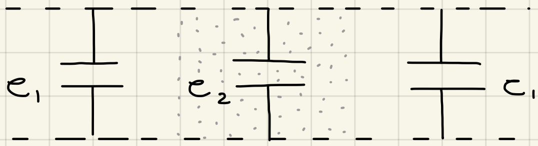
$$\alpha_c = \frac{R}{2Z_c} = \underline{\underline{0,0176\,\frac{Np}{m}}} \qquad \alpha_D = \frac{\pi}{\lambda}\frac{\varepsilon_r''}{\varepsilon_r'} = \frac{\pi}{\frac{c}{2f}}\frac{\varepsilon_r''}{\varepsilon_r'} = \underline{\underline{10,48\cdot 10^{-3}}}\,\frac{Np}{m}$$

oppure $\alpha_D = \frac{G Z_c}{2}$
× tutte le linee

- 

$\varepsilon_r = 4$ $\sigma_D = 5\cdot 10^{-4}\,\frac{S}{m}$ $h = 1\,\text{mm}$

Calcolare t affinché $Z_c = 60\,\Omega$ (trascurando gli effetti di bordo).



$$C_1 = \frac{\varepsilon_0 t}{h} \qquad C_2 = \varepsilon_0 \varepsilon_r \frac{t}{h}$$

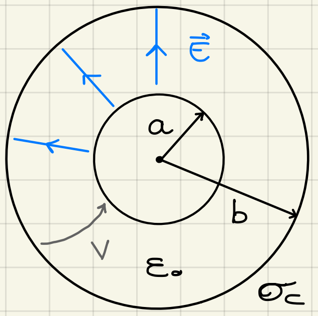
$$C = 2C_1 + C_2 = 6\,\frac{\varepsilon_0 t}{h}$$

$$Z_c = 60\,\Omega = \sqrt{\frac{L_0}{C}} = \sqrt{\frac{\mu_0 h \cdot h}{3t6\varepsilon_0 t}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{h}{t} \cdot \frac{1}{\sqrt{18}} \longrightarrow \underline{\underline{t = 1{,}48\ \text{cm}}}$$

Calcolare $\alpha_D \left(\frac{dB}{m}\right)$ dovuta alle perdite nel dielettrico.

$$G = \underset{\substack{\downarrow \\ \text{ho perdite solo nel dielettrico}}}{\frac{\sigma_D}{\varepsilon} C_2} = \sigma_D \frac{t}{h} = 7{,}4 \cdot 10^{-4}\,\frac{S}{m} \qquad \alpha_D = \frac{G Z_c}{2} = 0{,}022\,\frac{Np}{m}$$

$$= \underline{\underline{0{,}191\,\frac{dB}{m}}}$$



$$b = 0{,}5\ \text{cm} \qquad \sigma_c = 5{,}8 \cdot 10^{7}\,\frac{S}{m}$$

$$\frac{b}{a} = 3{,}5 \longrightarrow a = 0{,}143\ \text{cm}$$

Calcolare la massima potenza nel cavo coassiale, sapendo che $|E_{MAX}| = 30\,\frac{kV}{cm}$ adottando un fattore di sicurezza 2.

Calcolare la potenza specifica dissipata, nei conduttori esterno ed interno, con $f = 1\,GHz$.

$$Z_c = \frac{\eta_0}{2\pi} \ln \frac{b}{a} = 75\,\Omega \qquad E(r) = \frac{V}{\ln \frac{b}{a}} \cdot \frac{1}{r} \qquad E_{MAX} = E(r=a)$$

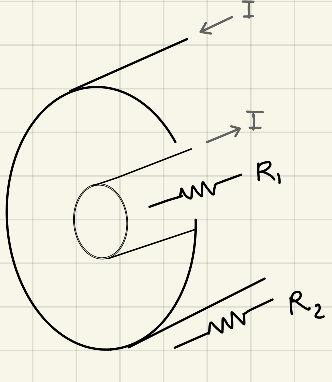
$$= \frac{30\,kV}{cm} \cdot \frac{1}{2}$$

$$= 1{,}5 \cdot 10^{6}\,\frac{V}{m}$$

$$V_{MAX} = E_{MAX}\, a \ln \frac{b}{a} = 2690\ V$$

$$P_{MAX} = \frac{1}{2} \frac{|V_{MAX}|^2}{Z_c} = \underline{\underline{48\,kW}}$$

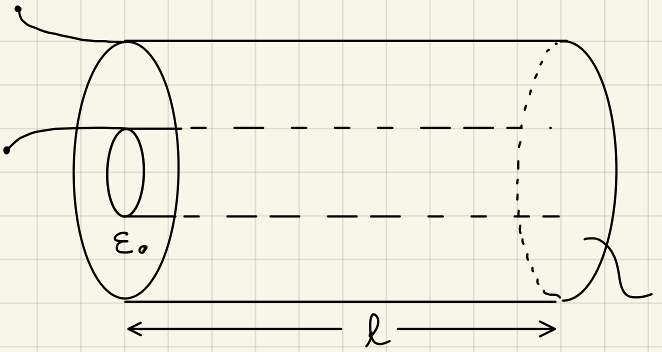
$$I = \sqrt{\frac{2P_{MAX}}{Z_c}} = 35{,}8\ A = \frac{V_{MAX}}{Z_c} \qquad R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \frac{1}{\sigma \delta} = 0{,}00825\,\frac{\Omega}{\square}$$



$$R_1 = \frac{R_s}{2\pi a} = 0{,}918\,\frac{\Omega}{m} \qquad R_2 = \frac{R_s}{2\pi b} = 0{,}263\,\frac{\Omega}{m}$$

$$P_1 = \frac{1}{2} |I|^2 R_1 = \underline{\underline{588}}\,\frac{W}{m} \qquad P_2 = \frac{1}{2} |I|^2 R_2 = \underline{\underline{168\,\frac{W}{m}}}$$

molto elevata!



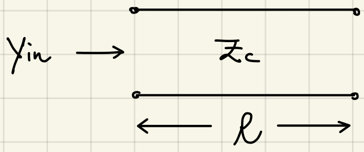
$Z_c = 50\ \Omega$

Determinare ℓ in modo da avere una capacità <u>elettrostatica</u> $C_0 = 6\,pF$.

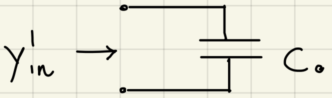
$$Z_c = \frac{\sqrt{\mu_0\varepsilon_0}}{\mathcal{C}} \longrightarrow \mathcal{C} = 66{,}7\ \frac{pF}{m}$$

$$C_0 = \mathcal{C}\cdot\ell \longrightarrow \underline{\underline{\ell = 0{,}09\,m}}\ \ (9\,cm)$$

Determinare la frequenza massima alla quale si può usare il "condensatore", per cui la capacità cambia del 5%



$Y_{in} = j\,Y_c\,tg\,\beta\ell$



$Y'_{in} = j\omega C_0$

$Y_{in} = Y'_{in} \qquad \omega C_0 = Y_c\,tg\,\beta\ell$

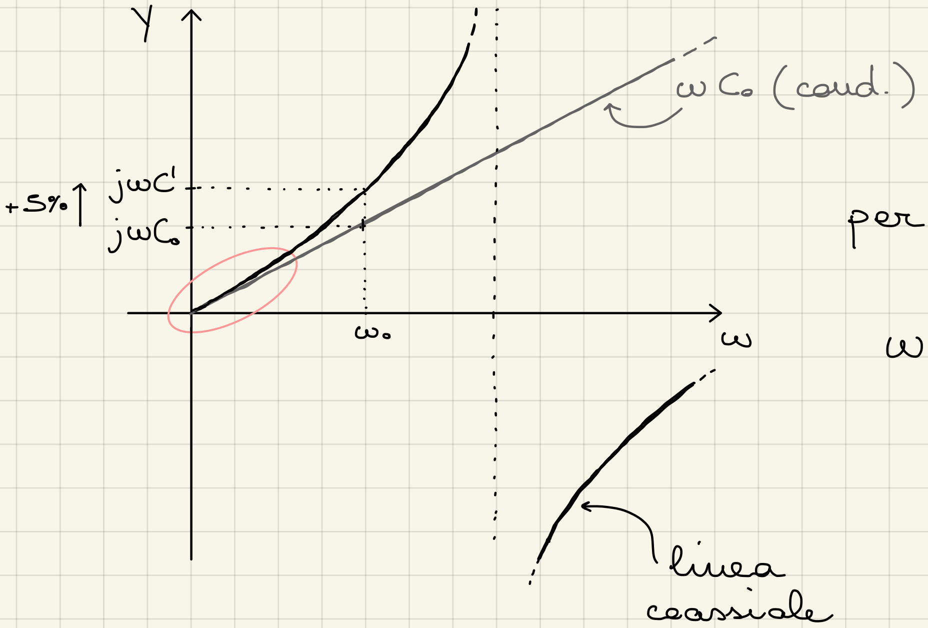
con $\beta\ell = \frac{2\pi}{\lambda}\ell \ll 1 \quad (\omega \to 0,\ \lambda \to +\infty)$

quindi $\underline{\underline{tg\,\beta\ell \approx \beta\ell}}$

$$\omega C_0 \simeq Y_c\,\frac{2\pi}{\lambda}\ell = Y_c\,\frac{2\pi}{\frac{2\pi c}{\omega}}\cdot\ell \longrightarrow \frac{Y_c}{c}\cdot\ell = C_0 \longrightarrow \ell = 9\,cm$$

come prima!

(la soluzione dinamica per $\omega \to 0$ equivale alla soluzione statica)



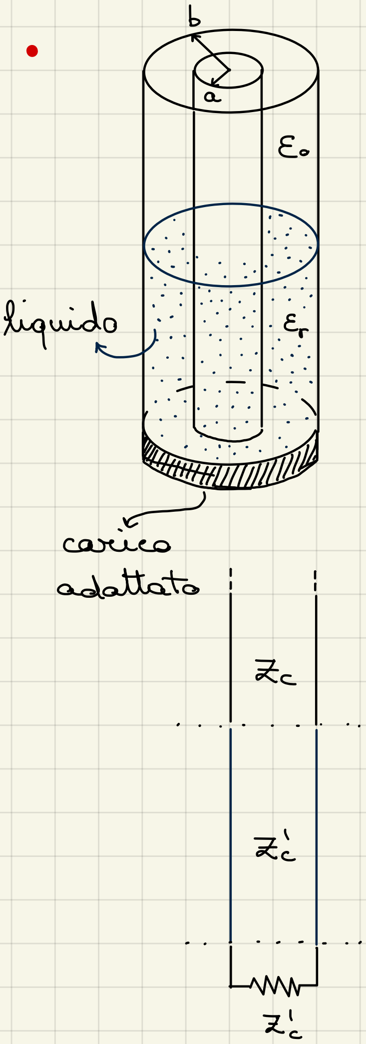
per $\omega = \omega_0 \qquad C' = C_0(1+5\%)$
$= 6{,}3\,pF$

$$\omega_0 C' = Y_c\,tg\left(\frac{\omega_0}{c}\cdot\ell\right)$$

$\big\downarrow$ risoluzione numerica

$$\omega_0 = 1{,}2\cdot10^9\ \frac{rad}{s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \underline{190\ MHz}$$



$|V_{MAX}| = 2V$ $|V_{min}| = 0,6\ mV$

Ricavare la ε_r del liquido

$$\begin{cases} Z_c' = \dfrac{\eta_0}{2\pi\sqrt{\varepsilon_r}} \ln \dfrac{b}{a} \\ Z_c = \dfrac{\eta_0}{2\pi} \ln \dfrac{b}{a} \end{cases}$$

$$ROS = \frac{|V_{MAX}|}{|V_{min}|} = 3,33$$

$$|\Gamma| = \frac{ROS - 1}{ROS + 1} = 0,54$$

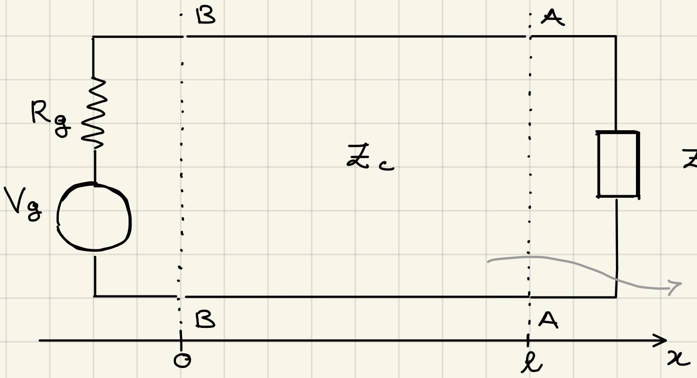
$$\Gamma = \frac{Z_c' - Z_c}{Z_c' + Z_c}$$

Z_c' e Z_c reali $\searrow$ Γ reale e <u>negativo</u>

$Z_c' \leqslant Z_c$ $\nearrow$

$$\Rightarrow \frac{Z_c' - Z_c}{Z_c' + Z_c} = -0,54 \longrightarrow \sqrt{\varepsilon_r} = 3,33$$

$$\underline{\underline{\varepsilon_r \simeq 11}}$$



$Z_L = 20 - j\,30\ [\Omega]$

no attenuazione (solo sfasamento)

$V_g = 10V$

$R_g = 50\Omega$ } adattam.

$Z_c = 50\Omega$

$\ell = 3,5m$

$f = 100\ MHz$

Calcolare P_L, $|V_{AA}|$, $|I_{AA}|$, $|V_{MAX}|$, $|V_{min}|$ e V_{BB}.

$$P_d = P_m^+ = \frac{|V_g|^2}{8R_g} = 0,25\ W$$ potenza disponibile

$\uparrow$ adattamento tra generatore e linea

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = -0,207 - j\,0,517 \qquad |\Gamma_L| = 0,557$$

$$P_L = P_d(1 - |\Gamma_L|^2) = 0,172\ W$$

$$P_L = \frac{1}{2}|V_{AA}|^2 \operatorname{Re}\{Y_L\} = \frac{1}{2}|I_{AA}|^2 \operatorname{Re}\{Z_L\}$$

$\hookrightarrow$ $P_L = \frac{1}{2}|V_{AA}|^2 \operatorname{Re}\{0,0154 + j\,0,023\}$ $P_L = \frac{1}{2}|I_{AA}|^2 \operatorname{Re}\{20 - j\,30\}$

$|V_{AA}| = 4,37\ V \qquad |I_{AA}| = 0,131\ A$

Altro metodo:

adattamento tra generatore e linea

$$|V^+(0)| = \frac{V_g}{2} = 5V$$

linea non attenuativa

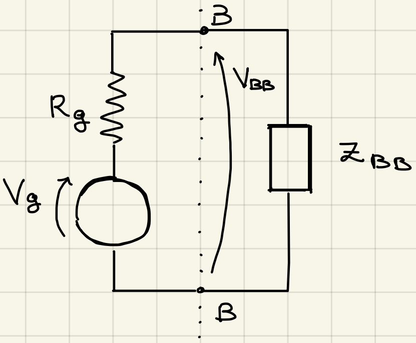
$$|V_L| = |V_{AA}| = |V^+(\ell) + V^-(\ell)| = |V^+(\ell)|\,|1+\Gamma_L| = |V^+(0)|\,|1+\Gamma_L| = $$
$$= 5 \cdot |1 - 0,207 - j\,0,517| = 4,73\,V$$

$$|I_{AA}| = |I^+(\ell) + I^-(\ell)| = |I^+(\ell)| \cdot |1 - \Gamma_L| = \frac{|V^+(0)|}{Z_c}\,|1-\Gamma_L| = 0,131\,A$$

$$\lambda = \frac{c}{f} = 3m$$

$$|V_{MAX}| = |V^+(0)| + |V^-(0)| = |V^+(0)|\,(1+|\Gamma_L|)$$
$$= 7,78\ V$$

$$|V_{min}| = |V^+(0)| - |V^-(0)| = |V^+(0)|\,(1-|\Gamma_L|)$$
$$= 2,21\,V$$



$$Z_{BB} = Z_c\,\frac{Z_L + j\,Z_c\,tg(\beta\ell)}{Z_c + j\,Z_L\,tg(\beta\ell)} = 17,2 + j\,22\ [\Omega]$$

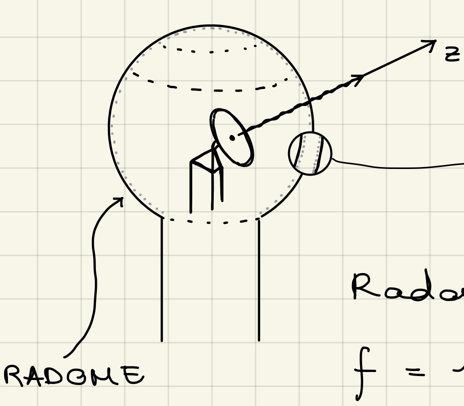
(Altro metodo: uso la carta di Smith)

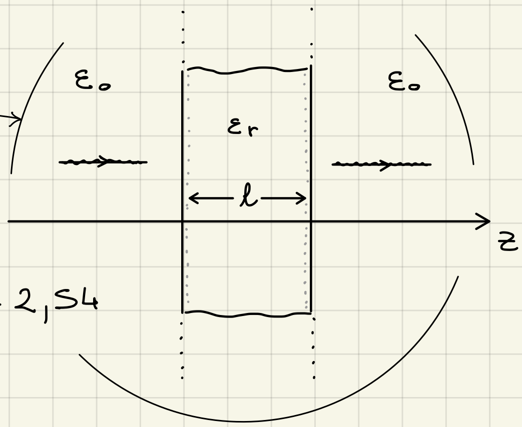
$$V_{BB} = V_g\,\frac{Z_{BB}}{R_g + Z_{BB}} = 3,27 + j\,2,19\ [V]$$

Altro metodo:

$$V_{BB} = V(0) = V^+(0) + V^-(0) = V^+(0)\,(1+\Gamma(0)) = 3,27 + j\,2,19\ [V]$$

$$\Gamma(0) = \Gamma_L\,e^{-2j\beta\ell} = -0,344 + j\,0,438$$





Radome in polystirene $\varepsilon_r = 2,54$

$f = 10\ GHz$

Calcolare ℓ in modo che il radome sia trasparente per il segnale radar.

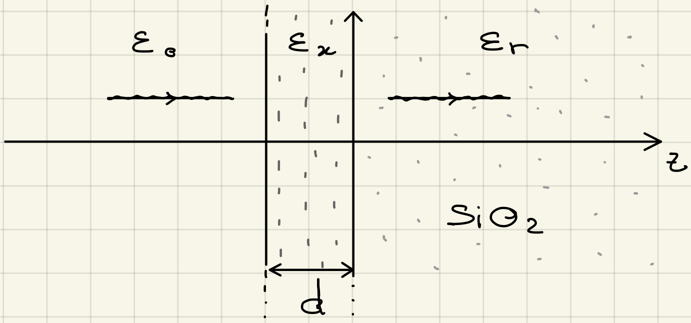
$\eta_0 = 377\,\Omega$

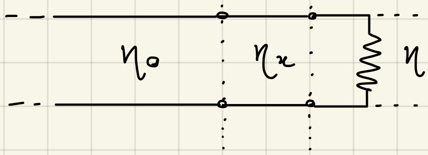
$$\ell = \frac{\lambda}{2} = \frac{\lambda_0}{\sqrt{\varepsilon_r}} \cdot \frac{1}{2} = \frac{c}{\sqrt{\varepsilon_r}\, f} \cdot \frac{1}{2} = 9{,}4 \text{ mm}$$

↳ da A a B devo compiere un giro completo (0,5 λ) sulla carta di Smith

- Laser ad argon ($\lambda_0 = 488$ nm) incide su silice fusa ($\varepsilon_r = 1{,}46$)

Dimensionare un setto antiriflesso (ε_x ? d ?)



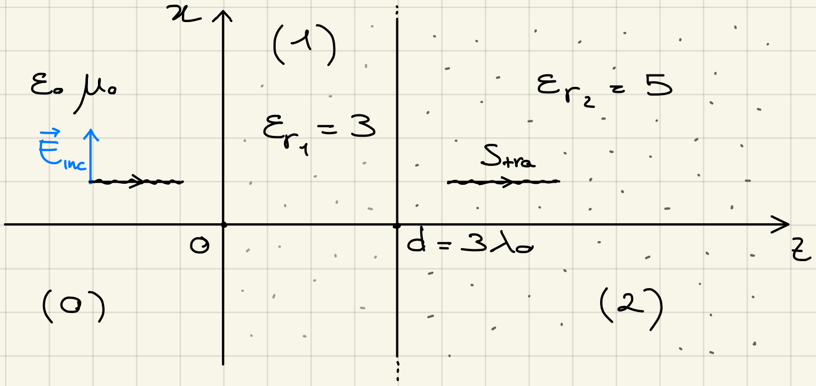


$$\eta_3 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = 312\,\Omega$$

→ Trasformatore $\frac{\lambda}{4}$: $\eta_x = \sqrt{312 \cdot 377} = 343\,\Omega$

$$\eta_x = \frac{\eta_0}{\sqrt{\varepsilon_x}} \rightarrow \varepsilon_x = 1{,}21 \geq 1 \text{ ok } \checkmark$$

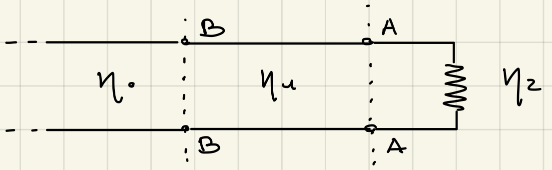
$$d = \frac{\lambda}{4} = \frac{\lambda_0}{\sqrt{\varepsilon_x}} \cdot \frac{1}{4} = 110 \text{ nm}$$

- 

$f = 2$ GHz

$|\vec{E}_{inc}| = 10 \frac{V}{m}$

$S_{tra} = ?$



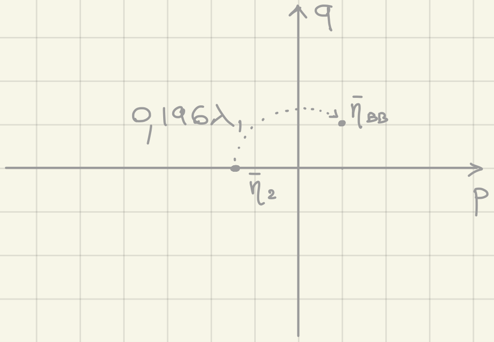
$\eta_0 = 377\,\Omega$

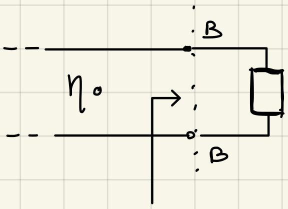
$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} = 217{,}7\,\Omega$

$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = 168{,}8\,\Omega$

$\bar{\eta}_2 = \frac{\eta_2}{\eta_1} = 0{,}775$

$\frac{d}{\lambda_1} = 3\lambda_0 \frac{1}{\frac{\lambda_0}{\sqrt{\varepsilon_{r_1}}}} = 3\sqrt{\varepsilon_{r_1}} = 0{,}196 \longrightarrow \bar{\eta}_{BB} = 1{,}2 + j\,0{,}195$



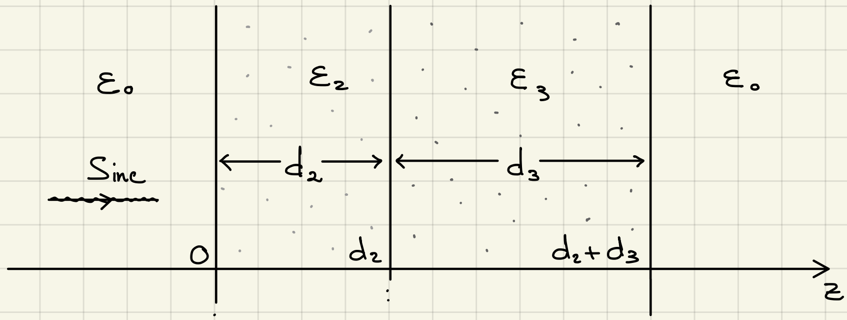


$\eta_{BB} = 261{,}7 + j\,42{,}4\ [\Omega]$

$\Gamma_{BB} = \frac{\eta_{BB} - \eta_0}{\eta_{BB} + \eta_0} = -0{,}175 + j\,0{,}078$

$S_{inc} = \frac{1}{2}\frac{|\vec{E}_{inc}|^2}{\eta_0} = 0{,}133\ \frac{W}{m^2}$

$S_{tra} = S_{inc}\left(1 - |\Gamma_{BB}|^2\right) = 0{,}128\ \frac{W}{m^2}$



$\varepsilon_2 = 2{,}54\,\varepsilon_0$

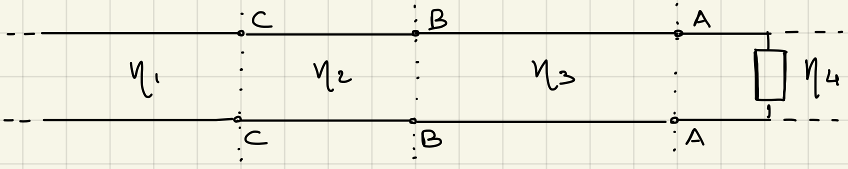
$\varepsilon_3 = 4 \cdot \varepsilon_0$

$d_2 = 2\,mm$

$d_3 = 3\,mm$

$f = 10\,GHz$

$S_{tra} = ?$



$\eta_4 = \eta_1 = \eta_0 = 377\,\Omega$

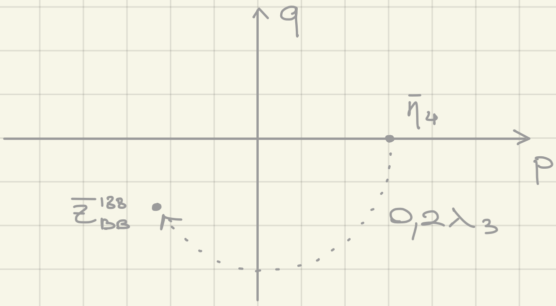
$\eta_3 = \frac{\eta_0}{\sqrt{\varepsilon_{r_3}}} = 188{,}5\,\Omega$

$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = 236{,}5\,\Omega$

$\bar{\eta}_4 = \frac{\eta_4}{\eta_3} = 2$

$\frac{d_3}{\lambda_3} = \frac{d_3}{\lambda_0}\sqrt{\varepsilon_{r_3}} = \frac{3\sqrt{\varepsilon_{r_3}}}{30} = 0{,}2$

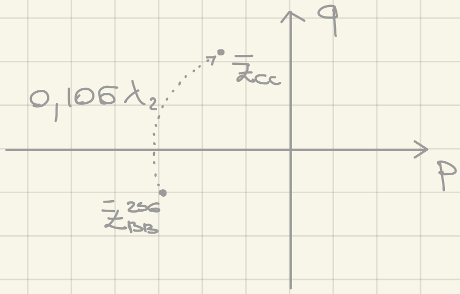
$\frac{d_2}{\lambda_2} = \frac{d_2}{\lambda_0}\sqrt{\varepsilon_{r_2}} = 0{,}106$



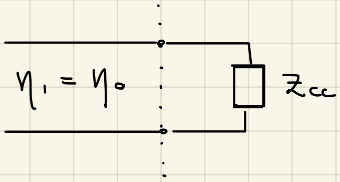
$\bar{z}_{BB}^{188} = 0,55 - j\,0,235$ $\qquad$ $\bar{z}_{BB}^{256} = \bar{z}_{BB}^{188} \cdot \frac{\eta_3}{\eta_2} = 0,438 - j\,0,187$

rinormalizzo

$\bar{z}_{cc} = 0,49 + j\,0,38$



$z_{cc} = 166 + j\,90\ [\Omega]$

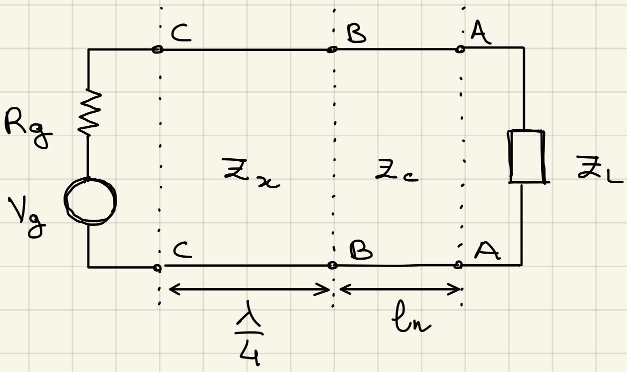


$$|\Gamma_{cc}| = \left|\frac{z_{cc} - \eta_0}{z_{cc} + \eta_0}\right| = 0,55$$

$$S_{tra} = S_{inc} \cdot \left(1 - |\Gamma_{cc}|^2\right) = 0,7\ S_{inc}$$

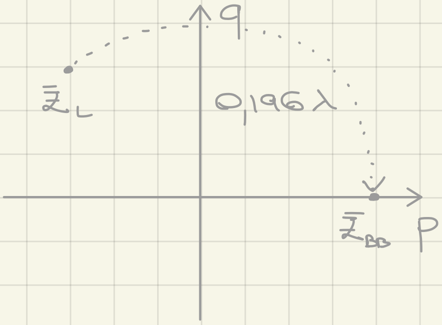
$z_L = 40 + j\,30\ [\Omega]$ da adattare a un generatore con impedenza interna $R_g = 20\Omega$ e $V_g = 10V$ tramite trasformatore $\frac{\lambda}{4}$. Linea di neutralizzazione $z_c = 100\ \Omega$

Calcolare $|V_{AA}|$, $|V_{BB}|$ e $|V_{cc}|$.



$\bar{z}_L = \frac{z_L}{z_c} = 0,4 + j\,0,3$

$l_n = ?$ $\qquad$ $z_x = ?$



$\bar{z}_{BB} = 2,76$

$l_n = 0,196\lambda$

$z_{BB} = \bar{z}_{BB}\, z_c = 276\ \Omega$

$z_x = \sqrt{z_{BB} \cdot R_g} = 74,3\Omega$

$P_L = P_d = \frac{|V_g|^2}{8\,R_g} = 0,625\,W$ $\qquad$ $P_L = \frac{1}{2}|V_{cc}|^2 \operatorname{Re}\{Y_{cc}\}$ $\quad \frac{1}{20\Omega}$

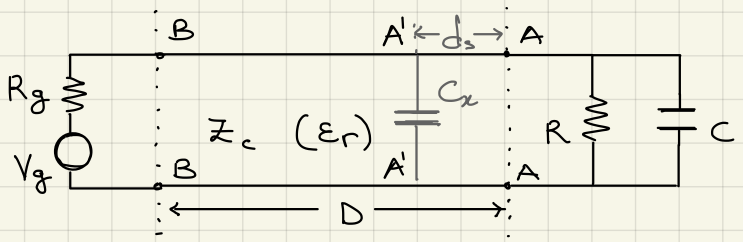
adattamento

$\longrightarrow |V_{cc}| = 5V$

$P_L = \frac{1}{2}|V_{BB}|^2 \operatorname{Re}\{Y_{BB}\}$ $\quad \frac{1}{276\Omega}$ $\qquad$ $P_L = \frac{1}{2}|V_{AA}|^2 \operatorname{Re}\{Y_{AA}\}$ $\quad \frac{1}{z_L}$

$\longrightarrow |V_{BB}| = 18,5\ V$ $\qquad$ $\longrightarrow |V_{AA}| = 8,8\ V$

$\hookrightarrow$ è (molto) più alto di V_g!



R = 100 Ω

D = 16 m

C = 20 pF

ε_r = 4

R_g = 75 Ω

f = 100MHz

P_d = 1W

Z_c = 50 Ω

a) Calcolare la potenza assorbita dal carico

$$Y_{AA} = \frac{1}{R} + j\omega C = \frac{1}{100\Omega} + j2\pi f C =$$

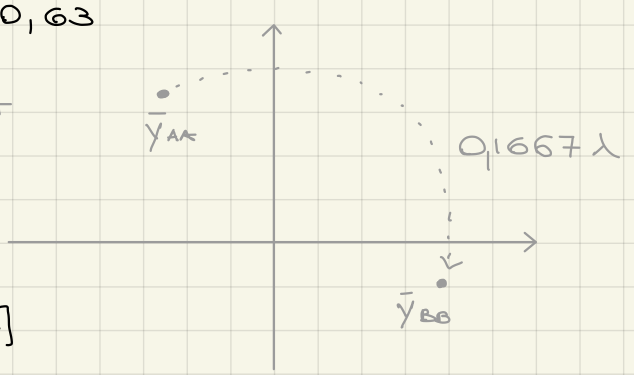
$$= 0{,}01 + 0{,}0126 \ [\Omega^{-1}]$$

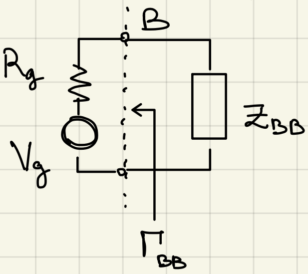
$$\bar{Y}_{AA} = \frac{Y_{AA}}{Y_c} = Y_{AA} \cdot Z_c = 0{,}5 + j\,0{,}63$$

$$\frac{D}{\lambda} = \frac{D}{\lambda_0}\sqrt{\varepsilon_r} = 10{,}6667 = 0{,}1667$$

$$\bar{Y}_{BB} = 2{,}6 - j\,0{,}8$$

$$Z_{BB} = \frac{1}{\bar{Y}_{BB}} \cdot Z_c = 17{,}5 + j5{,}5 \ [\Omega]$$





$$P_d = \frac{|V_g|^2}{8R_g}$$

$$P_L = P_d\left(1 - |\Gamma_{BB}|^2\right) = 0{,}611 \text{ W}$$

$$\text{con} \quad |\Gamma_{BB}| = \frac{Z_{BB} - R_g}{Z_{BB} + R_g} = 0{,}6234$$

b) Adattare il carico alla linea, utilizzando un <u>condensatore</u> posto ad opportuna distanza d_s dalla sezione AA in parallelo alla linea

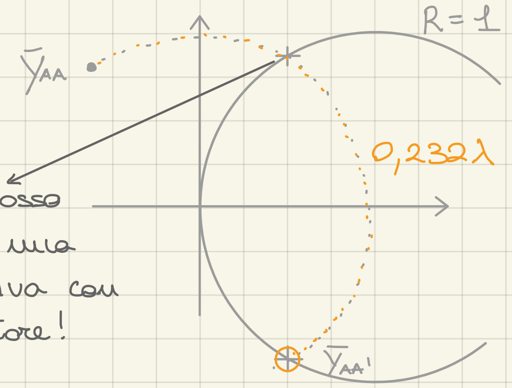
$$Y_{AA} \xrightarrow[d_s]{} 1 + jB$$

$$d_s = 0{,}232\,\lambda$$

$$\bar{Y}_{AA'} = 1 - j1{,}1$$

$$Y_s = +j\,1{,}1\,Y_c$$
$$= j\,0{,}022 \ [\Omega^{-1}]$$

$$Y_s = j\,\omega C_x \longrightarrow C_x = 35\text{pF}$$



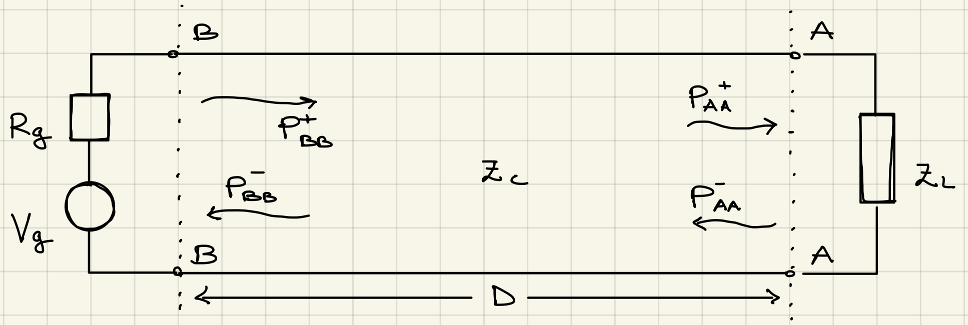
non posso compensare una suscettanza positiva con un condensatore!

c) Calcolare la potenza assorbita dal carico con la capacità appena inserita.

$$\Gamma_{BB} = \frac{50\Omega - 75\Omega}{50\Omega + 75\Omega} = 0{,}2$$

$$P_L = P_d\left(1 - |\Gamma_{BB}|^2\right) = 0{,}96\text{W}$$

•



$R_g = 50\,\Omega$

$Z_c = 50\Omega$ } linea adattata al generatore

$Z_L = 170\,\Omega$

$D = 25\,\Omega$

coeff. di attenuazione ← $\alpha_{dB} = 8\,dB/100m$

$P_d = 163\,W$

Calcolare:

P_L (assorbita dal carico)

P_{dis} (dissipata dalla linea)

P_g (erogata dal generatore).

$$\alpha = \frac{8}{100} \cdot \frac{1}{8{,}686} = 0{,}0092\,\frac{Np}{m}$$

$P^+_{BB} = P_d$ $\qquad$ $P^+_{AA} = P^+_{BB}\, e^{-2\alpha \cdot D} = 103\,W$

$|\Gamma_{AA}| = \left|\dfrac{Z_L - Z_c}{Z_L + Z_c}\right| = 0{,}545$ $\qquad$ $P_L = P^+_{AA}\left(1 - |\Gamma_{AA}|^2\right) = \underline{\underline{72{,}3\,W}}$

$P^-_{AA} = P^+_{AA}\,|\Gamma_{BB}|^2 = 30{,}7\,W$ $\qquad$ $P^-_{BB} = P^-_{AA}\, e^{-2\alpha D} = 19{,}3\,W$

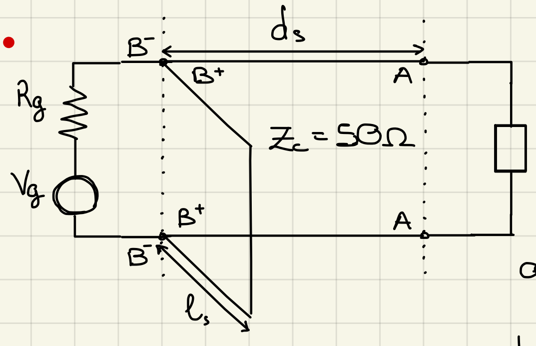
$P_{dis} = \Delta P^+ + \Delta P^- = (163 - 103) + (30{,}7 - 19{,}3) = 71{,}4\,W$

$P_g = P_d - P^-_{BB} = 163 - 19{,}3 = 143{,}7\,W$ $\quad$ (oppure $P_g = P_d\left(1 - |\Gamma_{BB}|^2\right)$ con $|\Gamma_{BB}| = |\Gamma_{AA}|e^{-2\alpha D}$)

Calcolare P_L nel caso di linea <u>non</u> attenuativa $(\alpha = 0)$

$P^+_{AA} = P^+_{BB} = P_d$ $\qquad$ $P_L = P^+_{AA}\left(1 - |\Gamma_{AA}|^2\right) = 114{,}6\,W$

•



$Z_L = 30 - j50\ [\Omega]$

$R_g = 83\,\Omega$

$V_g = 1\,V$

a) Determinare d_s, l_s per l'adattamento

b) Calcolare P_L, $|V_{AA}|$, $|V_{BB}|$

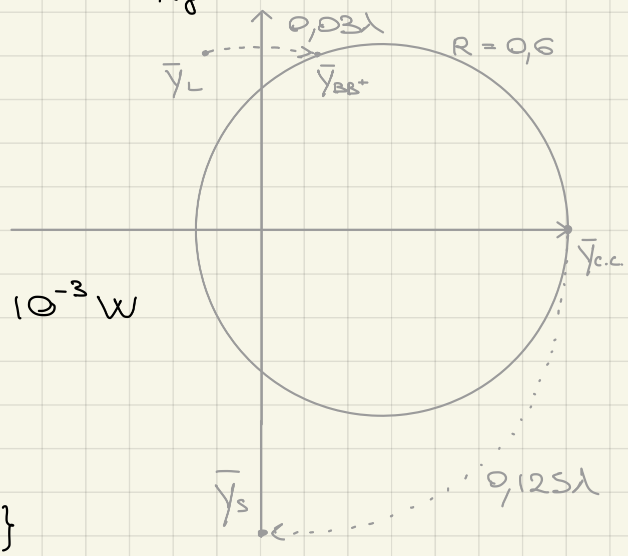
c) Tracciare l'andamento della tensione lungo lo stub.

a) $\bar{y}_L = \frac{Z_c}{Z_L} = 0{,}44 + j\,0{,}73$

Dobbiamo finire nel punto $\bar{y}_g = \frac{Z_c}{R_g} \simeq 0{,}6$

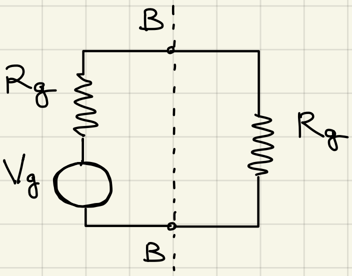
$d_s = 0{,}03\,\lambda$ $\quad \bar{y}_{BB^+} = 0{,}6 + \textcircled{1j}$

$\longrightarrow \bar{y}_s = -1j \quad l_s = 0{,}125\,\lambda$



b) Adattato $\quad P_L \overset{\text{adattato}}{=} P_d = \frac{|V_g|^2}{8R_g} = 1{,}5\cdot10^{-3}$ W

$|V_{BB}| = \frac{V_g}{2} = 0{,}5$ V

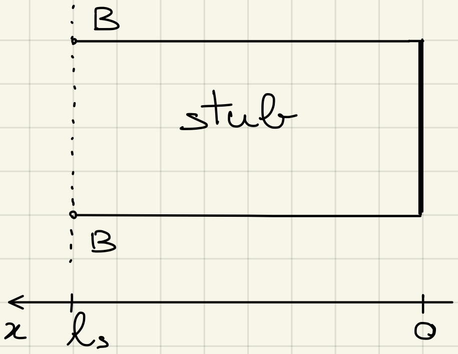


$P_L = \frac{1}{2}|V_{AA}|^2\,\mathrm{Re}\{y_L\}$

0,0088 + j 0,0147

$\longrightarrow |V_{AA}| = 0{,}584$ V

c)



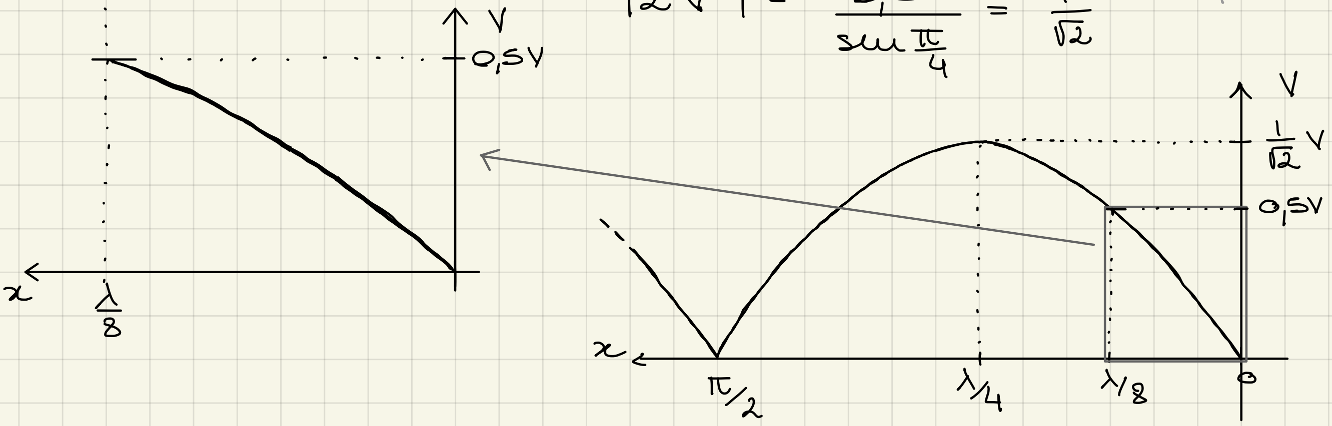
$|V(x)| = |2V^+|\,|\mathrm{sen}(\beta x)|$

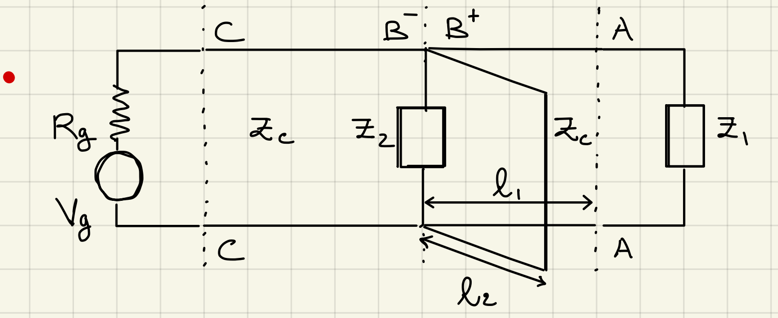
Cond. al contorno:

$|V(l_s)| = |V_{BB}| = 0{,}5 =$

$= |2V^+|\,\left|\mathrm{sen}\left(\underbrace{\overset{\frac{2\pi}{\lambda}\cdot\frac{\lambda}{8}}{\beta l_s}}_{\frac{\pi}{4}}\right)\right|$

$|2V^+| = \frac{0{,}5}{\mathrm{sen}\frac{\pi}{4}} = \frac{1}{\sqrt{2}}$





$R_g = 50\,\Omega$ $\quad Z_2 = (100 - j100)\,\Omega$

$V_g = 100$ V $\quad Z_1 = (100 + j\,50)\,\Omega$

$Z_c = 50\,\Omega$

a) Determinare ℓ_1 e ℓ_2 in modo da avere adattamento in BB

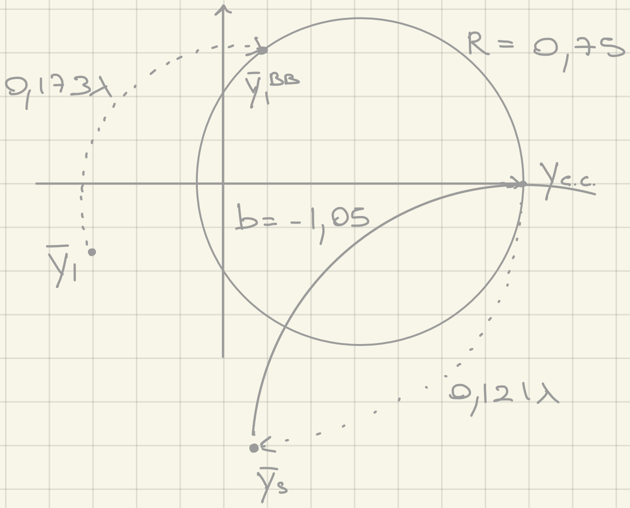
b) Calcolare P_{L1} e P_{L2}

a) $\bar{y}_1 = \frac{Z_c}{Z_1} = 0,4 - j0,2$ $\qquad$ $\bar{y}_2 = \frac{Z_c}{Z_2} = 0,25 + j0,25$

la metto a posto con lo stub.

$\bar{y}_2 \,/\!/\, \bar{y}_1^{BB} = 1 + jb \longrightarrow 0,25 + j0,25 + \bar{y}_1^{BB} = 1 + jb$

$\Longrightarrow \mathrm{Re}\{\bar{y}_1^{BB}\} = 0,75$



$\bar{y}_1^{BB} = 0,75 + j0,8$

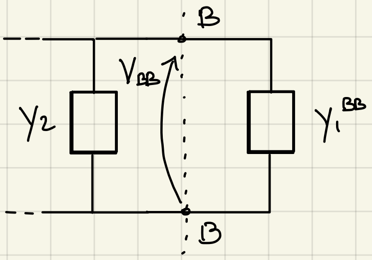
$\ell_1 = 0,173\lambda$

$\bar{y}_2 \,/\!/\, \bar{y}_1^{BB} = 1 + j\boxed{1,05}$

$\longrightarrow y_s = -1,05j$

$\ell_2 = 0,121\lambda$

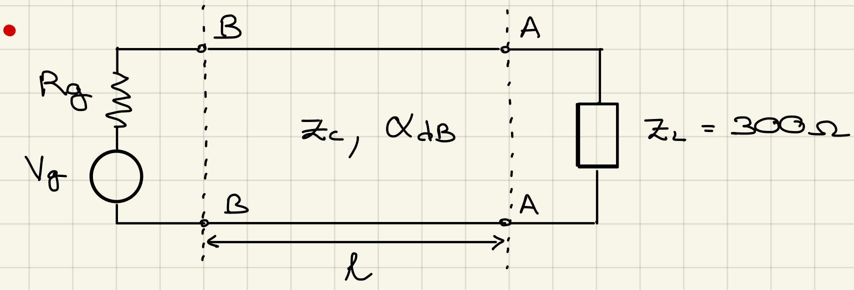
b) Adattato: $\boxed{P_{L1} + P_{L2} = P_d}$ con $P_d = \frac{|V_g|^2}{8R_g} = 25W$



$$\frac{P_{L1} = \frac{1}{2}|V_{BB}|^2\,\mathrm{Re}\{y_1^{BB}\}}{P_{L2} = \frac{1}{2}|V_{BB}|^2\,\mathrm{Re}\{y_2\}} \Rightarrow \frac{P_{L1}}{P_{L2}} = \frac{\mathrm{Re}\{y_1^{BB}\}}{\mathrm{Re}\{y_2\}}$$

$P_{L1} = P_d \cdot 0,75$ $\qquad$ $P_{L2} = P_d \cdot 0,25$

$P_{L1} = 18,75 W$ $\qquad$ $P_{L2} = 6,25$



$R_g = 75\Omega$ $\qquad$ $f = 100MHz$

$V_g = 10V$ $\qquad$ $\ell = 100m$

$\alpha_{dB} = 2dB/100m$

Calcolare: P_L, P_{diss}, P_{gen}, V_{AA}.

$\bar{Z}_L = \frac{Z_L}{Z_c} = 6$ $\qquad$ $\lambda = 3m$ $\qquad$ $\frac{\ell}{\lambda} = \cancel{33} + \frac{1}{3}$

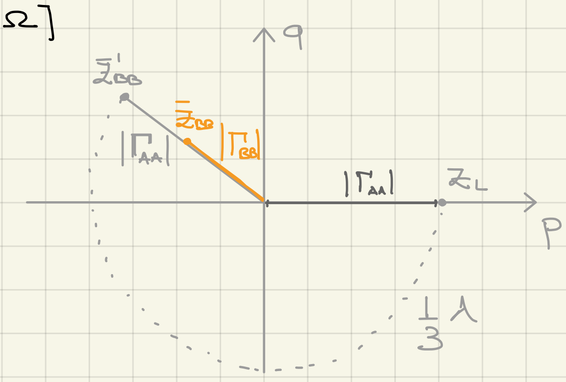
$|\Gamma_{BB}| = |\Gamma_{AA}|\, e^{-2\alpha l}$ con $\alpha = \frac{2}{100} \cdot \frac{1}{8,686} = 2,3 \cdot 10^{-3}\, \frac{Np}{m}$

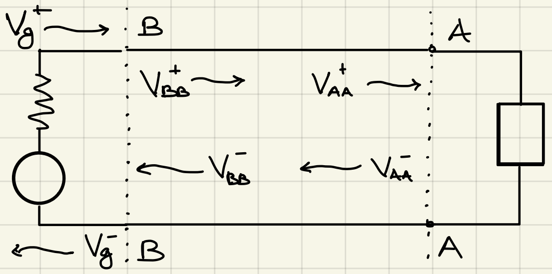
$|\Gamma_{AA}| = \left| \frac{Z_L - Z_c}{Z_L + Z_c} \right| = 0,714 \longrightarrow |\Gamma_{BB}| = 0,45$

$\bar{Z}_{BB} = 0,48 + j\,0,48 \qquad Z_{BB} = 24 + j\,24\ [\Omega]$

$|\Gamma_{BB}^{R_g}| = \left| \frac{Z_{BB} - R_g}{Z_{BB} + R_g} \right| = 0,55$

$$P_{gen} = \underset{\overset{\uparrow}{\frac{|V_g|^2}{8R_g} = \frac{1}{6}\,W}}{P_d} \left(1 - |\Gamma_{BB}^{R_g}|^2\right) = 0,116\,W$$





$V_g^+ = \frac{V_g}{2} = 5\,V$

$V_{AA}^- = \Gamma_{AA}\, V_{AA}^+$ con $\Gamma_{AA} = 0,714$

$V_{BB}^- = V_{AA}^-\, e^{-\gamma l}$ con $\gamma = \alpha + j\beta$
$= \alpha + j\frac{2\pi}{\lambda}$

$\gamma = 2,3 \cdot 10^{-3} + j\,2,094\ \left[\frac{1}{m}\right] \qquad V_{BB}^+ = V_{AA}^+\, e^{\gamma l}$

In BB: $V_g^+ + V_g^- = V_{BB}^+ + V_{BB}^-$ con $V_g^- = V_g^+ \cdot \Gamma_{BB}^{R_g}$ e

$\Gamma_{BB}^{R_g} = -0,431 + j\,0,347$

$V_g^+ \left(1 + \Gamma_{BB}^{R_g}\right) = V_{AA}^+ \left(e^{\gamma l} + \Gamma_{AA}\, e^{-\gamma l}\right)$

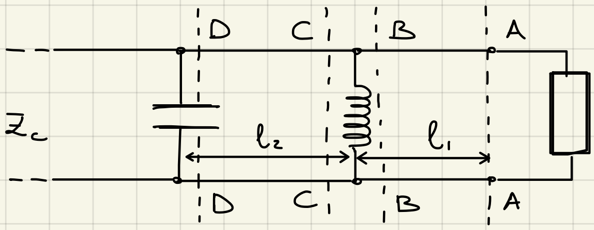
$\hookrightarrow V_{AA}^+ = -1,23 - j\,2,93\ [V]$

$V_{AA} = V_{AA}^+ \left(1 + \Gamma_{AA}\right) = -2,1 - 5j\ [V]$

$P_L = \frac{1}{2} |V_{AA}|^2\, Re\{Y_L\} = 0,0496\ W$

$P_{diss} = P_{gen} - P_L = 0,0664\,W$

•



$Z_L = 9 - j\,32,5\ [\Omega]$

$Z_c = 50\,\Omega$

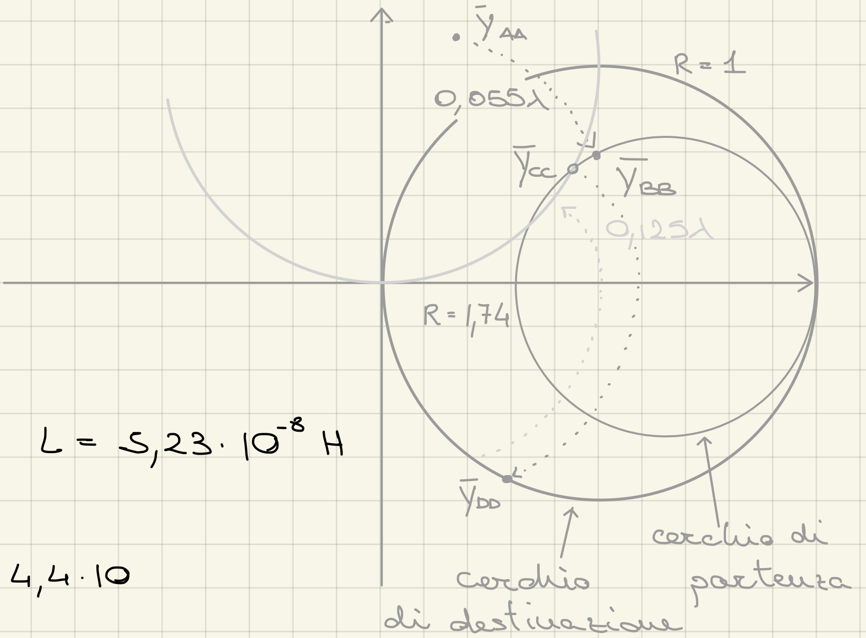
$f = 100\ MHz$

$l_1 = 0,165\,m$

$l_2 = 0,375\,m$

Determinare C ed L in modo da adattare il carico alla linea.

$\bar{Y}_L = \bar{Y}_{AA} = \frac{Z_c}{Z_L} = 0{,}4 + j\,1{,}43$ $\frac{\ell_1}{\lambda} = 0{,}055$ $\frac{\ell_2}{\lambda} = 0{,}125$

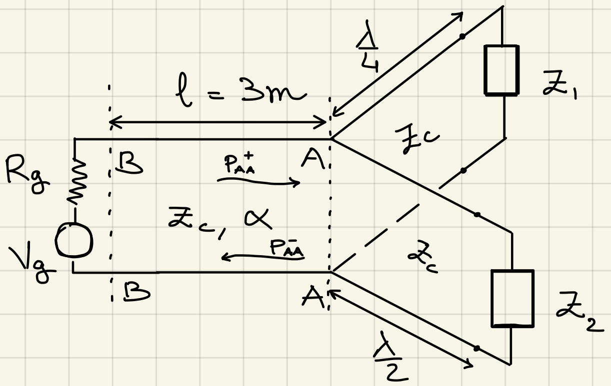


$L \left(\begin{array}{l} \bar{Y}_{BB} = 1{,}74 + j\,3{,}17 \\ \bar{Y}_{CC} = 1{,}74 + j\,1{,}65 \end{array} \right.$

$C \left(\begin{array}{l} \bar{Y}_{DD} = 1 - j\,1{,}38 \\ \bar{Y}_c \end{array} \right.$

$\frac{1}{j\omega L} = j\,\frac{(1{,}65 - 3{,}17)}{Z_c} \longrightarrow L = 5{,}23 \cdot 10^{-8}\,H$

$j\omega C = j\,\frac{1{,}38}{Z_c} \longrightarrow C = 4{,}4 \cdot 10$

- 

$Z_c = 50\,\Omega$ $R_g = 50\,\Omega$

$\alpha_{dB} = 50\,dB/100m$ $P_d = 100\,W$

$Z_1 = (25 - j25)\,\Omega$

$Z_2 = (50 - j100)\,\Omega$

$\alpha_{Np} = 0{,}05756\,\frac{Np}{m}$

Calcolare P_{L_1}, P_{L_2}, P_{diss}.

$y_2^{AA} = y_2 = \frac{1}{Z_2} = (0{,}004 + j\,0{,}008)\,\Omega^{-1}$ → giro di $\frac{\lambda}{2}$ → stessa imped.

$y_1^{AA} = \frac{y_c^2}{y_1} = \frac{Z_1}{Z_c^2} = (0{,}01 - j\,0{,}01)\,\Omega^{-1}$ → trasformatore $\frac{\lambda}{4}$

$Y_{AA} = y_1^{AA} + y_2^{AA} = (0{,}014 - j\,0{,}002)\,\Omega^{-1} \neq Y_c$

$|\Gamma_{AA}| = \left| \frac{Y_c - Y_{AA}}{Y_c + Y_{AA}} \right| = \left| \frac{Z_{AA} - Z_c}{Z_{AA} + Z_c} \right| = 0{,}186$

generatore adattato

$P_{AA}^+ = P_d\, e^{-2\alpha \ell} = 70{,}8\,W$ $P_L = P_{AA}^+ (1 - |\Gamma_{AA}|^2) = 68{,}3\,W$

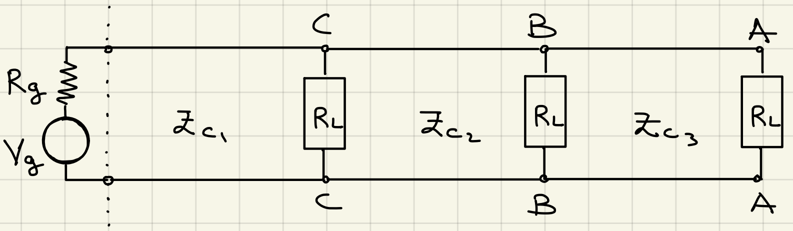
$P_{AA}^- = P_{AA}^+ |\Gamma_{AA}|^2 = 2{,}5\,W$ $P_{L_1} + P_{L_2} = P_L$

$P_{L_1} = \frac{1}{2} |V_{AA}|^2\, Re\{y_1^{AA}\}$ $P_{L_2} = \frac{1}{2} |V_{AA}|^2\, Re\{y_2^{AA}\}$

$$\begin{cases} P_{L_1} + P_{L_2} = P_L \\ \dfrac{P_{L_1}}{P_{L_2}} = \dfrac{Re\{Y_1^{AA}\}}{Re\{Y_2^{AA}\}} \end{cases} \Longrightarrow \quad P_{L_1} = 48{,}8\,W \qquad P_{L_2} = 19{,}5\,W$$

$$P_{BB}^- = P_{AA}^- e^{-2\alpha l} \qquad P_{diss} = (P_d - P_{AA}^+) + (P_{AA}^- - P_{BB}^-) = 29{,}9\,W$$

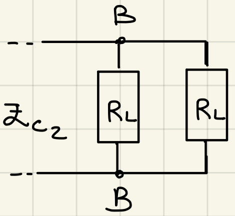
•



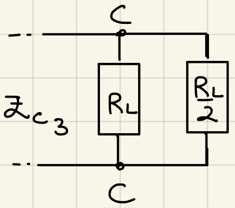
$R_L = 75\,\Omega$

Determinare Z_{c_1}, Z_{c_2}, Z_{c_3} e R_g in modo da non avere riflessione qualunque sia la lunghezza delle linee (= qualunque sia la frequenza del generatore).

$Z_{c_3} = R_L = 75\,\Omega$ (linea BA adattata a R_L^{AA})



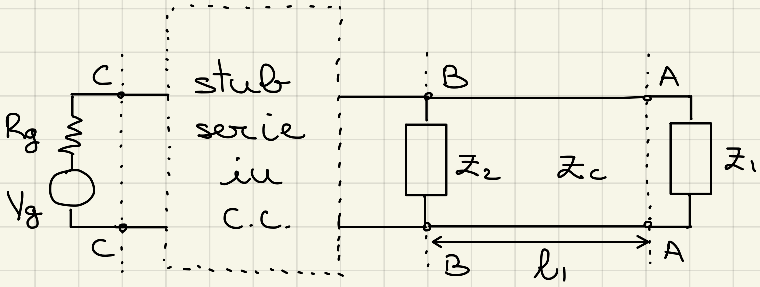
$$\frac{R_L}{2} = Z_{c_2} = 37{,}5\,\Omega$$



$$\frac{R_L}{2} /\!/ R_L = Z_{c_1} = 25\,\Omega$$

$$R_g = Z_{c_1} = 25\,\Omega$$

•



$Z_c = 50\,\Omega$

$V_g = 50\,V$ $\quad R_g = 50\,\Omega$

$Z_1 = 25\,\Omega$

$Z_2 = (50 + j50)\,\Omega$

$f = 600\,MHz$

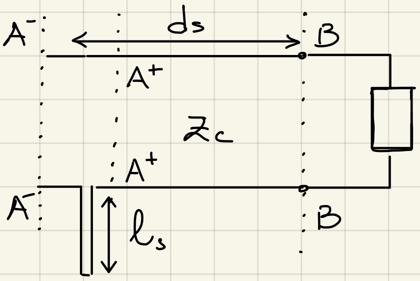
Determinare l_1 in modo che $P_{L_1} = P_{L_2}$ e dimensionare lo stub.

Per avere $P_{L_1} = P_{L_2}$ deve essere $Re\{Y_1^{BB}\} = Re\{Y_2\}$

$\bar{y}_1 = \frac{Z_c}{Z_1} = 2 \qquad \bar{y}_2 = \frac{Z_c}{Z_2} = \underline{\underline{0,5}} - j0,5$

$\bar{y}_1^{BB} = 0,5 \qquad l_1 = \frac{\lambda}{4}, \quad \lambda = 0,5m$

$\bar{y}_{BB} = \bar{y}_2 + \bar{y}_1^{BB} = 1 - j0,5 \; (\neq y_c)$



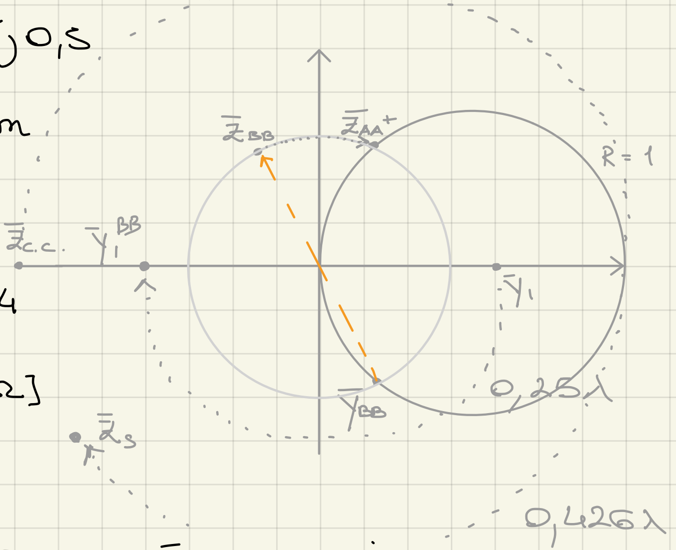
$\bar{Z}_{BB} = 0,8 + j0,4$

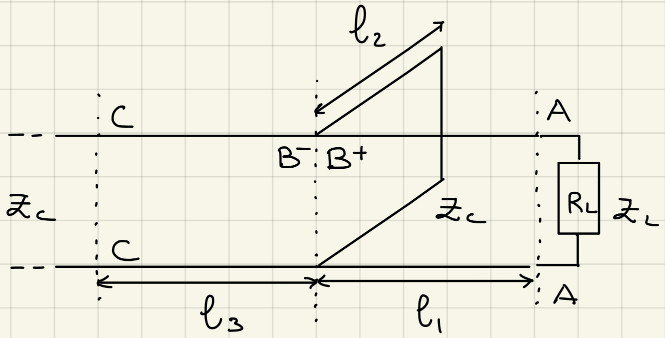
$Z_{BB} = 40 + j20 \, [\Omega]$

$d_s = 0,038\lambda$

$\bar{Z}_{AA^+} = 1 + j0,5 \rightarrow \bar{Z}_s = -0,5j$

$l_s = 0,426\lambda$





$l_1 = 1,05 \, m$

$l_2 = 0,67 m$

$l_3 = 1m$

$Z_c = 50 \Omega$

$f = 100 MHz \qquad (\lambda = 3m)$

In $\overline{CC}$ si trova il MINIMO della tensione. Il ROS è 3. Determinare Z_L.

$ROS = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 3 \longrightarrow |\Gamma| = 0,5$

In $\overline{CC}$ Γ è <u>reale negativo</u> $\rightarrow \Gamma_{cc} = -0,5$

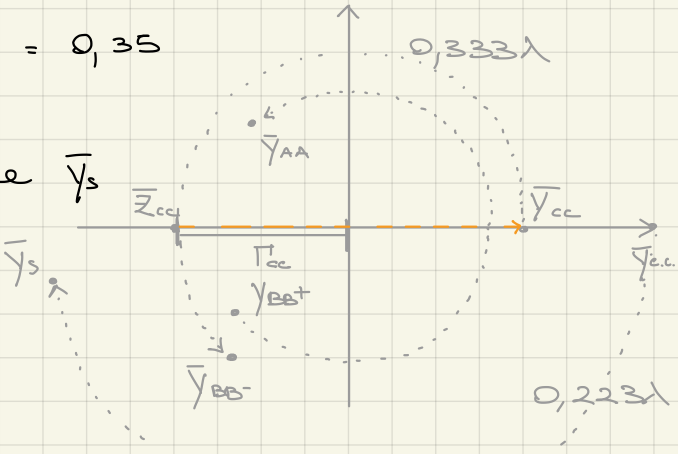
$\frac{l_3}{\lambda} = 0,333 \qquad \frac{l_2}{\lambda} = 0,223 \qquad \frac{l_1}{\lambda} = 0,35$

$\bar{Y}_{BB^-} = 0,42 - j0,50$ bisogna sottrarre $\bar{Y}_s$

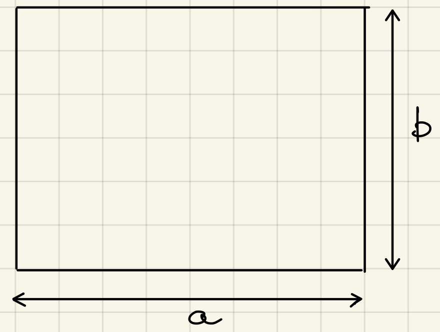
$\bar{Y}_s = -j0,17 \rightarrow \bar{Y}_{BB^+} = 0,42 - j0,33$

$\bar{Y}_{AA} = \bar{Y}_L = 0,5 + 0,5j$

$Z_L = \frac{Z_c}{Y_L} = 50 - j50 \, [\Omega]$



<u>NB</u>: passare da impedenze (z) a suscettanze (y) o viceversa sulla carta di Smith significa trovare il punto speculare al centro.



$a = 10$ cm

$b = 7{,}5$ cm

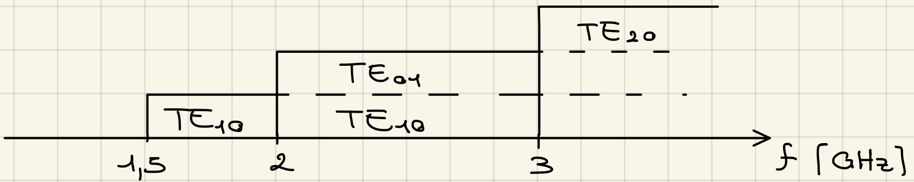
Determinare la banda di funzionamento monomodale

TE_{10}: $\lambda_c = 2a = 0{,}2$ m $\qquad f_c = \frac{c}{\lambda_c} = 1{,}5$ GHz

TE_{01}: $\lambda_c = 2b = 0{,}15$m $\qquad f_c = 2$ GHz

TE_{20}: $\lambda_c = a = 0{,}1$ m $\qquad f_c = 3$ GHz

TE_{02}: $\lambda_c = b = 0{,}075$m $\qquad f_c = 4$ GHz

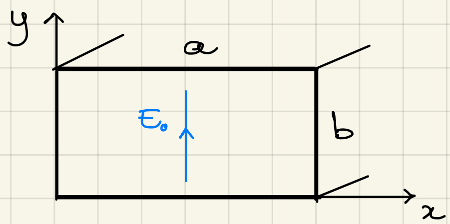


banda monomodale: 1,5 - 2 GHz

Qual è il valore di ε_r, in modo che $f_c = 1$GHz?

$\lambda_c = 0{,}2$m $\qquad f_c = \frac{c}{\sqrt{\varepsilon_r}\cdot\lambda_c} = \underset{(TE_{10})}{1\text{GHz}} \longrightarrow \sqrt{\varepsilon_r} = 1{,}5 \rightarrow \varepsilon_r = 2{,}25$

$\lambda_c = 0{,}15$m $\qquad f_c = \frac{c}{\sqrt{\varepsilon_r}\cdot\lambda_c} = \underset{(TE_{01})}{1{,}33\text{GHz}} \Longrightarrow$ nuova banda monomodale 1 - 1,33 GHz



$a = 3$ cm $\qquad f_0 = 7$ GHz

$b = 1{,}5$cm $\qquad$ Rigidità dielettrica dell'aria: 30 kV/cm

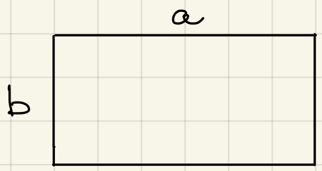
Calcolare la potenza massima, in condizioni di adattamento, con un coefficiente di sicurezza 2.

TE_{10}: $\lambda_c = 2a = 0{,}06$m $\qquad f_c = 5$ GHz $\longrightarrow$ 5GHz ⊢—⊣ 10GHz banda monomod

$|E_{MAX}| = 30\,\frac{kV}{cm}\cdot\frac{1}{2} = 15\,\frac{kV}{cm} = 1{,}5\,\frac{MV}{m}$

$E_y(x) = \underset{\overset{\downarrow}{E_0}}{|E_{MAX}|}\, \text{sen}\left(\frac{\pi x}{a}\right) \qquad P^+ = \frac{|E_{MAX}|^2\, a\cdot b}{4\, Z_{TE_{10}}}$

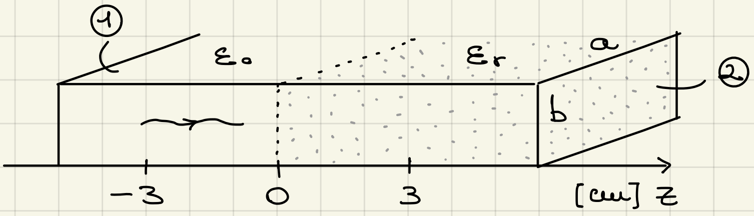
$$Z_{TE_{10}} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}} = \frac{377\,\Omega}{\sqrt{1-\left(\frac{5}{7}\right)^2}} = 538{,}6\,\Omega \Longrightarrow P^+_{MAX} = 470\text{ kW}$$



$a = 10$ cm

$b = 5$ cm

$\varepsilon_r = 4$



$f_0 = 2$ GHz $\qquad |E_0| = 20\,\frac{mV}{m} = |E^+(0)|$

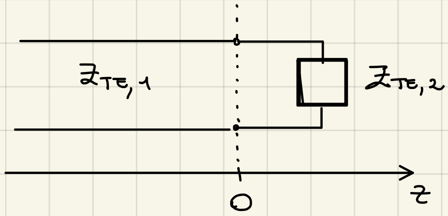
Calcolare $|E|$ in $z = 0$, $z = 3$ cm e $z = -3$ cm.

$\lambda_c = 2a = 0{,}2\,m$

$f_{c,1} = \frac{c}{\lambda_c} = 1{,}5$ GHz (aria) $\quad (1{,}5 - 3)$ GHz

$f_{c,2} = \frac{c}{\sqrt{\varepsilon_r}\,\lambda_c} = 0{,}75$ GHz (dielettrico) $\quad (0{,}75 - 1{,}5)$ GHz

$$\underset{TE_{10}}{Z_{TE,1}} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_{c,1}}{f_0}\right)^2}} = 570\,\Omega \qquad Z_{TE,2} = \frac{\eta_0}{\sqrt{\varepsilon_r}}\frac{1}{\sqrt{1-\left(\frac{f_{c,2}}{f_0}\right)^2}} = 203\,\Omega$$



$$\Gamma = \frac{Z_{TE,2} - Z_{TE,1}}{Z_{TE,2} + Z_{TE,1}} = -0{,}475$$

Tratto ② (z positive): ← solo onda progressiva

$$|E(0)| = |E^+(0)|\,\underbrace{|1+\Gamma|}_{T} = 10{,}5\,\frac{mV}{m}$$

$$|E(3\,cm)| = |E(0)| = 10{,}5\,\frac{mV}{m}$$

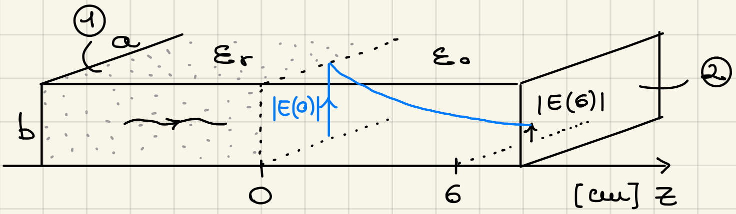
Tratto ① (z negative):

$$\lambda_{g,1} = \frac{\overset{=\lambda_0}{\lambda}}{\sqrt{1-\left(\frac{f_{c,1}}{f_0}\right)^2}} = 0{,}227\,m \qquad \beta_z = \frac{2\pi}{\lambda_g} = 27{,}706\,\frac{rad}{m}$$

$$E(z) = E^+(z) + E^-(z) = E^+(0)\,e^{-j\beta_z z} + E^-(0)\,e^{+j\beta_z z} =$$

$$= E^+(0)\left(e^{-j\beta_z z} + \Gamma\,e^{+j\beta_z z}\right) \underset{\overset{\uparrow}{z\,=\,-3cm}}{=} (7{,}07 + j\,21{,}7)\,\frac{mV}{m}$$

$$|E(-3\,cm)| = 23\,\frac{mV}{m}$$



$a = 10$ cm $\quad |E_0| = |E^+(0)| = 10 \frac{mV}{m}$

$b = 5$ cm $\quad f_0 = 1 GHz$

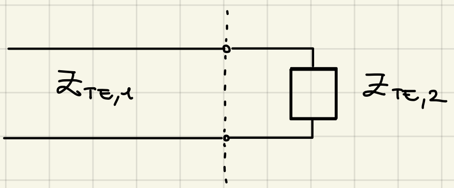
$\varepsilon_r = 4$ m

Calcolare $|E(z)|$ in $z=0$ e $z = 6$ cm.

$f_{c,1} = 0{,}75 GHz \qquad f_{c,2} = 1{,}5 GHz > f_0\ !$

$$Z_{TE,1} = \frac{\eta \; {\scriptstyle = \eta_0/\sqrt{\varepsilon_r}}}{\sqrt{1-\left(\frac{f_{c,1}}{f_0}\right)^2}} = 285\,\Omega \qquad Z_{TE,2} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_{c,2}}{f_0}\right)^2}} = \frac{377\,\Omega}{\underbrace{\sqrt{1-(1{,}5)^2}}_{\pm j1{,}22}} = j\,337$$

$$\beta = K\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \pm j\,1{,}22 \cdot K \leftarrow \alpha = j\beta = \mp 1{,}22\cdot K$$



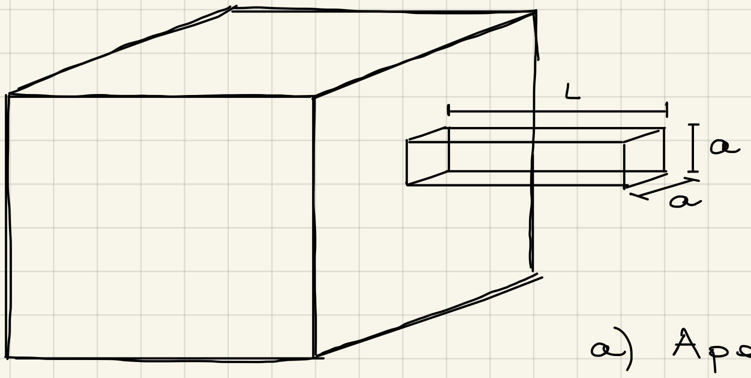
$$\Gamma = \frac{Z_{TE,2} - Z_{TE,1}}{Z_{TE,2} + Z_{TE,1}} = 0{,}166 + j\,0{,}986$$

$$|\Gamma| = 1 \qquad E(0) = E^+(0) + E^-(0) = E^+(0)\,(1+\Gamma)$$

$$|E(0)| = |E^+(0)|\,|1+\Gamma| = 15{,}3 \frac{mV}{m}$$

$$\alpha_{z,2} = \frac{2\pi}{\lambda_c}\sqrt{1-\left(\frac{f_0}{f_{c,2}}\right)^2} = 23{,}41 \frac{Np}{m} \qquad |E(z)| = |E(0)|\,e^{-\alpha_{z,2} z} \underset{\substack{\uparrow \\ z = 6\,cm}}{=} 3{,}75 \frac{mV}{m}$$

"gabbia di Faraday"



$f_{MAX} = 100 MHz$

Attenuazione minima di 60dB

a) Apertura $\underline{a}$ massima consentita per $L = 2$ cm

attenuazione sotto f_c

$$|E(\ell)| = |E(0)|\,e^{-\alpha \ell} \quad \text{per } \ell = L \qquad \frac{|E(L)|}{|E(0)|} = \frac{1}{\underset{\substack{\| \\ -60dB}}{1000}} = e^{-\alpha L}$$

$$-\alpha L = -6{,}9 \qquad \alpha = 345 \frac{Np}{m}$$

$$\frac{2\pi}{\lambda_c}\sqrt{1-\left(\frac{f_{MAX}}{f_c}\right)^2} = \alpha \longrightarrow \left(\frac{\alpha\lambda_c}{2\pi}\right)^2 = 1-\left(\frac{\lambda_c f_{MAX}}{c}\right)^2 \longrightarrow$$

$\lambda_c = 0{,}0182$ cm

$$a = \frac{\lambda_c}{2} = 9{,}1\,mm$$

b) Lunghezza L se l'apertura deve essere 10 × 10 cm

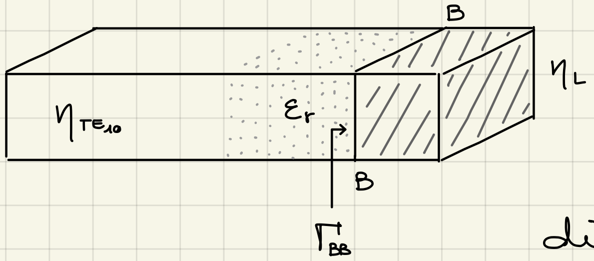
↓ a = 10 cm

$\lambda_c = 0{,}2\ m = 2a$ $\quad f_c = 1{,}5\ GHz$

$\alpha = \frac{2\pi}{0{,}2}\sqrt{1-\left(\frac{0{,}1}{1{,}5}\right)^2} = 31{,}3\ \frac{Np}{m}$ $\quad \alpha \cdot L = 6{,}9$ (per garantire i -60dB)

$L = 0{,}22\ m$

●



$a = 2$ cm $\quad f = 11$ GHz

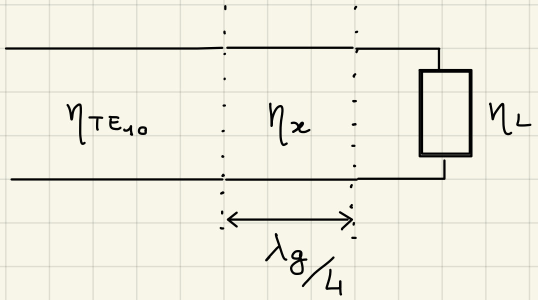
$b = 1$ cm $\quad \Gamma_{BB} = 0{,}3$

Dimensionare una struttura dielettrica adattante. ($\varepsilon_r = ?$)

$\Gamma_{BB} = \frac{\eta_L - \eta_{TE_{10}}}{\eta_L + \eta_{TE_{10}}} = 0{,}3$ $\quad \eta_L = \eta_{TE_{10}} \cdot 1{,}857$ (senza adattamento)

$\lambda_c = 2a = 0{,}04\ m$ $\quad f_c = 7{,}5\ GHz$

$\eta_{TE_{10}} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 515\Omega$ $\quad \eta_L = 956\ \Omega$



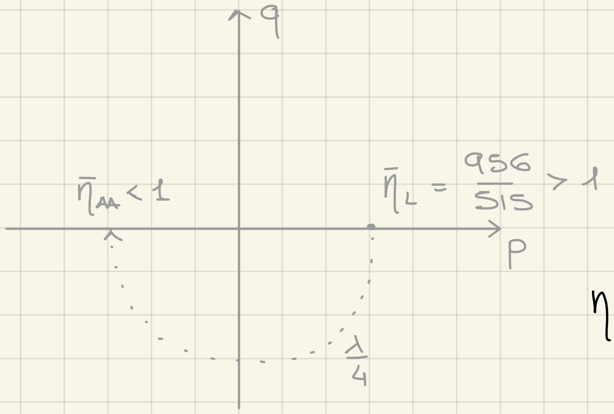
$\eta_x = \sqrt{\eta_L \cdot \eta_{TE_{10}}} = 700\Omega$

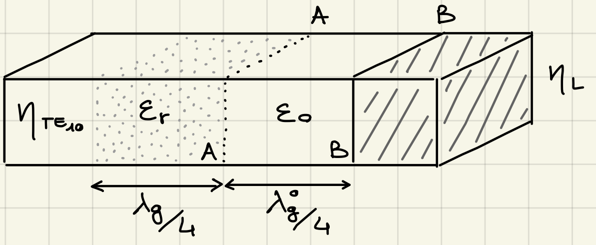
$$\eta_x = \frac{\eta_0}{\sqrt{\varepsilon_r}} \cdot \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \cdot \frac{1}{\sqrt{1-\frac{1}{\varepsilon_r}\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$f_c = \frac{1}{\lambda_c} \cdot \frac{c}{\sqrt{\varepsilon_r}}$

$f = \frac{c}{\lambda_0}$

$\Longrightarrow \varepsilon_r = \left(\frac{\eta_0}{\eta_x}\right)^2 + \left(\frac{\lambda_0}{\lambda_c}\right)^2 = 0{,}754 < 1$ → non va bene

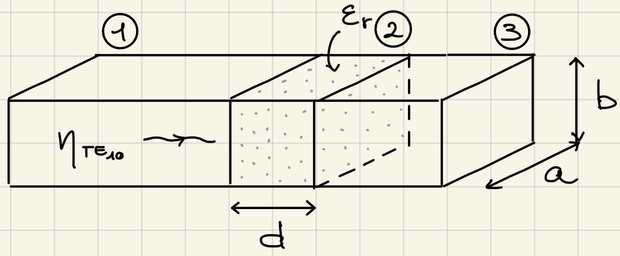




$\eta_{AA} = \frac{\eta_{TE_{10}}^2}{\eta_L} = 277\Omega$ $\quad \eta_x = \sqrt{\eta_{TE_{10}} \cdot \eta_{AA}} = 378\Omega$

$\varepsilon_r = \left(\frac{377}{378}\right)^2 + \left(\frac{2{,}73}{4}\right)^2 = 1{,}46 > 1$

$$\frac{\lambda_g}{4} = \frac{1}{4}\frac{\lambda_0}{\sqrt{\varepsilon_r}} \cdot \frac{1}{\sqrt{1-\frac{\lambda_0^2}{\varepsilon_r \lambda_c^2}}} = 0,684 \text{ m} \qquad \frac{\lambda_g^0}{4} = \frac{\lambda_0}{4}\frac{1}{\sqrt{1-\frac{\lambda_0^2}{\lambda_c^2}}} = 1,367 \text{ cm}$$



$a = 5$ cm $d = 15$ cm

$b = 2,5$ cm $\varepsilon_r = 2$

Calcolare la frazione di potenza riflessa alla f_0 (centro banda TE_{10} - guida vuota).

TE_{10}: $\lambda_c = 2a = 10$ cm $f_c = 3$ GHz

TE_{01}: $\lambda_c = 2b = 5$ cm $f_c = 6$ GHz

TE_{20}: $\lambda_c = a = 5$ cm $f_c = 6$ GHz

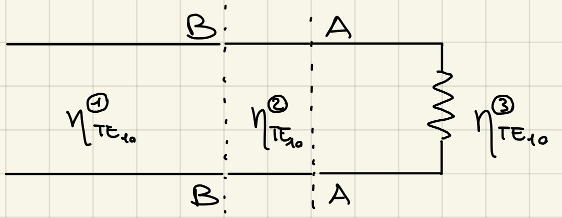
$\Big\}$ $f_0 = 4,5$ GHz

$(\lambda_0 = 6,66$ cm$)$

$$\eta^{①}_{TE_{10}} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}} = 506\,\Omega = \eta^{③}_{TE_{10}}$$

<u>Setto dielettrico</u>: $\lambda_c = 10$ cm $f_c = 2,12$ GHz

$$\eta^{②}_{TE_{10}} = \frac{\eta_0}{\sqrt{\varepsilon_r}}\frac{1}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}} = 302\,\Omega$$



$$\Gamma_{AA} = \frac{\eta^{③}_{TE_{10}} - \eta^{②}_{TE_{10}}}{\eta^{③}_{TE_{10}} + \eta^{②}_{TE_{10}}} = 0,252$$

$$\lambda_g^{②} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\frac{1}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}} = 5,36 \text{ cm}$$

$$\frac{d}{\lambda_g^{②}} = \frac{15}{5,36} = 2,8067 \approx 0,3067$$

$$\bar{\eta}_L = \frac{\eta^{③}_{TE_{10}}}{\eta^{②}_{TE_{10}}} = 1,67 = \bar{\eta}_{AA} \xrightarrow{d} \bar{\eta}_{BB} = 0,649 + j\,0,23$$

$$\eta_{BB} = \bar{\eta}_{BB} \cdot \eta^{②}_{TE_{10}} = (196 + j\,69,3)\,\Omega$$

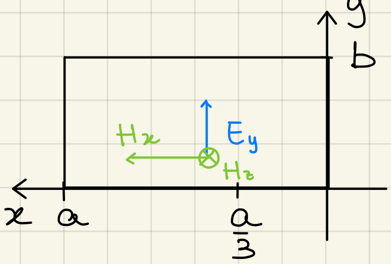
$$\Gamma_{BB} = \frac{\eta_{BB} - \eta^{①}_{TE_{10}}}{\eta_{BB} + \eta^{①}_{TE_{10}}} = -0,43 + j\,0,14 \qquad P_{rif} = P^+ |\Gamma_{BB}|^2 = P^+ \cdot \underline{\underline{0,203}}$$

- Dimensionare una guida d'onda rettangolare con le seguenti caratteristiche:

1) banda monomodale: 1 ⟼ 1,5 GHz

2) dimensione del lato maggiore $a < 10$ cm.

Si calcoli l'ampiezza del campo elettrico e magnetico totale per $x = \frac{a}{3}$, se nella guida si propaga un'onda a $f_0 = 1,25$ GHz, $P^+ = 1$W



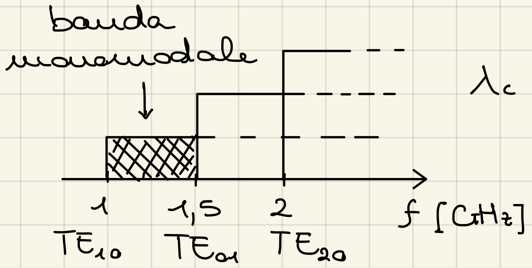
Guida vuota, $a = 10$ cm:

$\lambda_c = 2a = 20$ cm $f_c = 1,5$ Hz $> f_0$!

f_0 non può propagarsi in una guida vuota.

Guida con dielettrico, $\boxed{a = 10\text{ cm}}$:

$\lambda_c = 20$cm $f_c = \frac{c}{\sqrt{\varepsilon_r}\cdot\lambda_c} = 10^9 \longrightarrow \boxed{\varepsilon_r = 2,25}$



$$\lambda_c = \frac{c}{\sqrt{\varepsilon_r}\, f_c} \quad (TE_{01})$$

$$= 13,3 \text{ cm} = 2b \rightarrow \boxed{b = 6,66\text{cm}}$$

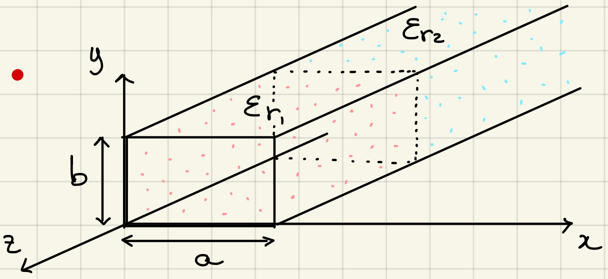
$$P^+ = \frac{|E_0|^2 ab}{4\,\eta_{TE_{10}}} = 1W \quad \text{con} \quad \eta_{TE_{10}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \cdot \frac{1}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}} = 419\,\Omega$$

$$\Longrightarrow \quad |E_0| = 500 \ V/m$$

$$E_y(x) = E_0 \operatorname{sen}\left(\frac{\pi x}{a}\right) \xrightarrow{x = \frac{a}{3},\ z = 0} E_y\left(\frac{a}{3}\right) = 433 \ \frac{V}{m}$$

$$H_x(x) = -\frac{E_y(x)}{\eta_{TE_{10}}} \longrightarrow H_x\left(\frac{a}{3}\right) = -1 \ \frac{A}{m}$$

$$H_z(x) = j\,\frac{E_0}{\eta_0}\left(\frac{\lambda}{2a}\right)\cos\left(\frac{\pi x}{a}\right) \longrightarrow H_z\left(\frac{a}{3}\right) = j\,0,796 \ \frac{A}{m}$$

- 

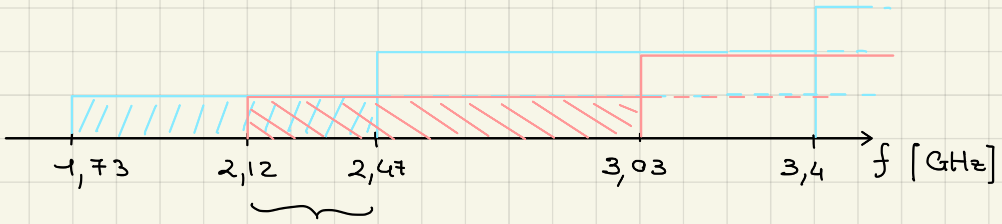
$\varepsilon_{r_1} = 2$ $a = 5$ cm

$\varepsilon_{r_2} = 3$ $b = 3,5$ cm

a) Calcolare la banda monomodale (TE_{10}) per l'<u>intera</u> struttura.

b) Alla frequenza f_0 (centro banda), si propaga un'onda con $P^+ = 100\,W$; trovare il valore/i di x per cui il modulo del campo elettrico nel secondo dielettrico vale $6{,}43\ \frac{kV}{m}$

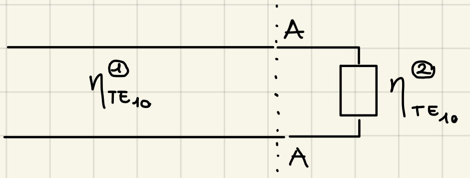
		Guida 1	Guida 2
TE_{10}:	$\lambda_c = 2a = 10\,cm$	$f_c^{①} = 2{,}12\,GHz$	$1{,}73\,GHz = f_c^{②}$
TE_{01}:	$\lambda_c = 2b = 7\,cm$	$3{,}03\,GHz$	$2{,}47\,GHz$
TE_{20}:	$\lambda_c = a = 5\,cm$	$4{,}24\,GHz$	$3{,}4\,GHz$



banda monomodale $2{,}12 \longmapsto 2{,}47\ GHz$

$\downarrow$

$f_0 = 2{,}295\,GHz$



$$\eta^{①}_{TE_{10}} = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} \cdot \frac{1}{\left(1-\left(\frac{f_c^{①}}{f_0}\right)^2\right)^{1/2}} = 696\,\Omega$$

$$\eta^{②}_{TE_{10}} = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} \cdot \frac{1}{\left(1-\left(\frac{f_c^{②}}{f_0}\right)^2\right)^{1/2}} = 331\,\Omega$$

$$\Gamma_{AA} = \frac{\eta^{②}_{TE_{10}} - \eta^{①}_{TE_{10}}}{\eta^{②}_{TE_{10}} + \eta^{①}_{TE_{10}}} = -0{,}355 \qquad T_{AA} = 1 + \Gamma_{AA} = 0{,}645$$

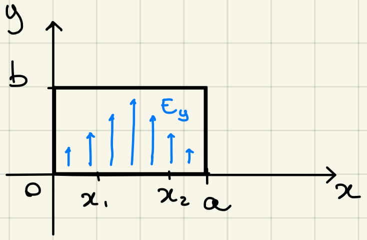
$$P^+ = \frac{|E_0|^2\, a \cdot b}{4\, \eta^{①}_{TE_{10}}} = 100\,W \longrightarrow |E_0| = 12{,}6\ \frac{kV}{m}$$

$$E_{II} = E_0 \cdot T_{AA} = 8{,}13\ \frac{kV}{m}$$

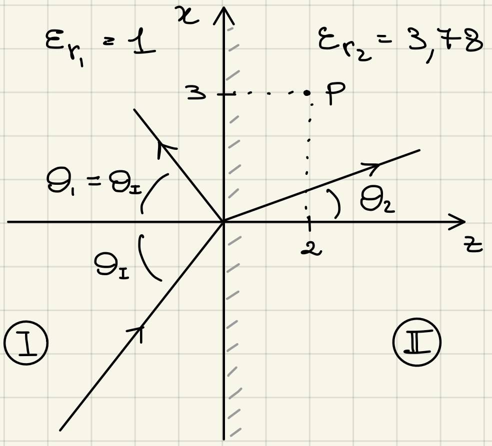
Nel tratto ②: $E_y(x) = E_{II}\, \text{sen}\left(\frac{\pi x}{a}\right) = 6{,}43\ \frac{kV}{m}$

$$\text{sen}\left(\frac{\pi x}{a}\right) = 0{,}79$$

$$\downarrow$$



$$\begin{cases} x_1 = 1{,}45\,cm \\ x_2 = 3{,}55\,cm \end{cases}$$

- 

$\theta_I = 60°$

$S_{inc} = 0{,}5 \; \frac{mW}{m^2}$

$f = 100 \; MHz$

- Calcolare la frazione di densità di potenza trasmessa e riflessa

Calcolare i campi $\bar{E}$ e $\bar{H}$ in $P(2,3)$ [m]

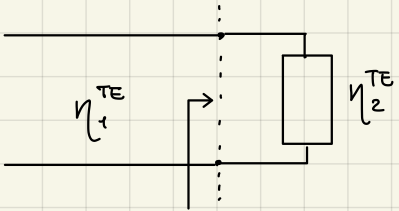
Incidenza TE o TM

$n_1 \operatorname{sen}\theta_1 = n_2 \operatorname{sen}\theta_2$

$\operatorname{sen}60° = 1{,}94 \operatorname{sen}\theta_2 \quad \longrightarrow \quad \theta_2 = 26{,}5°$

$\eta_1 = \eta_0 = 377 \, \Omega \qquad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = 194 \, \Omega$

Incidenza TE



$\eta_1^{TE} = \eta_1 \frac{1}{\cos\theta_1} = 754 \, \Omega$

$\eta_2^{TE} = \eta_2 \frac{1}{\cos\theta_2} = 217 \, \Omega$

$\Gamma = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = -0{,}553 \qquad T = 1 + \Gamma = 0{,}447$

$S_{inc} = \frac{1}{2} \frac{|E_1^+|^2}{\eta_1} = 0{,}5 \cdot 10^{-3} \frac{W}{m^2} \quad \longrightarrow \quad |E_1^+| = 0{,}614 \; \frac{V}{m}$

$E_2^+ = E_1^+ \cdot T = 0{,}275 \; \frac{V}{m}$ cui corrisponde $S_{tra} = \frac{1}{2} \frac{|E_2^+|^2}{\eta_2} =$

$= 1{,}95 \cdot 10^{-4} \; \frac{W}{m^2}$

$\gamma_2 = j\omega\sqrt{\mu\varepsilon} = j\beta = j\,4{,}0752 \; m^{-1}$

$\gamma_{2x} = \gamma_2 \operatorname{sen}\theta_2 = j\,1{,}818 \; m^{-1}$

$\gamma_{2z} = \gamma_2 \cos\theta_2 = j\,3{,}647 \; m^{-1}$

$z = 2, \; x = 3, \;$ fase in $(0,0) = 0$

$E_2^+(x,z) = E_2^+(0,0) \, e^{-\gamma_{2x} x} e^{-\gamma_{2z} z} = -0{,}117 - j0{,}25 \; \left[\frac{V}{m}\right], \quad H_2^+ = \frac{E_2^+}{\eta_2}$

$= -0{,}603 - j\,1{,}289 \left[\frac{mA}{m}\right]$

Incidenza TM

$\eta_1^{TM} = \eta_1 \cdot \cos\theta_1 = 188{,}5 \, \Omega \qquad \eta_2^{TM} = \eta_2 \cos\theta_2 = 174 \, \Omega$

$$\Gamma = -\frac{\eta_2^{TM} - \eta_1^{TM}}{\eta_2^{TM} + \eta_1^{TM}} = 0,04 \qquad T = 1 + \Gamma = 1,04$$

$$|H_1^+| = \sqrt{\frac{2 S_{inc}}{\eta_1}}$$

$$H_2^+ = H_1^+ \cdot T = 1,69 \cdot 10^{-3} \frac{A}{m} \qquad S_{tra} = \frac{1}{2}|H_2^+|^2 \eta_2 = 2,78 \cdot 10^{-4} \frac{W}{m}$$

anche se $T \simeq 1$ la potenza trasmessa è $\sim \frac{1}{2}$ la potenza incidente perché solo la componente // all'asse z viene trasmessa

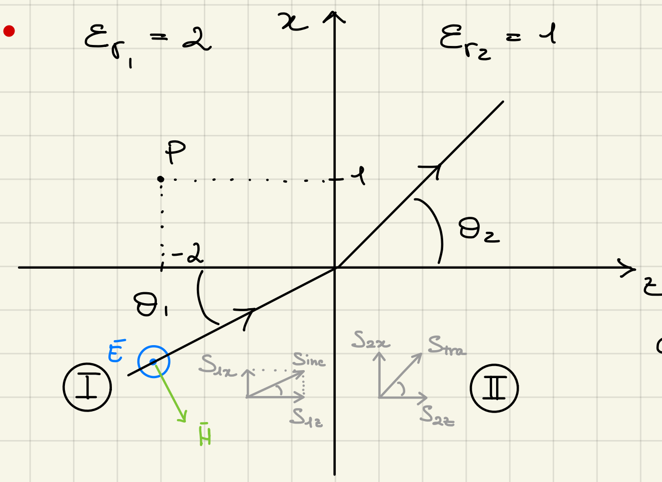
$$H_2^+(x,z) = H_2^+(0,0)\, e^{-\gamma_{2x}x} e^{-\gamma_{2z}z} =$$

$z = 2,\ x = 3$

$$= (-7,19 - j\,15,3)\,10^{-4} \left[\frac{A}{m}\right]$$

$$E_2^+(x,z) = H_2^+(x,z) \cdot \eta_2 = -0,14 - j\,0,297 \left[\frac{V}{m}\right]$$

- $\varepsilon_{r_1} = 2$ $\qquad \varepsilon_{r_2} = 1$



$S_{1z} > S_{2z}$
ma
$S_{inc} < S_{tra}$

$$\bar{E}_1^+ = E_0 e^{-j(x + \sqrt{3}z)} \vec{u}_y$$

con $E_0 = 3 \frac{V}{m}$

a) Determinare frequenza, θ_1 e θ_2.

$\bar{E} \,//\, \vec{u}_y \rightarrow$ Incidenza TE

$$\bar{E}_1^+ = E_1^+(0,0)\, e^{-j\beta_{1x}x - j\beta_{1z}z} \cdot \vec{u}_y$$

$$\begin{cases} \beta_{1z} = \beta_1 \cos\theta_1 = \sqrt{3}\ m^{-1} \\ \beta_{2x} = \beta_1 \operatorname{sen}\theta_1 = 1\,m^{-1} \end{cases} \Longrightarrow \frac{\beta_{1x}}{\beta_{2z}} = \operatorname{tg}\theta_1 = \frac{1}{\sqrt{3}} \longrightarrow \theta_1 = 30°$$

$$n_1 \operatorname{sen}\theta_1 = n_2 \operatorname{sen}\theta_2 \longrightarrow \theta_2 = 45°$$

$$\beta_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi\sqrt{\varepsilon_{r_1}}}{c} \cdot f \quad \text{ma} \quad \beta_1 = \frac{\beta_{1z}}{\cos\theta_1} = 2\ m^{-1} \longrightarrow f = 67,6\ MHz$$

b) Calcolare la densità di potenza trasmessa nel mezzo Ⅱ

$$\eta_1^{TE} = \frac{\eta_1}{\cos\theta_1} = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} \cdot \frac{1}{\cos\theta_1} = 308\,\Omega$$

$$\eta_2^{TE} = \frac{\eta_0}{\cos\theta_2} = 533\,\Omega \qquad \Gamma = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0,267$$

$$S_{inc} = \frac{1}{2}\frac{|E_1^+|^2}{\eta_1} = 0,0169 \frac{W}{m^2} \qquad \text{dove} \quad \eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} = 267\,\Omega$$

$$S_{1z} = S_{inc} \cdot \cos\vartheta_1 = 0{,}0169 \cdot \cos 30^\circ = 0{,}0146 \ \frac{W}{m^2}$$

$$S_{2z} = S_{1z}\left(1 - |\Gamma|^2\right) = 0{,}0136 \ \frac{W}{m^2}$$

$$S_{tra} = \frac{S_{2z}}{\cos\vartheta_2} = 0{,}0192 \ \frac{W}{m}$$

$> S_{inc}$! è possibile poiché in II cambia la direzione di propagazione dell'onda

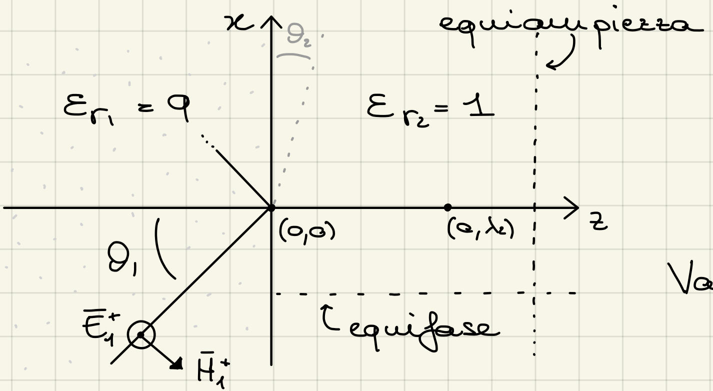
c) Calcolare i campi nel punto $P(1,-2)$

$$E_1(x,z) = E_1^+(x,z) + E_1^-(x,z) = E_1^+(0,0)\, e^{-j\beta_{1x}x - j\beta_{1z}z} + E_1^-(0,0)\, e^{-j\beta x x + j\beta_{1z}z}$$

con $E_1^-(0,0) = E_1^+(0,0)\,\Gamma$

$$\longrightarrow \quad E_1(x,z) = 3\left[e^{-j(x+\sqrt{3}z)} + 0{,}267\, e^{-jx + j\sqrt{3}z}\right]$$

$$E_1(1,-2) = (-2{,}53 + j\,2{,}67) \ \frac{V}{m}$$



$\vartheta_1 = 45^\circ$

$|E_1^+| = 1 \ \frac{V}{m}$

Valutare $|E|$ in $\begin{pmatrix} z=0 \\ x=0 \end{pmatrix}$ e $\begin{pmatrix} z=\lambda_2 \\ x=0 \end{pmatrix}$

$n_1 \operatorname{sen}\vartheta_1 = n_2 \operatorname{sen}\vartheta_2 \quad \rightarrow \quad \operatorname{sen}\vartheta_2 = 2{,}12 > 1$!

Incidenza oltre l'angolo critico → riflessione totale

Incidenza TE

$$\eta_1^{TE} = \frac{\eta_1}{\cos\vartheta_1} = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} \cdot \frac{1}{\cos\vartheta_1} = 178\,\Omega$$

$$\eta_2^{TE} = \frac{\eta_2}{\cos\vartheta_2} = \frac{\eta}{\sqrt{1 - \operatorname{sen}^2\vartheta_2}} = \frac{377\,\Omega}{(\pm)\, j\,1{,}87}$$

$$= j\,200\,\Omega$$

Il campo E deve attenuarsi al crescere di z:

$$\beta_{2z} = \beta_2 \cos\vartheta_2 = \frac{2\pi}{\lambda_2}\sqrt{1 - \operatorname{sen}^2\vartheta_2}$$

$$= \frac{2\pi}{\lambda_2}(\pm j\,1{,}87)$$

$$e^{-j\beta_{2z}z} = e^{-\alpha_{2z}z}$$

$$\alpha_{2z} = j\beta_{2z} > 0 \Rightarrow \boxed{-j\,1{,}87}$$

$$\Gamma = \frac{\eta_2^{TE} - \eta_1^{TE}}{\eta_2^{TE} + \eta_1^{TE}} = 0{,}116 + j\,0{,}99$$

$$T = 1 + \Gamma = 1{,}116 + j\,0{,}99 \qquad E(0,0) = \underset{1\,V/m}{E^+(0,0)}\, T = (1{,}116 + j\,0{,}99)\frac{V}{m}$$

$|E(0,0)| = 1{,}5 \frac{V}{m}$.

$\boxed{\beta_{1x} = \beta_{2x}} \rightarrow$ campi tg. continui

$E_2^+(x,z) = E_2^+(0,0)\, e^{-j\beta_{2x}x}\, e^{-\alpha_{2z}z}$

$x = \text{const.} \leftrightarrow$ sup. equifase $\quad z = \text{const} \leftrightarrow$ sup. equi-ampiezza

varia solo l'ampiezza

$E_2^+(0,\lambda_2) = E_2^+(0,0)\, e^{-\alpha_{2z}\lambda_2} \longrightarrow |E_2^+(0,\lambda_2)| = 7{,}89 \cdot 10^{-6} \frac{V}{m}$

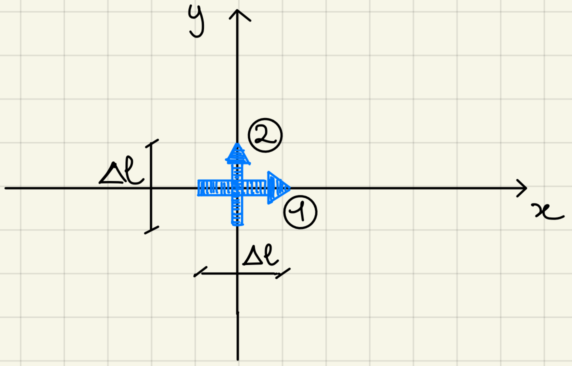
- $$\begin{cases} E_x = 10\, e^{-j\beta z} \left[\frac{V}{m}\right] \\ E_y = j10\, e^{-\frac{\pi}{2}}\, e^{-j\beta z} \left[\frac{V}{m}\right] \end{cases}$$ Polarizzazione ?

Piano trasverso (ad es. $z = 0$):

$$\begin{cases} E_x = 10 \left[\frac{V}{m}\right] \\ E_y = j10\, e^{\frac{\pi}{2}} \left[\frac{V}{m}\right] \end{cases} \rightarrow \quad \begin{aligned} E_x &= 10\cos(\omega t) \\ E_y &= \mathrm{Re}\left\{10\, e^{j\frac{\pi}{2}} e^{j\frac{\pi}{2}} e^{j\omega t}\right\} = 10\cos(\omega t + \pi) \end{aligned}$$

$\varphi = \pi \Longrightarrow$ polarizzazione <u>lineare</u>

Due dipoli hertziani trasmittenti di uguale lunghezza Δl

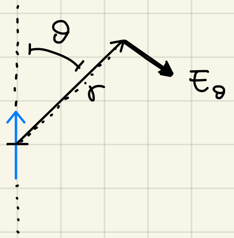


$I_1 = j2$ A

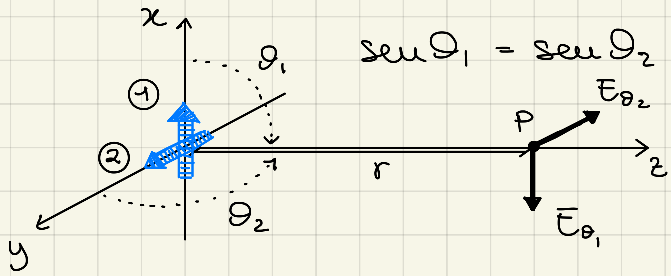
$I_2 = -j4$ A

Determinare la polarizzazione dell'onda irradiata in direzione asse z.

$E_\theta(\theta, r) = j\frac{\omega\mu I \cdot l\, e^{-j\beta r}}{4\pi r} \operatorname{sen}\theta$



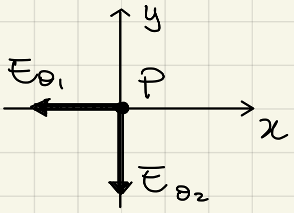
$\operatorname{sen}\theta_1 = \operatorname{sen}\theta_2 = 1$



$E_{\theta_1} = j\frac{\omega\mu I_1 l_1 e^{-j\beta r}}{4\pi r} = K I_1 = Kj2$

$E_{\theta_2} = j\frac{\omega\mu I_2 l_1 e^{-j\beta r}}{4\pi r} = K I_2 = -Kj4$

In P:



E_{θ_1} e E_{θ_2} sono sfasati di π ($+j$ e $-j$)

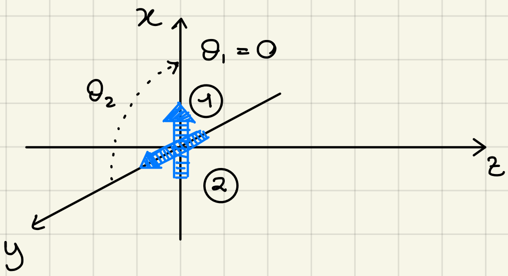
$\longrightarrow \varphi = \pi \longrightarrow$ polarizzazione <u>lineare</u>

Se $I_1 = 2A$, $I_1 = j2A$. Polarizzazione?

$E_{\theta_1} = K I_1 = K 2 A$ $E_{\theta_2} = K I_2 = jK2A$

$\longrightarrow \varphi = \frac{\pi}{2}$, $|E_{\theta_1}| = |E_{\theta_2}| \longrightarrow$ polarizzazione <u>circolare sinistra</u>

Determinare la polarizzazione dell'onda irradiata in direzione asse x

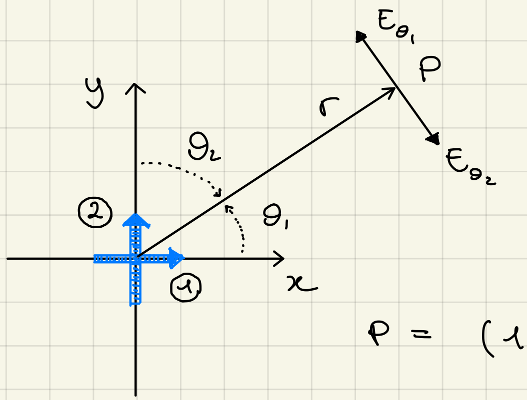


$\theta_1 = 0° \rightarrow \operatorname{sen}\theta_1 = 0$

$\theta_2 = 90° \rightarrow \operatorname{sen}\theta_2 = 1$

Polarizzazione <u>lineare</u> (c'è solo il dipolo ② che irradia lungo x)

- Due dipoli hertziani trasmittenti di uguale lunghezza e intensità.



$l = \frac{\lambda}{10}$ $I = 1A$

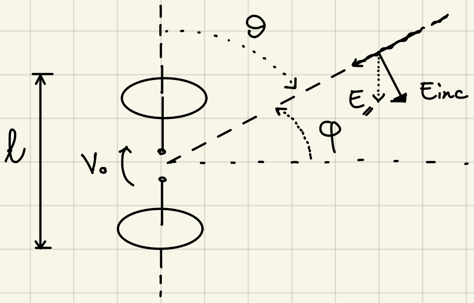
$f = 600MHz$ ($\lambda = 0{,}5m$)

Calcolare $\bar{E}(100, 100)$ (di sola radiazione)

$P = (100, 100) \rightarrow \theta_1 = \theta_2 = \frac{\pi}{4}$

$$E_\theta = j\omega\mu \frac{Il}{4\pi r} e^{-j\beta r} \operatorname{sen}\theta \Longrightarrow E_{\theta_1} = E_{\theta_2} \text{ in } P$$

$\bar{E}(100, 100) = 0$

- 

$\varphi = 30°$ $\rightarrow$ $\theta = 60°$ $f = 100\ MHz$ ($\lambda = 3m$)

$|\bar{E}_{inc}| = 10^{-2} \frac{V}{m}$ $l = \frac{\lambda}{10} = 0{,}3\ m$

$|V_0| = ?$

Metodo 1:

$E_{/\!/} = E_{inc} \cdot \cos\varphi = 8,66 \cdot 10^{-3} \frac{V}{m}$ $\quad V_0 = E_{/\!/} \cdot \ell = 2,6 \cdot 10^{-3}\ V$

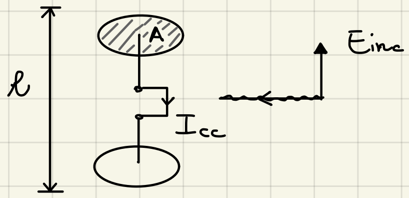
Metodo 2: (hp: adattamento di polarizzazione)

$f(\theta) = \sin^2\theta \qquad A_e = \frac{\lambda^2}{8\pi} \qquad R_R = \frac{2}{3}\pi\eta_0\left(\frac{\ell}{\lambda}\right)^2 = 7,9\,\Omega$

$S_{inc} = \frac{|E_{inc}|^2}{2\eta_0} = 1,326 \cdot 10^{-7} \frac{W}{m^2} \qquad A_e = 1,074\ m^2$

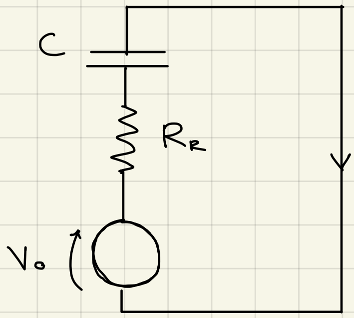
$P_d = S_{inc} \cdot A_e \cdot f(\theta) = 1,06 \cdot 10^{-7}\ W$ ma $P_d = \frac{|V_0|^2}{8R_R}$

$\longrightarrow |V_0| = 2,6 \cdot 10^{-3}\ V$

- 

$|E_{inc}| = 1 \frac{V}{m} \qquad \ell = 1m \qquad A = 10\ m^2$

Calcolare $|I_{cc}|$ a $f = 30\ MHz$ $(\lambda = 10m)$.



$\omega = 2\pi f$

$R_R = \frac{2}{3}\pi\eta_0\left(\frac{\ell}{\lambda}\right)^2 = 7,9\,\Omega$

$C = \varepsilon_0 \frac{A}{\ell} = 88,6\ pF$

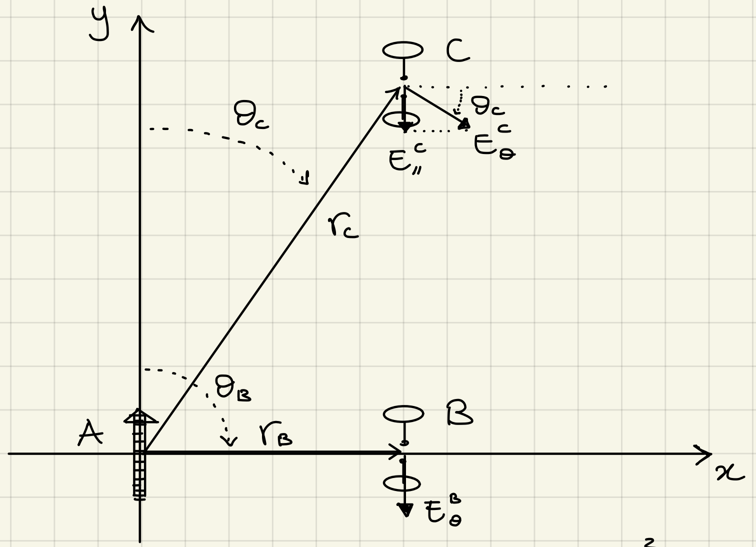
$|V_0| = |E_{inc}| \cdot \underset{\ell \text{ per il dipolo}}{\underset{\parallel}{\ell_e}} = 1V$

$$|I_{cc}| = \frac{|V_0|}{\left|R_R + \frac{1}{j\omega C}\right|} = 16,5\ mA$$

Si può ottenere una $|I| \gg |I_{cc}|$ inserendo un componente (passivo) al posto del corto circuito?

Sì, inserendo un induttore tale che $\frac{1}{j\omega C} + j\omega L = 0$

$$\Longrightarrow L = \frac{1}{\omega^2 C} = 317\ nH \rightarrow |I| = \frac{|V_0|}{R_R} = 126,6\ mA$$



$P_T = 1 W$ $f = 1 GHz$

$B = (30,0)$ $C = (30,60)$

$\ell = \frac{\lambda}{10}$

Calcolare la tensione a vuoto di B e C.

$$P_T = \frac{\pi}{3}\eta_0 |I_A|^2 \left(\frac{\ell}{\lambda}\right)^2 \longrightarrow |I_A| = 0,5 A$$

$$|E_\theta^B| = \left| \frac{j\omega\mu I_A \ell e^{-j\beta r_B}}{4\pi r_B} \operatorname{sen}\theta_B \right| = \frac{2\pi f \mu_0 I_A \ell}{4\pi r_B} = 0,314 \frac{V}{m}$$

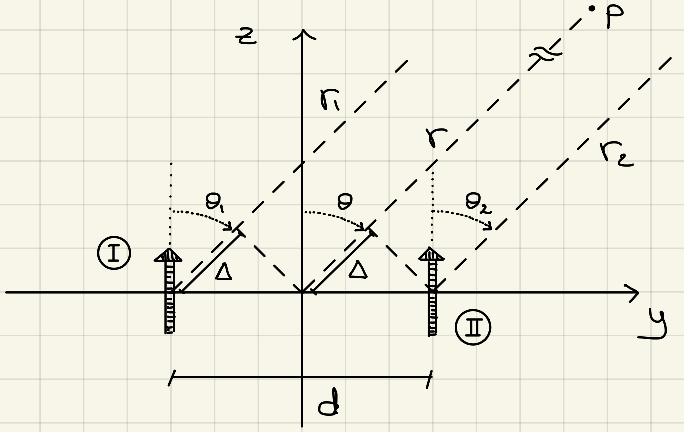
$$|V_B| = |E_\theta^B| \cdot \ell = 0,0094 V$$

$$\theta_c = 26,6° = \operatorname{arctg}\left(\frac{30}{60}\right) \qquad r_c = 67 m$$

$$|E_\theta^c| = \frac{2\pi f \mu_0 I_A \ell}{4\pi r_c} \operatorname{sen}\theta_c = 0,063 \frac{V}{m}$$

$$|V_c| = |E_{//}^c| \cdot \ell = |E_\theta^c| \operatorname{sen}\theta_c \cdot \ell = 8,4 \cdot 10^{-4} V$$

Dipoli hertziani (trasmittenti) in gruppo



Campo totale $\bar{E}$ irradiato?

$\boxed{r \gg d}$ $(\gg \lambda)$

$I_1 = I_2 = I$

$\ell_1 = \ell_2 = \ell$

$$E_{\theta_1}^{I} = \frac{j\omega\mu I_1 \ell_1 e^{-j\beta r_1}}{4\pi r_1} \operatorname{sen}\theta_1 \qquad E_{\theta_1}^{II} = j\frac{\omega\mu I_2 \ell_2 e^{-j\beta r_2}}{4\pi r_2} \operatorname{sen}\theta_2$$

Se $r \gg d \longrightarrow r_1 // r_2 // r \longrightarrow \theta_1 \simeq \theta_2 \simeq \theta$

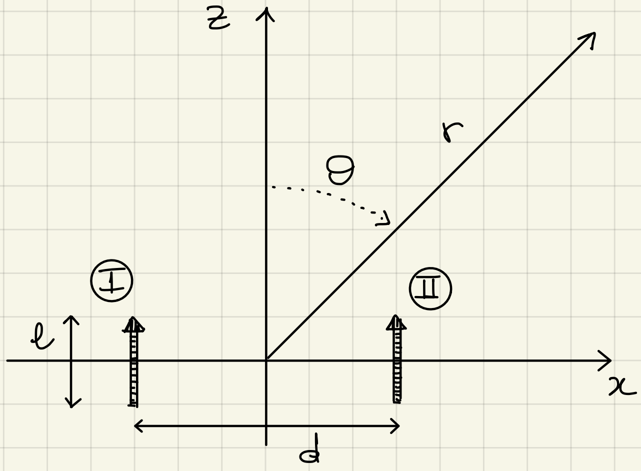
$r_1 = r + \Delta$, $\quad r_2 = r - \Delta \quad$ con $\quad \Delta = \frac{d}{2}\,\text{sen}\vartheta \quad (\Delta \ll r)$

Ai fini del denominatore: $\; r_1 \simeq r_2 \simeq r$

$$E_\vartheta^{I} = j\frac{\omega\mu I l\, e^{-j\beta r}}{4\pi r}\, e^{-j\beta\frac{d}{2}\text{sen}\vartheta}\; \text{sen}\vartheta$$

$$E_\vartheta^{II} = j\frac{\omega\mu I l\, e^{-j\beta r}}{4\pi r}\, e^{+j\beta\frac{d}{2}\text{sen}\vartheta}\; \text{sen}\vartheta$$

$$\left\{ E_\vartheta = \underbrace{E_\vartheta^{I} + E_\vartheta^{II}}_{\text{paralleli}} = j\frac{\omega\mu I l\, e^{-j\beta r}}{4\pi r}\,\text{sen}\vartheta \underbrace{\left[e^{-j\frac{\beta}{2}d\,\text{sen}\vartheta} + e^{j\frac{\beta}{2}d\,\text{sen}\vartheta}\right]}_{2\cos\left(\beta\frac{d}{2}\text{sen}\vartheta\right)} \right\}$$



$l = \frac{\lambda}{20} \qquad I_1 = I_2 = I$

$f = 100\,\text{MHz}$

Determinare la minima distanza d in modo che la radiazione in direzione $\vartheta = 45°$ sia nulla.

$$\boxed{E_\vartheta(\vartheta = 45°) = 0}$$

$$E_\vartheta^{tot} \simeq j\frac{\omega\mu I l \cdot e^{-j\beta r}}{2\pi r}\,\text{sen}\vartheta\, \underbrace{\cos\left(\beta\frac{d}{2}\,\text{sen}\vartheta\right)}_{\text{unico termine che si può annullare}}$$

$\cos\left(\beta\frac{d}{2}\,\text{sen}\vartheta\right) = 0 \longrightarrow \beta\frac{d}{2}\,\text{sen}\vartheta = \frac{\pi}{2} \quad$ con $\quad \beta = \frac{2\pi}{\lambda}$

$\frac{2\pi}{\lambda}\,\frac{d}{2}\,\text{sen}\vartheta = \frac{\pi}{2} \longrightarrow d = \frac{\lambda}{2\,\text{sen}\vartheta} = 2{,}12\text{ m}$ (le soluzioni sono infinite, questa ha il d minore)

$\text{sen}\vartheta = \frac{1}{\sqrt{2}}$, $\lambda = 3\text{m}$

Se invece $I_1 = I$, $I_2 = -I$ determinare il nuovo $d\ (>0)$.

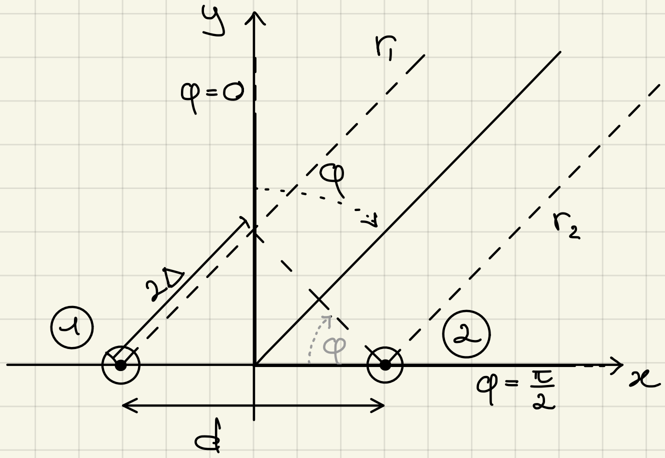
$|E_\vartheta^{I}| = |E_\vartheta^{II}| \qquad \angle II:\ \beta r_2 + \pi \qquad \angle I:\ \beta r_2 + \beta 2\Delta$

Affinché $E_\vartheta^{I} = -E_\vartheta^{II}$ devo avere $\angle I - \angle II = \pi$

$\beta 2\Delta - \pi = \pi$

$\frac{2\pi}{\lambda} \cdot d\,\text{sen}\vartheta = 2\pi$

$\Longrightarrow d = \frac{\lambda}{\text{sen}\vartheta} = 4{,}24\text{ m}$



$I_1 = 1A$ $\ell = \frac{\lambda}{10}$

$|I_2| = |I_1|$ $f = 300MHz$

a) Determinare I_2 e d in modo che il campo in direzione $\varphi = 0$ e $\varphi = \frac{\pi}{2}$ sia <u>nullo</u> (simultaneamente)

Dipoli // asse z → $\underline{\theta_1 = \theta_2 = 90°}$

Per $\varphi = 0$, d é ininfluente $(r_1 = r_2)$ → $I_2 = -I_1 = -1A$

Per $\varphi = \frac{\pi}{2}$, $2\Delta = d$: $\beta 2\Delta = \beta d = 2\pi$ → $d = \lambda = 1m$

↖ E_0^1 e E_0^2 devono essere IN FASE per annullarsi poiché sono opposti

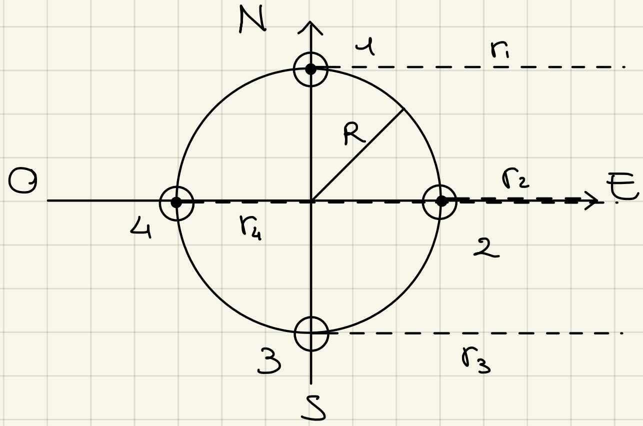
b) Determinare la direzione di massima radiazione

E_0^1 in fase con E_0^2 $2\Delta = d \operatorname{sen}\varphi$ $\beta 2\Delta = \pi$

$\frac{2\pi}{\lambda} d \operatorname{sen}\varphi = \pi$

$\longrightarrow \varphi = \operatorname{arcsen}\left(\frac{\lambda}{2d}\right) = \operatorname{arcsen}\left(\frac{1}{2}\right)$

$= 30°$



$f = 300MHz$

Stessa ℓ, stessa I.

Determinare il raggio R <u>minimo</u> per il quale non c'é radiazione <u>simultaneamente</u> nelle direzioni N, S, E, O.

<u>Est</u>: $r_1 = r_3$ $(\forall R)$ $|E_0| = |E_0^1| = |E_0^2| = |E_0^3| = |E_0^4|$

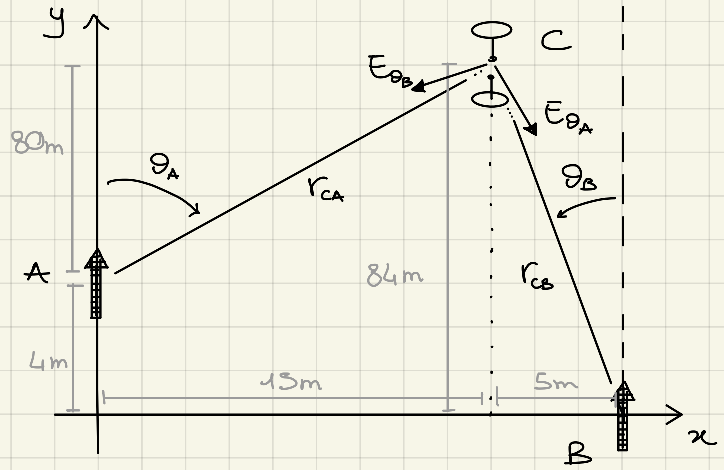
Il campo totale generato da 1 e 3 vale $2|E_0|$

La differenza $r_1 - r_2 \cong R$ → $\beta R = \pi$ → annulla E_0^1

" " $r_3 - r_4 \cong R$ → $\beta R = \pi$ → annulla E_0^3

$R = \frac{\lambda}{2} = 0,5m$

per simmetria anche le altre direzioni hanno radiaz. nulla

- 

A (0,4) P_A = 25mW

B(20,0) P_B = 15mW

C (15,84)

f = 6 GHz (λ = 5cm)

$\ell = \frac{\lambda}{20} = 2{,}5$mm

I_A e I_B in fase. Calcolare $|V_c|$ (tensione a vuoto)

$$P = \frac{\pi}{3}\eta_0 |I|^2 \left(\frac{\ell}{\lambda}\right)^2 < \begin{matrix} I_A = 0{,}159\,A \\ I_B = 0{,}123\,A \end{matrix}$$

$\theta_A = \arccos\left(\frac{80}{r_{CA}}\right) = 10{,}6°$ $\theta_B = \arccos\left(\frac{84}{r_{CB}}\right) = 3{,}4°$

$r_{CA} = \sqrt{(84-4)^2 + 15^2} = 81{,}3941\,m$

(λ = 0,05m ---- 7° per mm
se tronco troppe cifre decimali rischio
di commettere un errore notevole)

$r_{CB} = \sqrt{84^2 + 5^2} = 84{,}1487\,m$

$E_{\theta_A} = j\frac{\omega\mu I_A \ell}{4\pi r_{AC}} e^{-j\beta r_{CA}} \operatorname{sen}\theta_A$ $E_{\theta_B} = j\frac{\omega\mu I_B \ell}{4\pi r_{CB}} e^{-j\beta r_{CB}} \operatorname{sen}\theta_B$

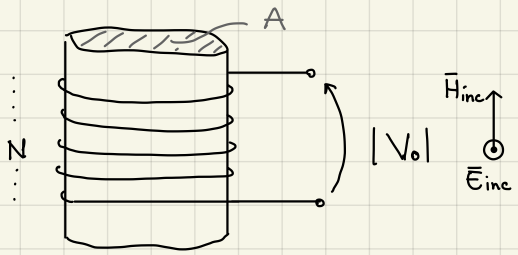
$E_{yA} = -E_{\theta_A} \operatorname{sen}\theta_A = (4{,}198 - j\,4{,}585)\cdot 10^{-4}\,\frac{V}{m}$

$E_{yB} = E_{\theta_B} \operatorname{sen}\theta_B = (8{,}01 - j\,47{,}33)\cdot 10^{-6}\,\frac{V}{m}$

$|E_{tot}| = |E_{yA} + E_{yB}| = 6{,}629 \cdot 10^{-4}\,\frac{V}{m}$

$|V_c| = |E_{tot}| \cdot \underset{\underset{\ell}{\parallel}}{\ell_e} = \underline{\underline{1{,}657\,\mu V}}$

- Onde radio (bassa frequenza)



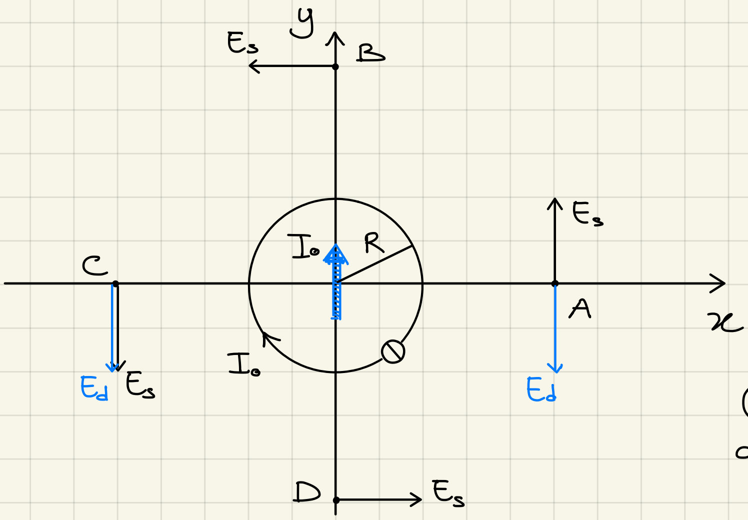
N = 1000 spire A = 1cm² (ferrite) μ_r = 100

$|E_{inc}| = 1\,\frac{mV}{m}$ f = 1MHz (λ = 300m)

Determinare $|V_o|$. $\rightarrow$ $|e| = N \frac{d\Phi}{dt} = N\, j\omega\mu H \cdot A$

$V_o = N \cdot j\omega\mu H A$ con $H = H_{inc}$

$$|V_o| = |N \cdot 2\pi f \mu_0 \mu_r A| \underbrace{|H_{inc}|}_{\frac{|E_{inc}|}{\eta_0}} = 0{,}21\,mV$$

- 

$\ell = 1m$

$I_0 = 1A$

$R = 1m$

$f = 10\,MHz$ $(\lambda = 30\,m)$

Calcolare $|E|$ in A, B, C e D a distanza $r = 1000\,m$

$$E_s = -j\beta\, \frac{j\omega\mu I_o S}{4\pi r}\, e^{-j\beta r} \quad \text{con } S = \pi R^2$$

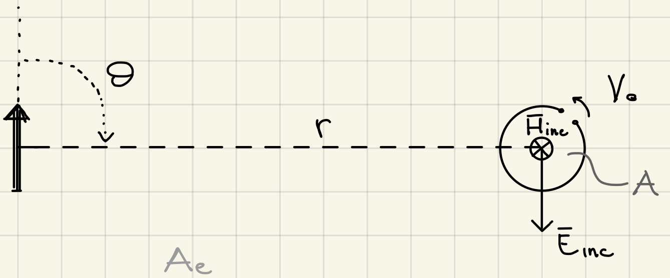
$$E_d = j\omega\mu_0 \frac{I_0 \ell}{4\pi r}\, e^{j\beta r}$$

In B e D: $|E| = |E_s| = 4{,}1 \cdot 10^{-3}\, \frac{V}{m}$ (il dipolo non irradia lungo il suo asse)

In A e C: E_s e E_d sono in quadratura (sfasati di $\frac{\pi}{2}$) ↓ uno è Re e l'altro è Im

$$|E| = |E_d \pm E_s| = |j6{,}3 \cdot 10^{-3} \pm 4{,}1\, 10^{-3}| =$$
↳ irrilevante

$$= 7{,}5 \cdot 10^{-3}\, \frac{V}{m}$$

- 

$\ell = \frac{\lambda}{10}$ $\quad r = 100\lambda$

$A = 10^{-3}\lambda^2$ $\quad |I_0| = 1A$

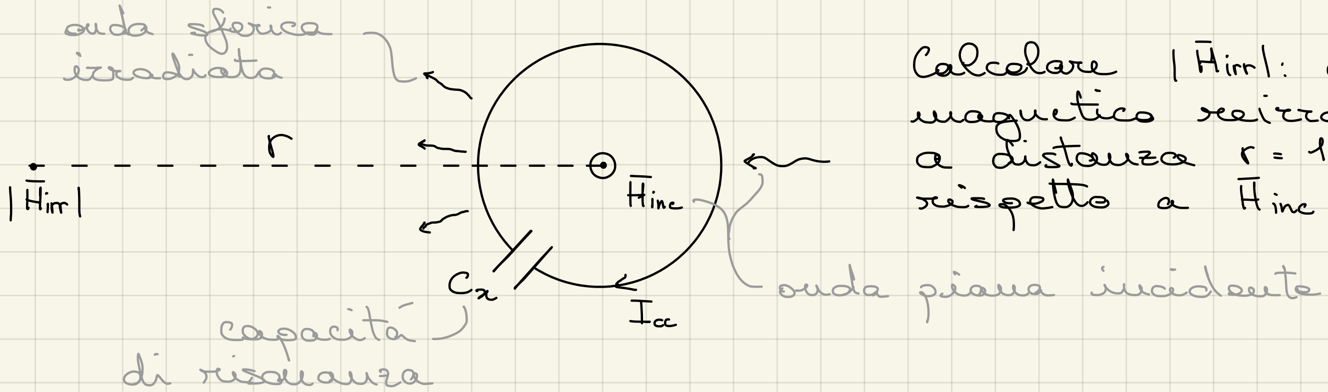
$\theta = \frac{\pi}{2}$ $\quad V_o = ?$

$$V_o = \overbrace{j\omega\mu_0 A}^{A_e} \boxed{H_\perp} = j\omega\mu_0 A \boxed{\frac{E_{inc}}{\eta_0}}$$

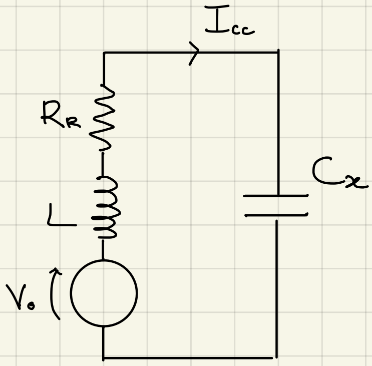
$$|E_{inc}| = \frac{\omega\mu|I d\ell|}{4\pi r} = 2\pi f \mu_0 \frac{\lambda |I_0|}{10} \frac{1}{4\pi\, 100\,\lambda}$$

$$|V_0| = 2\pi f \mu_0 10^{-3} \lambda^2 \frac{2\pi f \mu_0 |I_0|}{1000 \cdot 4\pi} \qquad \text{con} \quad \lambda = \frac{c}{f}$$

$$= 1{,}18 \cdot 10^{-3}\ \text{V}$$

- 

Calcolare $|\bar{H}_{irr}|$: campo magnetico reirradiato a distanza $r = 100$m, rispetto a $\bar{H}_{inc}$.



$$\boxed{j\omega L = -\frac{1}{j\omega C_x}} \longrightarrow |I_{cc}| = \frac{|V_0|}{R_R}$$

$$|I_{cc}| = \frac{|j\beta A \cdot E_{inc}|}{R_R} = \frac{\beta A |H_{inc}| \eta_0}{R_R}$$

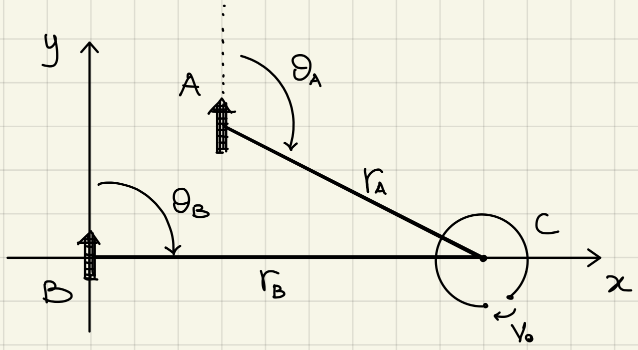
$$|H_{irr}| = \left| \frac{j\beta}{\eta_0} \frac{j\omega\mu I_{cc} A}{4\pi r} e^{-j\beta r} \operatorname{sen}\theta \right| \qquad \text{con} \quad \theta = \frac{\pi}{2}$$

$$= \frac{\beta}{\eta_0} \frac{\omega\mu A}{4\pi\, 100\,\lambda} \cdot \frac{\beta A \eta_0}{R_R} |H_{inc}|$$

dove $R_R = \eta_0 \frac{8}{3}\pi^2 \left(\frac{A}{\lambda^2}\right)^2$, $\omega = 2\pi f = \frac{2\pi c}{\lambda}$, $\beta = \frac{2\pi}{\lambda}$

$$\Longrightarrow \quad |H_{irr}| = 3 \frac{(2\pi)^3}{\lambda^3} \frac{1}{8\pi^3} \frac{\lambda^4 |H_{inc}|}{4\pi\, 100\,\lambda} = 2{,}4 \cdot 10^{-3} |H_{inc}|$$

$$\frac{|H_{irr}|}{|H_{inc}|} = 2{,}4 \cdot 10^{-3} = -52{,}4\,\text{dB}$$

- 

B(0,0) A(5,5) C(15,0)

$f = 500$MHz $\quad P_T = 1$W $\quad \ell = \frac{\lambda}{10}$

$a = 3$cm $\quad I_A$ e I_B in fase

Calcolare $|V_0|$.

$\theta_B = \frac{\pi}{2}$ $\quad$ $\bar{H}_A$ e $\bar{H}_B$ sono // tra loro e $\perp$ alla spira

$P_T = \frac{\pi}{3} \eta_0 |I|^2 \left(\frac{\ell}{\lambda}\right)^2 = 1\,W \longrightarrow |I_A| = |I_B| = 0{,}503\,A$

$\lambda = 0{,}6\,m \qquad \ell = \frac{\lambda}{10} = 0{,}06\,m$

$|\bar{H}_A| = \left| \frac{j\omega\mu I_A \ell}{4\pi r_A \eta_0} \operatorname{sen}\theta_A \right|$ con $\theta_A = \frac{\pi}{2} + \operatorname{arctg}\frac{5}{10}$

$\hookrightarrow \operatorname{sen}\theta_A = 0{,}894$

$r_A = \sqrt{10^2+5^2} = 11{,}18034\,m$

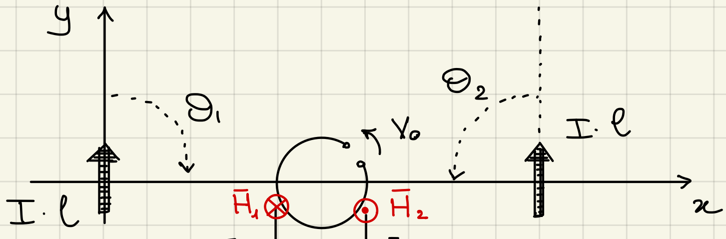
$\Longrightarrow H_A = 0{,}002012\,\frac{A}{m} \qquad H_B = \left| \frac{j\omega\mu I_B \ell}{4\pi r_B \eta_0} \right| = 0{,}001677\,\frac{A}{m}$

$r_B = 15m = 25\lambda \longrightarrow \bar{H}_B$ reale positivo

$r_A = \cancel{18}\lambda + 0{,}634\lambda \longrightarrow \bar{H}_A$ complesso

$H_{tot} = |\bar{H}_A + \bar{H}_B| = |H_A e^{-j\beta 0{,}3804} + H_B| = 1{,}537 \cdot 10^{-3}\,\frac{A}{m}$

$|V_0| = |j\omega\mu A_e H_{tot}| = 0{,}171\,V$ $\quad$ con $A = \pi a^2$



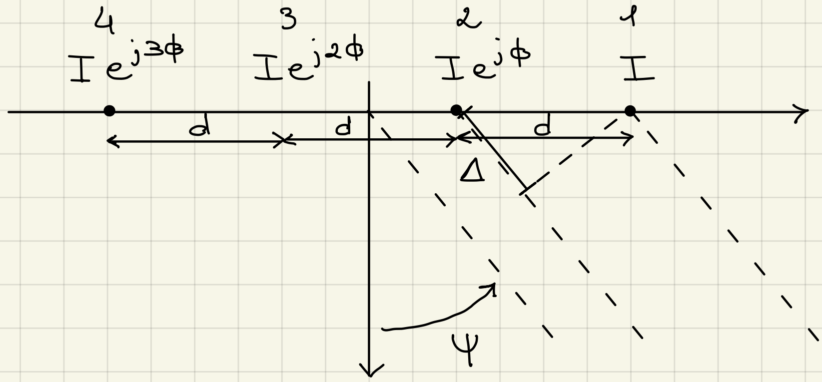
Calcolare $|V_0|$.

$E_{tot} = E_1 + E_2 = 2E_1$

ma

$H_{tot} = H_1 - H_2 = 0$

$\Longrightarrow |V_0| = 0$



$\phi = 20°$

$d = \frac{\lambda}{2}$

a) Determinare ψ_{max} di massima radiazione.

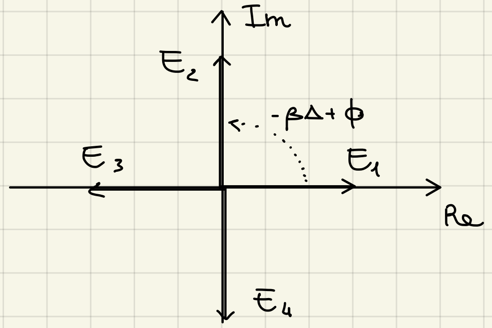
Affinché i vari dipoli si sommino in fase devo imporre:

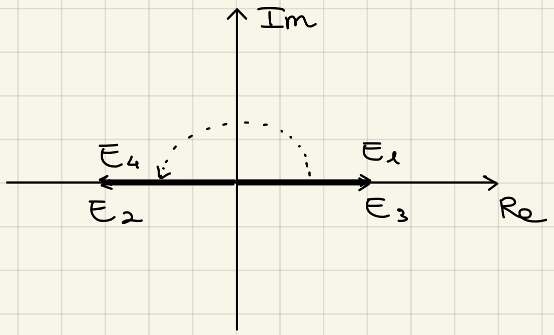
$e^{-j\beta\Delta}$. $e^{j\phi}$

ritardo di propagazione = anticipo di alimentazione

$$\beta\Delta = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\,\text{sen}\,\psi_{MAX} = \phi \Longrightarrow \psi_{MAX} = 6{,}4°$$

b) Determinare ψ_0 di radiazione nulla

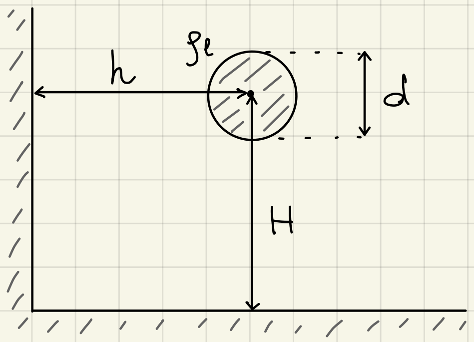




Entrambe le soluzioni sono valide (l'importante è che lo sfasamento fra dipoli limitrofi sia lo stesso)

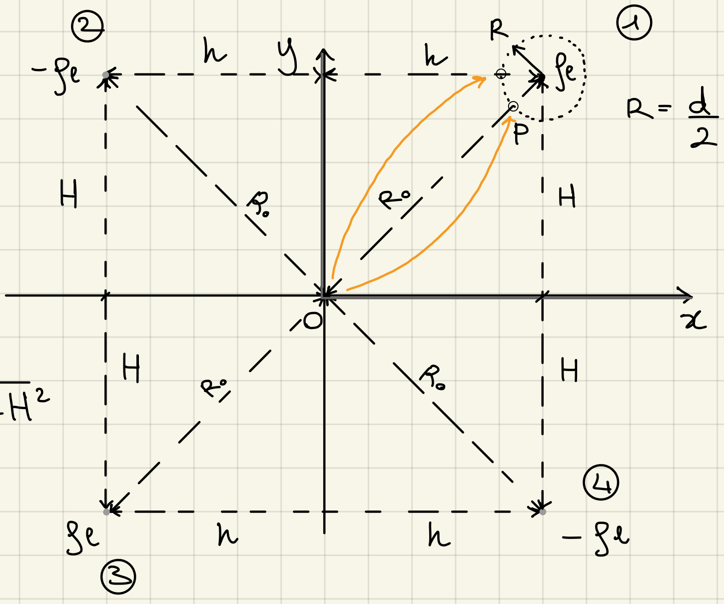
Differenze di fase tra elementi limitrofi deve essere ±k90° (k = 1, 2)

$$K = +1: \quad -\beta\frac{\lambda}{2}\,\text{sen}\,\psi_0 + \phi = 90° \Longrightarrow \psi_0 = -22{,}9°$$

- 

$h = 5\text{ cm} = H \qquad d = 1\text{ cm}$

Calcolare Z_c (approx conduttori sottili)



$V(P) = -\frac{\rho_l}{2\pi\varepsilon_0}\Big[\ln\left(\frac{R}{R_0}\right)$ ① $-$

② $-\ln\left(\frac{2h-R}{R_0}\right) +$

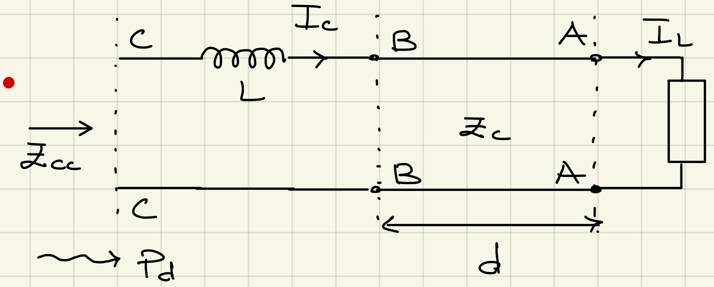
$+\ln\left(\frac{2R_0-R}{R_0}\right)$ ③

④ $-\ln\left(\frac{2H-R}{R_0}\right)\Big]$

$R_0 = \sqrt{h^2+H^2}$

$$V(P) = -\frac{f\ell}{2\pi\varepsilon_0}\ln\left[\frac{R}{\cancel{R_0}}\cdot\frac{\cancel{R_0}}{2h-R}\cdot\frac{2R_0-R}{\cancel{R_0}}\cdot\frac{\cancel{R_0}}{2H-R}\right]$$

$$\mathcal{C} = \frac{f\ell}{V(P)} = 21{,}54\ \frac{pF}{m} \qquad Z_c = \frac{\sqrt{\mu_0\varepsilon_0}}{\mathcal{C}} = 154{,}38\ \Omega$$



$Z_L = (20 - j30)\ \Omega$

$Z_c = 50\ \Omega$ $\qquad R_g = 50\ \Omega$

$P_d = 1W$ $\qquad f = 100\ MHz$

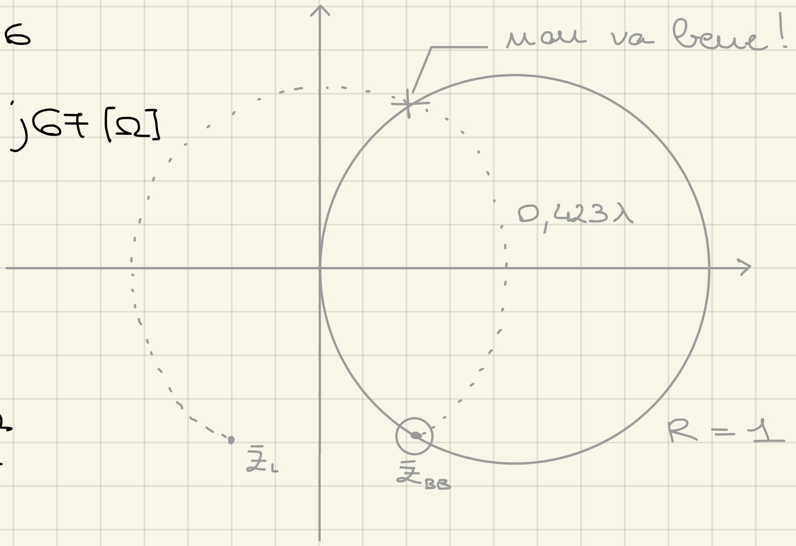
Determinare L e d per avere adattamento in $\bar{C}\bar{C}$
Determinare $|V_{AA}|$ e $|V_{BB}|$, $|I_L|$ e $|I_c|$

Collegamento serie → usiamo le impedenze

$$\bar{Z}_L = \frac{20 - j30}{50} = 0{,}4 - j\,0{,}6$$

$$\bar{Z}_{BB} = 1 - j\,1{,}34 \qquad Z_{BB} = 50 - j67\ [\Omega]$$

deve essere negativa per permettere all'induttanza di compensarla



$$\rightarrow \quad L = 1{,}07\cdot10^{-7}\ H = \frac{67\Omega}{2\pi f}$$

$$\rightarrow \quad d = 0{,}423\,\lambda$$

$$P_d = 1W = \frac{1}{2}|V_{BB}|^2\,\mathrm{Re}\left\{\frac{1}{Z_{BB}}\right\} = \frac{1}{2}|V_{AA}|^2\,\mathrm{Re}\left\{\frac{1}{Z_L}\right\}$$

$$\longrightarrow \quad |V_{BB}| = 16{,}7\ V, \qquad |V_{AA}| = 11{,}4\ V$$

$$P_d = \frac{1}{2}|I_L|^2\,\mathrm{Re}\{Z_L\} = 1W \longrightarrow |I_L| = 0{,}316\ A$$